QUESTION PAPER
MATHEMATICS

Time Allowed : 3.00 Hours **Maximum Marks : 80**

General Instructions :

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub-parts.

Section A

(Multiple Choice Questions) Each question carries 1 mark

1. From the set $\{1, 2, 3, 4, 5\}$, two numbers a and b$(a \neq b)$ are chosen at random. The probability that $\frac{a}{b}$ is an integer, is :

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{3}{5}$

2. If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$, then the value of x is :

(a) 3 (b) 0 (c) -1 (d) 1

3. The graph of the inequality $2x + 3y > 6$ is :

(a) half plane that contains the origin

(b) half plane that neither contains the origin nor the points of the line $2x + 3y = 6$

(c) whole XOY-plane excluding the points on the line $2x + 3y = 6$

(d) entire XOY plane

4. If A is a square matrix of order 3, such that $A(\text{adj } A) = 10\, I$, then $|\text{adj } A|$ is equal to :

(a) 1 (b) 10 (c) 100 (d) 101

5. If $\vec{a} = 3\hat{i} + 2\hat{j} + 5\hat{k}$ and $\vec{b} = 6\hat{i} - \hat{j} - 5\hat{k}$ then $\left(\vec{a} + \vec{b}\right) . \left(\vec{a} - \vec{b}\right)$ is :

(a) 24 (b) -24 (c) 18 (d) 10

6. The two lines x = ay +b, z = cy + d; and x = a'y+b', z=c'y + d' are perpendicular to each other, if :

(a) $\frac{a}{a'}+\frac{c}{c'}=1$ (b) $\frac{a}{a'}+\frac{c}{c'}=-1$ (c) aa' + cc' = 1 (d) aa' + cc'= -1

7. If $\vec{a}$ is a non zero vector, then $\left(\vec{a}\cdot\hat{i}\right)\hat{i}+\left(\vec{a}\cdot\hat{j}\right)\hat{j}+\left(\vec{a}\cdot\hat{k}\right)\hat{k}$ is equals to :

(a) $\vec{a}$ (b) $2\vec{a}$ (c) $3\vec{a}$ (d) $\vec{0}$

8. The direction ratios of the line passing through two points (2, - 4, 5) and (0, 1, -1) is :

(a) (-2,-6,5) (b) (-2,5-6) (c) (5, - 2,-6) (d) (-6, -2,5)

9. If A is a 3 × 3 matrix such that | A | = 8, then |3A| is equals to :

(a) 8 (b) 24 (c) 72 (d) 216

10. $\int\frac{2^{x+1}-5^{x-1}}{10^x}dx$ is equal to :

$(a)\frac{1}{5}\log 2\left(2^{-x}\right)-2\log 5\left(5^{-x}\right)+C$ $(b)\frac{1}{6}\log\left(2^{-x}\right)-2\log 5\left(5^{-x}\right)+C$

$(c)\frac{1}{5}\log\left(2^{-x}\right)+3\log 5\left(5^{-x}\right)+C$ (d) None of the above

11. The integrating factor of the differential equation $x\frac{dy}{dx}+2y=x^2$ is :

(a) x $(b)\,x^2$ (c) 3x (d) xy

12. If $y=\cos^{-1}x$, then $(1-x^2)y_2$ is equal to :

(a) xy (b) xy_1 (c) xy_2 (d) x^2y

13. In an LPP if the objective function Z = ax + by has the same maximum value on two corner points of the feasible region, then the number of points at which Z_{max} occurs is

(a) 0 (b) 2 (c) finite (d) infinite

14. The number of points of discontinuity of f defined by f(x) = |x| - |x + 1| is :

(a) 1 (b) 2 (c) 0 (d) 5

15. The degree of the differential equation $1+\left(\frac{dy}{dx}\right)^2=x$ is :

(a) 1 (b) 2 (c) 3 (d) 4

16. If $\begin{vmatrix}2 & 2\\ 2 & 3\end{vmatrix}=\begin{vmatrix}3x & 1\\ 4x & 2\end{vmatrix}$, then x is equals to :

(a) 1 (b) 2 (c) 3 (d) 4

17. If $\vec{a}\cdot\vec{b}=\frac{1}{2}|\vec{a}||\vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is :

(a) 0° (b) 30° (c) 60° (d)90°

18. $\int_0^{\pi/8} \tan^2(2x)dx$ is equal to :

$(a)\frac{4-\pi}{8}$ $(b)\frac{4+\pi}{8}$ (c) $\frac{4-\pi}{4}$ $(d)\frac{4-\pi}{2}$

Direction : (Q.19 to Q.20) : In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

19. **Assertion (A) :** We can write $\sin^{-1} x = (\sin x)^{-1}$

Reason (R) : Any value in the range of principal value branch is called principal value of that inverse trigonometric function.

20. **Assertion (A) :** A 2×2 matrix $A = [a_{ij}]$, whose elements are given by $a_{ij} = i \times j$, is $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$.

Reason (R) : If A is a 4×2 matrix, then the number of elements in matrix A are 5.

Section B

(This section comprises of very short answer type questions (VSA) of 2 marks each)

21. Check if the relation R on the set

$A = \{1,2,3,4,5,6\}$ defined as $R = \{(x, y): y \text{ is divisible by } x\}$ is

(i) symmetric (ii) transitive.

OR

Find the value of

$$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$$

22. If $x = a\cos\theta$ and $y = b\sin\theta$, then find $\frac{d^2y}{dx^2}$.

23. Find $|\vec{a}|$ and $|\vec{b}|$, if $|\vec{a}| = 2|\vec{b}|$ and $(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 12$.

OR

Find the unit vector perpendicular to each of the vectors $\vec{a} = 4\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$.

24. Show that the function f defined by $f(x) = (x - 1)e^x + 1$ is an increasing function for all $x>0$.

25. If $\vec{a} = \hat{i} + \hat{j} + 3\hat{k}$ *and* $\vec{b} = 2\hat{i} - \hat{j} + \lambda\hat{k}$, then find the value of λ, if the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are orthogonal.

Section C

(This section comprises of short answer type questions (SA) of 3 marks each)

26. Evaluate : $\int \frac{x}{x^2+3x+2}dx$

27. Evaluate : $\int_0^1 x(1-x)^n dx.$

OR

Evaluate $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx.$

28. Find the particular solution of : $(x+1)\frac{dy}{dx} = 2e^{-y}+1; y=0$ when $x=0.$

OR

Find the particular solution of : $x\sin\left(\frac{y}{x}\right)\frac{dy}{dx}+x-y\sin\left(\frac{y}{x}\right)=0; y=\frac{\pi}{2}$ when $x=1.$

29. Three rotten apples are mixed with seven fresh apples. Find the probability distribution of the number of rotten apples, if three apples are drawn one by one with replacement. Find the mean of the number of rotten apples.

OR

In a shop X, 30 tins of ghee of type A and 40 tins of ghee of type B which look alike, are kept for sale. While in shop Y, similar 50 tins of ghee of type A and 60 tins of ghee of type B are there. One tin of ghee is purchased from one of the randomly selected shop and is found to be of type B. Find the probability that it is purchased from shop Y.

30. Evaluate $\int_1^2 \left[\frac{1}{x}-\frac{1}{2x^2}\right]e^{2x}dx.$

31. If $Z = 2x + 3y$, subject to constraints $x + 2y \leq 10$, $2x + y \leq 14$, $x, y \geq 0$, then find the corner point of feasible region.

Section D

(This section comprises of long answer type questions (LA) of 5 marks each)

32. If $A=\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$ then find A^{-1}.

Using A^{-1}, solve the following system of equations:

$2x-3y+5z=11$

$3x+2y-4z=-5$

$x+y-2z=-3$

33. Find the vector and certesian equations of the line which is perpendicular to the lines with equations $\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4}$ and $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and passes through the point (1, 1, 1). Also, find the angle between the given lines.

OR

Find the shortest distance between the lines given by

$\vec{r}=(2+\lambda)\hat{i}-(3+\lambda)\hat{j}+(5+\lambda)\hat{k}$ and $\vec{r}=(2\mu-1)\hat{i}+(4\mu-1)\hat{j}+(5-3\mu)\hat{k}$.

34. Prove that the relation R on Z, defined by R {(x, y);(x - y) is divisible by 5} is an equivalence relation.

OR

Show that the relation R in the set A of points in a plane, given by R = {{P,Q): distance of the point P from the origin is same as the distance of the point Q from the origin}, is an equivalence reation. Further, show that the set of all points related to a point $P \neq (0,0)$ is the circle passing through P with origin as centre.

35. Find the area of the region lying in the first quadrant and enclosed by the X -axis, the line y = x and the circle $x^2 + y^2 = 32$,

Section E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1,1, 2 respectively. The third case study question has two sub-parts of 2 marks each)

36. $P(x) = -6x^2 + 120x + 25000$ (in Rs.) is the total profit function of a company where x denotes the production of the company.

Based on the above information, answer the following questions.

(i) Find the profit of the company, when the production is 3 units.

(ii) Find P'(5).

(iii) Find the interval in which the profit is strictly increasing.

OR

Find the production, when the profit is maximum.

37. In a college, an architecture design a auditorium for its cultural activities purpose. The shape of the floor of the auditorium is rectangular and it has a fixed perimeter, say P.

Based on the above information, answer the following questions.

(i) If l and b represents the length and breadth of the rectangular region, then find the 'relationship between l, b, P.

(ii) Find the area (A) of the floor, as a function of l.

(iii) College manager is interested in maximising the area of the floor A.

For this purpose, find the value of l.

OR

Find the maximum area of the floor.

38. In an office three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay process 50% of the forms. Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 0.06, Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03.

Based on the above information answer the following questions.

(i) The total probability of committing an error in processing the form.

(ii) The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is not processed by Vinay.