ANSWER KEY

Test # 01 **Test Pattern : BOARD**

Class-XII

Mathematics

1. **(c)** We have, set of numbers $\{1, 2, 3, 4, 5\}$.

Sample space of choosing two numbers = $^5C_2$

$$= \frac{5\times4}{1\times2} = 10$$

Favourable outcomes are $\left(\frac{2}{1}, \frac{3}{1}, \frac{4}{1}, \frac{5}{1}, \frac{4}{2}\right) = 5$

$\therefore$ Required probability $= \frac{5}{10} = \frac{1}{2}$

2. (c) We have, $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$

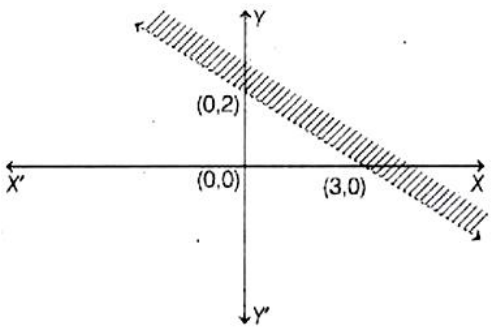
$\Rightarrow 2(x-9x)-3(x-4x) + 2(9x-4x) + 3 = 0$

$\Rightarrow -16x + 9x + 10x + 3 = 0$

$\Rightarrow 3x + 3 = 0$

$\Rightarrow x = -1$

3. (b) The inequality $2x + 3y > 6$ represent half plane that neither contains the origin'nor the points of the line $2x + 3y = 6$



4. (c) We know that

A (adj A) = |A|I Now, we have

A (adj A) = 10I

$\therefore |A| = 10$

Again, $|\text{adj } A| = |A|^{n-1}$

$\therefore |\text{adj } A| = |A|^{3-1} = |A|^2 = (10)^2 = 100$

5. (b) Given $\vec{a} = 3\hat{i} + 2\hat{j} + 5\hat{k}$

and $\vec{b} = 6\hat{i} - \hat{j} - 5\hat{k}$

Now, $\vec{a} + \vec{b} = 9\hat{i} + \hat{j}$

and $\vec{a} - \vec{b} = -3\hat{i} + 3\hat{j} + 10\hat{k}$

$\therefore \left(\vec{a} + \vec{b}\right)\cdot\left(\vec{a} - \vec{b}\right) = -27 + 3 = -24$

6. (d) We have,

$x = ay + b,\ z = cy + d$

and $x = a'y + b',\ z = c'y + d'$

$$\Rightarrow \frac{x-b}{a} = \frac{y}{1} = \frac{z-d}{c} \text{ and } \frac{x-b'}{a'} = \frac{y}{1} = \frac{z-d'}{c'}$$

Since, these lines are perpendicular.

$\therefore aa' + 1 + cc' = 0$

[$\therefore$ two lines are perpendicular, if $a_1 a_2 + b_1 b_2 + c_1 c_2 = 1$]

$\Rightarrow aa' + cc' = -1$

7. (a) Let $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$

$\therefore (\vec{a}\cdot\hat{i}) = (x\hat{i} + y\hat{j} + z\hat{k})\cdot\hat{i} = x$

Similarly, $(\vec{a}\cdot\hat{j}) = y$ *and* $(\vec{a}\cdot\hat{k}) = z$

Now, $(\vec{a}\cdot\hat{i})\hat{i} + (\vec{a}\cdot\hat{j})\hat{j} + (\vec{a}\cdot\hat{k})\hat{k} = x\hat{i} + y\hat{j} + z\hat{k} = \vec{a}$

8. (b) Let $A(x_1, y_1, z_1) = (2, -4, 5)$

and $B(x_2, y_2, z_2) = (0, 1, -1)$

Then, DR's of line AB is $(0-2, 1+4, -1-5)$

i.e, $(-2, 5, -6)$.

9. (d) $|KA| = K^n|A|$

$\therefore |3A| = 3^3|A|$ [$\therefore n = 3$]

$= 27 \times 8 = 216$

10. **(a)** Let $I = \int \frac{2^{x+1} - 5^{x-1}}{10^x} dx$

$$\Rightarrow I = \int \left(2\left(\frac{2}{10}\right)^x - \frac{1}{5}\left(\frac{5}{10}\right)^x \right) dx$$

$$\Rightarrow I = \int (2(5)^{-x} - \frac{1}{5}(2)^{-x}) dx$$

$$\Rightarrow I = -2 \cdot 5^{-x} \log 5 + \frac{1}{5} \cdot 2^{-x} \log 2 + C$$

$$\Rightarrow I = \frac{1}{5} \log 2 \left(2^{-x}\right) - 2 \log 5 \left(5^{-x}\right) + C$$

11. **(b)** We have

$$x\frac{dy}{dx} + 2y = x^2 \Rightarrow \frac{dy}{dx} + \frac{2}{x} y = x$$

$$\therefore \text{Integrating factor} = e^{\int \frac{2}{x} dx} = e^{2\log x} = e^{\log x^2} = x^2$$

12. (b) We have, $y = \cos^{-1} x$

$$\Rightarrow y_1 = \frac{-1}{\sqrt{1 - x^2}} \quad \ldots\text{(i)}$$

$$\Rightarrow y_2 = \frac{(\sqrt{1 - x^2}) \times 0 - (-1)\frac{1}{2\sqrt{1 - x^2}}(-2x)}{(1 - x^2)}$$

$$\Rightarrow y_2(1 - x^2) = \frac{-x}{\sqrt{1 - x^2}} \Rightarrow y_2(1 - x^2) = xy_1$$

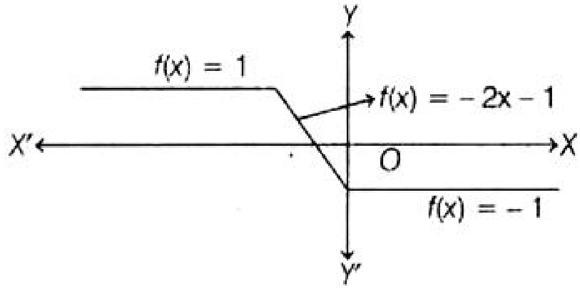
[using eq. (i)]

13. (d) An objective function has the same maximum value of two comer points the maximum value occur line joining two points.

$\therefore$ **Infinite maximum value at line joining two points.**

14. (c) We have, $f(x) = |x| - |x + 1|$

$$f(x) = \begin{cases} 1, & x < -1 \\ -2x - 1, & -1 \le x < 0 \\ -1, & x \ge 0 \end{cases}$$



Clearly, f(x) is continuous for all values of x. Hence, no discontinuous point exist.

15. (b) We have, $1 + \left(\frac{dy}{dx}\right)^2 = x$

$\therefore$ Degree = 2

16. (a) We have, $\begin{vmatrix} 2 & 2 \\ 2 & 3 \end{vmatrix} = \begin{vmatrix} 3x & 1 \\ 4x & 2 \end{vmatrix}$

$\therefore$ 6 – 4 = 6x - 4x

$\therefore$ 2 = 2x

$\therefore$ x = 1

17. (c) Given $\vec{a} \cdot \vec{b} = \frac{1}{2}|\vec{a}||\vec{b}|$

$$\Rightarrow |\vec{a}||\vec{b}|\cos\theta = \frac{1}{2}|\vec{a}||\vec{b}|$$

$$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

$\therefore$ *Angle between $\vec{a}$ and $\vec{b}$ is* 60°.

18. (a) Let $I = \int_0^{\frac{\pi}{8}} \tan^2(2x) dx$

$$\Rightarrow I = \int_0^{\frac{\pi}{8}} \left(\sec^2(2x) - 1\right) dx$$

$$\Rightarrow I = \left[\frac{\tan 2x}{2} - x\right]_0^{\frac{\pi}{8}}$$

$$\Rightarrow I = \left(\frac{\frac{\tan\pi}{4}}{2} - \frac{\pi}{8}\right) - (0-0) = \frac{1}{2} - \frac{\pi}{8} = \frac{4-\pi}{8}$$

19. (d) **Assertion** $\sin^{-1}x$ should not be confused with $(\sin x)^{-1}$. In fact $(\sin x)^{-1} = \frac{1}{\sin x}$ and similarly for other trigonometric functions.

The value of an inverse trigonometric function which lies in the range of principle branch, is called the principal value of that inverse trigonometric function. Hence, we can say that Assertion is false but Reason is true.

20. (c) **Assertion** In general, the matrix A of order 2×2 is given by $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$.

Now, $a_{ij} = i \times j$, i = 1, 2 and j = 1, 2

$\therefore a_{11} = 1, a_{12} = 2, a_{21} = 2$ and $a_{22} = 4$

Thus, matrix A is $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

If A is a 4×2 matrix, then A has $4 \times 2 = 8$ elements.

Hence, Assertion is true but Reason is false.

21. (i) **For symmetry** We observe that 6 is divisible by 2 . This means that $(2, 6) \in R$ but 2 is not divisible by 6 i.e. $(6,2) \notin R$. So, R is not symmetric. (1)

(ii) **For transitivity** Let $(x, y) \in R$ and $(y, z) \in R$. then z is divisible by x. i.e., $(x, z) \in R$

e.g. 2 is divisible by 1,4 is divisible by 2.

So, 4 is divisible by 1. So, R is transitive. (1)

OR

We know that ranges of principle values of $\tan^{-1}$, $\cos^{-1}$ and $\sin^{-1}$ are $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, $[0, \pi]$ and $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$, respectively.

Let $\tan^{-1}(1) = \theta_1$,

$\Rightarrow \tan\theta_1 = 1$

$\Rightarrow \tan\theta_1 = \tan\frac{\pi}{4} \Rightarrow \theta_1 = \frac{\pi}{4} \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ (1/2)

Again, let $\cos^{-1}\left(\frac{-1}{2}\right) = \theta_2 \Rightarrow \cos\theta_2 = \frac{-1}{2}$

$\Rightarrow \cos\theta_2 = -\cos\frac{\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\frac{2\pi}{3}$

$\Rightarrow \theta_2 = \frac{2\pi}{3} \in [0, \pi]$ (1/2)

Again, let $\sin^{-1}\left(\frac{-1}{2}\right) = \theta_3$

$\Rightarrow \sin\theta_3 = \frac{-1}{2} \Rightarrow \sin\theta_3 = -\sin\frac{\pi}{6}$

$\Rightarrow \sin\theta_3 = \sin\left(\frac{-\pi}{6}\right)$

$\Rightarrow \theta_3 = \frac{-\pi}{6} \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (1/2)

$\therefore \tan^{-1}(1) + \cos^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right)$

$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6} = \frac{3\pi}{4}$ (1/2)

22. We have $x = a\cos\theta$ and $y = b\sin\theta$

$\therefore \frac{dx}{d\theta} = -a\sin\theta$ and $\frac{dy}{d\theta} = b\cos\theta$

$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{b\cos\theta}{-a\sin\theta} = \frac{-b}{a}\cot\theta$ (1)

Again, $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

$= \frac{d}{dx}\left(\frac{-b}{a}\cot\theta\right) = \frac{-b}{a}\left(-\text{cosec}^2\theta\right).\frac{d\theta}{dx}$

$= \frac{b}{a}\text{cosec}^2\theta \times \frac{1}{(-a\sin\theta)} \quad \left[\because \frac{d\theta}{dx} = \frac{1}{\frac{dx}{d\theta}}\right]$

$= \frac{-b}{a^2}\text{cosec}^3\theta$ (1)

23. Given, $\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{a}-\vec{b}\right) = 12$ and $|\vec{a}| = 2|\vec{b}|$

$\Rightarrow |\vec{a}|^2 - |\vec{b}|^2 = 12$

$\Rightarrow \left(2|\vec{b}|\right)^2 - |\vec{b}|^2 = 12 \left[\text{given, } |\vec{a}| = 2|\vec{b}|\right]$

$\Rightarrow 4|\vec{b}|^2 - |\vec{b}|^2 = 12$ (1)

$\Rightarrow 3|\vec{b}|^2 = 12$

$\Rightarrow |\vec{b}|^2 = 4$

$\Rightarrow |\vec{b}| = 2$

$|\vec{a}| = 2|\vec{b}| = 2(2) = 4$ (1)

OR

Given, vectors are $\vec{a} = 4\hat{i}+3\hat{j}+\hat{k}$ and $\vec{b} = 2\hat{i}-\hat{j}+2\hat{k}$

Now, perpendicular vector to the given vector

is $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 3 & 1 \\ 2 & -1 & 2 \end{vmatrix}$

$= \hat{i}(6+1) - \hat{j}(8-2) + \hat{k}(-4-6)$

$= 7\hat{i} - 6\hat{j} - 10\hat{k}$ (1/2)

$|\vec{a}\times\vec{b}| = \sqrt{7^2 + (-6)^2 + (-10)^2}$

$= \sqrt{49+36+100} = \sqrt{185}$ (1/2)

$\therefore$ Required unit vector $= \pm\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$

$= \pm\frac{(7\hat{i}-6\hat{j}-10\hat{k})}{\sqrt{185}}$ (1)

24. We have, $f(x) = (x - 1)e^x + 1$

On differentiating w.r.t. x, we get

$f'(x) = (x-1)e^x + e^x$

$f'(x) = xe^x$ (1)

For all $x > 0$ $\Rightarrow f'(x) > 0$ (1)

$\therefore$ f(x) is an increasing function for all $x > 0$. (1)

25. Given, $\vec{a} = \hat{i}+\hat{j}+3\hat{k}$ and $\vec{b} = 2\hat{i}-\hat{j}+\lambda\hat{k}$

$\therefore \vec{a}+\vec{b} = 3\hat{i} + (\lambda+3)\hat{k}$

$\vec{a}-\vec{b} = -\hat{i} + 2\hat{j} + (3-\lambda)\hat{k}$

According to the question,

$(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = 0$

$\Rightarrow (3\hat{i} + (\lambda+3)\hat{k})\cdot(-\hat{i}+2\hat{j}+(3-\lambda)\hat{k}) = 0$

[$\therefore$ if two vectors $\vec{a}$ and $\vec{b}$ are orthogonal, then $\vec{a}\cdot\vec{b} = 0$]

$\Rightarrow -3 + (3+\lambda)(3-\lambda) = 0$

$\Rightarrow -3 + 9 - \lambda^2 = 0$

$\Rightarrow 6 = \lambda^2$

$\Rightarrow \lambda = \sqrt{6}$ (i)

26. Let $I = \int \frac{x}{x^2+3x+2}dx$

Again let $x = A\frac{d}{dx}\left(x^2+3x+2\right) + B$

$\Rightarrow x = (2x+3)A + B$

$\Rightarrow\ x = 2Ax + (3A + B)$

$\therefore\ 2A = 1$ and $3A + 8 = 0$

$\Rightarrow\ A = \frac{1}{2}$ and $B = -\frac{3}{2}$

$\therefore I = \frac{1}{2}\int \frac{(2x+3)\,dx}{x^2+3x+2} - \frac{3}{2}\int \frac{dx}{x^2+3x+2}$

$\Rightarrow I = \frac{1}{2}I_1 - \frac{3}{2}I_2 \ldots (i)$ (1)

Let $I_1 = \int \frac{2x+3}{x^2+3x+2}dx$ and $I_2 = \int \frac{dx}{x^2+Sx+2}$

Now $I_1 = \int \frac{2x+3}{x^2+3x+2}dx$

Put $x^2+3x+2 = t \Rightarrow (2x+3)dx = dt$

$\therefore I_1 = \int \frac{dx}{t} = \log|t| + C_1 = \log|x^2+3x+2| + C_1$

(1/2)

and $I_2 = \int \frac{dx}{x^2+3x+2} = \int \frac{dx}{x^2+3\times+\frac{9}{4}+2-\frac{9}{4}}$

$= \int \frac{dx}{\left(x+\frac{3}{2}\right)^2+2-\frac{9}{4}} = \int \frac{ox}{\left(X+\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}$

$= \frac{1}{2\times\frac{1}{2}}\log\left|\frac{x+\frac{3}{2}-\frac{1}{2}}{x+\frac{3}{2}+\frac{1}{2}}\right| + C_2$

$\Rightarrow\ I_2 = \log\left|\frac{x+1}{x+2}\right| + C_2$ (1/2)

On substituting the value of I_1 and I_2 in Eq. (i), we get

$I = \frac{1}{2}\log|x^2+3x+2| + \frac{1}{2}C_1 - \frac{3}{2}\log\left|\frac{x+1}{x+2}\right| - \frac{3}{2}C_2$

$= \frac{1}{2}\log|x^2+3x+2| - \frac{3}{2}\log\left|\frac{x+1}{x+2}\right| + C,$

where $C = \frac{1}{2}C_1 - \frac{3}{2}C_2$ (1)

27. Let $I = \int_0^1 x(1-x)^n dx$

$I = \int_0^1 (1-x)\{1-(1-x)\}^n dx$

$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$

$= \int_0^1 (1-x)x^n dx = \int_0^1 (x^n - x^{n+1})dx$ (1)

$= \left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2}\right]_0^1 = \left[\frac{1}{n+1} - \frac{1}{n+2}\right] - 0$

$= \frac{(n+2)-(n+1)}{(n+1)(n+2)} = \frac{1}{(n+1)(n+2)}$ (2)

OR

$Let\ I = \int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx \ \ldots(i)$

$I = \int_0^{\pi} \frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}dx = \int_0^{\pi} \frac{(\pi-x)\sin x dx}{1+\cos^2 x} \ \ldots(ii)$

On adding Eqs. (i) and (ii), we get

$2I = \pi\int_0^{\pi} \frac{\sin x dx}{1+\cos^2 x} \Rightarrow I = \frac{\pi}{2}\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}dx$ (1)

Using $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx, if\ f(2a-x) = f(x),$

we get

$$I=\frac{\pi}{2}\times 2\int_0^{\frac{\pi}{2}}\frac{\sin x}{1+\cos^2 x}dx \Rightarrow I=\pi\int_0^{\frac{\pi}{2}}\frac{\sin x}{1+\cos^2 x}dx$$

Put cosx = t, then - sinxdx = dt

When x = 0, then t = 1 and when x = $\frac{\pi}{2}$,

then t = 0 (1)

$$\therefore\ I=\pi\int_1^0\frac{-dt}{1+t^2}$$

$$I=-\pi[\tan^{-1}t]_1^0$$

$$I=-\pi[\tan^{-1}0-\tan^{-1}1]$$

$$I=-\pi\left[0-\frac{\pi}{4}\right]=\frac{\pi^2}{4}\ (1)$$

28. Given, differential equation is

$$(x+1)\frac{dy}{dx}=2e^{-y}+1.$$

$$\Rightarrow\ (x+1)\frac{dy}{dx}=\frac{2+e^y}{e^y}\Rightarrow\ \frac{e^y}{e^y+2}dy=\frac{dx}{x+1}$$

On integrating both sides, we get

$$\int\frac{e^y}{e^y+2}dy=\int\frac{dx}{x+1}$$

$$\Rightarrow\ \log\left(e^y+2\right)=\log(x+1)+\log C$$

$$\Rightarrow\ \log\left(e^y+2\right)=\log C(x+1)\ (1)$$

$$\Rightarrow\ e^y+2=C(x+1)\ ..(i)$$

Also given, $y=0$, when $x=0$

On putting $x=0$ and $y=0$ in Eq. (i), we get

$$e^0+2=C(0+1)\Rightarrow C=1+2=3\ (1)$$

On putting value of C in Eq. (1), we get

$$e^y+2=3(x+1)$$

$$\Rightarrow\ e^y=3x+3-2\Rightarrow e^y=3x+1$$

$$\Rightarrow y=\log(3x+1)\quad(1)$$

OR

Given, differential equation can be written as

$$\frac{dy}{dx}=\frac{y}{x}-\frac{1}{\sin\left(\frac{y}{x}\right)}$$

$$\text{Now, }(\lambda x,\ \lambda y)=\frac{\lambda y}{\lambda x}-\frac{1}{\sin\left(\frac{\lambda y}{\lambda x}\right)}=\lambda^0\left(\frac{y}{x}-\frac{1}{\sin\frac{y}{x}}\right)$$

$$=\lambda^0F(x,\ y)\ (1/2)$$

It is a homogeneous differential equation.

Now, put y = vx

$$\Rightarrow\ \frac{dy}{dx}=v+x\frac{dv}{dx}$$

$$\therefore\ v+x\frac{dv}{dx}=\frac{vx}{x}-\frac{1}{\sin\left(\frac{vx}{x}\right)}\Rightarrow v+x\frac{dv}{dx}=v-\frac{1}{\sin v}$$

$$\Rightarrow\ \sin v dv=-\frac{1}{x}dx \qquad (1/2)$$

On integrating both sides, we get

$$-\cos v=-\log|x|-C$$

$$\Rightarrow -\cos\left(\frac{y}{X}\right)=-\log|x|-C\ \text{[putting } v=\frac{y}{x}]$$

$$\Rightarrow \cos\left(\frac{y}{x}\right)=\log|x|+C\ \ldots(1)\ (1)$$

Given that $x=1$, when $y=\frac{\pi}{2}$

$$\therefore\ \cos\left(\frac{\pi}{2}\right)=\log|1|+C$$

$\Rightarrow 0 = 0 + C$

$\Rightarrow C \Rightarrow 0$

Putting C = 0 in Eq. (i), we get

$$\cos\left(\frac{y}{x}\right) = \log|x| + 0$$

$$\Rightarrow \cos\left(\frac{y}{x}\right) = \log|x| \quad (1)$$

29. Given, rotten apples = 3 and fresh apples = 7

Here, total number of apples = 3 + 7 = 10

Let X denotes the number of rotten apples.

Then, X takes the values 0, 1, 2, 3.

Let A be the event getting a rotten apple.

$\therefore$ P(A) = $\frac{3}{10}$ and P(A') = 1 - P(A) = 1 - $\frac{3}{10} = \frac{7}{10}$

Now, P(X = 0) = P (getting 0 rotten apple)

= P(A')P(A')P(A')

$$= \frac{7}{10} \times \frac{7}{10} \times \frac{7}{10} = \frac{343}{1000} \quad (1/2)$$

$P(X=1)$ = P(getting 1 rotten apple)

= 3P(A)P(A')P(A')

$$= 3 \times \frac{3}{10} \times \frac{7}{10} \times \frac{7}{10} = \frac{441}{1000}$$

$P(X=2)$ = P (getting 2 rotten apples)

$$= 3P(A)P(A)P(A')$$

$$= 3 \times \frac{3}{10} \times \frac{3}{10} \times \frac{7}{10} = \frac{189}{1000} \quad (1/2)$$

and $P(X=3)$ = P(getting 3 rotten apples)

= P(A)·P(A)·P(A)

$$= \frac{3}{10} \times \frac{3}{10} \times \frac{3}{10} = \frac{27}{1000}$$

$\therefore$ Probability distribution is as follows:

X	0	1	2	3
$P(X)$	$\frac{343}{1000}$	$\frac{441}{1000}$	$\frac{189}{1000}$	$\frac{27}{1000}$

Now, mean (μ) = ΣX . P(X)

$$= \frac{0 \times 343}{1000} + \frac{1 \times 441}{1000} + \frac{2 \times 189}{1000} + \frac{3 \times 27}{1000}$$

$$= 0 + \frac{441}{1000} + \frac{378}{7000} + \frac{81}{1000} = \frac{900}{1000} = \frac{9}{10} \quad (1)$$

OR

Let E_1 = Getting ghee from shop X

E_2 = Gettlng ghee from shop Y

A = Gettlng type B ghee

$$\therefore P(E_1) = P(E_2) = \frac{1}{2}$$

[$\because$ both shop have equal chances]

$P\left(\frac{A}{E_1}\right)$ = Probability that type B ghee is purchased from shop X (1)

$$= \frac{40}{70} = \frac{4}{7} \quad (1)$$

P(E_2/A) = Probability that type B ghee is purchased from shop Y

$$= \frac{60}{110} = \frac{6}{11} \quad (1)$$

Now, by Baye's theorem, we get

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2}\times\frac{6}{11}}{\frac{1}{2}\times\frac{4}{7}+\frac{1}{2}\times\frac{6}{11}} = \frac{\frac{6}{11}}{\frac{4}{7}+\frac{6}{11}}$$

$$= \frac{\frac{6}{11}}{\frac{44+42}{77}} = \frac{42}{86} = \frac{21}{43} \qquad (1)$$

30. Let $I = \int_1^2 \left[\frac{1}{x} - \frac{1}{2x^2}\right] e^{2x} dx$

Put $2x = t \Rightarrow x = \frac{1}{2}t \Rightarrow dx = \frac{1}{2}dt$

When $x = 1$, then $t = 2$

and when $x = 2$, then $t = 4$ (1)

$$\therefore I = \frac{1}{2}\int_2^4 \left[\frac{2}{t} - \frac{2}{t^2}\right] e^t dt = \int_2^4 \left(\frac{1}{t} + \frac{-1}{t^2}\right) e^t dt$$

$$= \left[\frac{1}{t}e^t\right]_2^4 \left[\because \int e^x \left(f(x) + f'(x)\right) dx = e^x f(x) + C\right]$$

(1)

$$= \frac{e^4}{4} - \frac{e^2}{2} = \frac{e^2}{4}\left(e^2 - 2\right) \qquad (1)$$

31. Given, $Z = 2x + 3y$

Subject to constraints

$x + 2y \le 10$ ...(i)}

$2x + y \le 14$ ...(ii)

and $x, y \ge 0$ ...(iii)

Shade the region to the right of Y-axis to show $x \ge 0$ and above X-axis to show $y \ge 0$,

Table for line $x + 2y = 10$ is

x	0	4	10
y	5	3	0

(1/2)

So, the line is passing through the points (0, 5), (4, 3)and (10, 0).

On putting (0, 0) in the inequality $x + 2y \ge 10$, we get $0 + 0 \ge 10$, which is true,

So, the half plane is towards the origin.

Table for line $2x + y = 14$ is

x	4	6	7
y	6	2	0

(1/2)

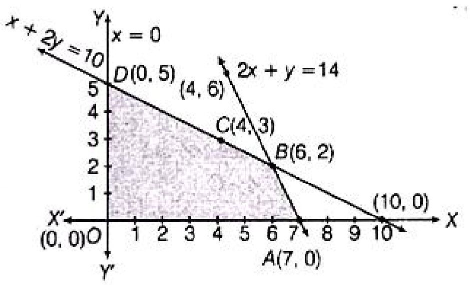
So, the line is passing through the points (4, 6), (6 2)and (7,0).

On putting (0,0) in the inequality $2x + y \le 14$, we get $0 + 0 \le 14$, which is true.

So, the half plane is towards the origin.

The intersection point of lines corresponding to Eqs. (i) and (ii) is S(6,2).

On shading the common region, we get the feasible region OABD.



(1)

The corner poitns are 0(0, 0), A(7, 0), B(6,2) and D(0,5). (1)

32. We have, $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$

$$= 2(-4+4) + 3(-6+4) + 5(3-2$$

$$= 2(0) + 3(-2) + 5(1) = -6 + 5 = -1 \neq 0$$

Thus A^{-1} exist. (1)

Now cofactor of |A| are

$$C_{11}=\begin{vmatrix}2 & -4\\1 & -2\end{vmatrix}=-4+4=0$$

$$C_{12}=\begin{vmatrix}3 & -4\\1 & -2\end{vmatrix}=-(-6+4)=2$$

$$C_{13}=\begin{vmatrix}3 & 2\\1 & 1\end{vmatrix}=3-2=1$$

$$C_{21}=-\begin{vmatrix}-3 & 5\\1 & -2\end{vmatrix}=-(6-5)=-1$$

$$C_{22}=\begin{vmatrix}2 & 5\\1 & -2\end{vmatrix}=(-4-5)=-9$$

$$C_{23}=-\begin{vmatrix}2 & -3\\1 & 1\end{vmatrix}=-(2+3)=-5$$

$$C_{31}=\begin{vmatrix}-3 & 5\\2 & -4\end{vmatrix}=12-10=2$$

$$C_{32}=-\begin{vmatrix}2 & 5\\3 & -4\end{vmatrix}=-(-8-15)=23$$

$$C_{33}=\begin{vmatrix}2 & -3\\3 & 2\end{vmatrix}=4+9=13$$

$$\therefore \text{adj}(A)=\begin{bmatrix}C_{11} & C_{32} & C_{13}\\C_{21} & C_{22} & C_{23}\\C_{31} & C_{32} & C_{33}\end{bmatrix}^T \quad (1)$$

$$=\begin{bmatrix}0 & 2 & 1\\-1 & -9 & -5\\2 & 23 & 13\end{bmatrix}^T=\begin{bmatrix}0 & -1 & 2\\2 & -9 & 23\\1 & -5 & 13\end{bmatrix}$$

$$\therefore A^{-1}=\frac{\text{adj}(A)}{|A|}$$

$$=\frac{1}{-1}\begin{bmatrix}0 & -1 & 2\\2 & -9 & 23\\1 & -5 & 13\end{bmatrix}$$

$$\Rightarrow A^{-1}=\begin{bmatrix}0 & 1 & -2\\-2 & 9 & -23\\-1 & 5 & -13\end{bmatrix} \quad \ldots(i) \qquad (1)$$

Now, to find the solution of system equations

$2x-3y+5z=11$

$3x+2y-4z=-5$

$x+y-2z=-3$

Given, system of equations can be written in matrix form as

$AX=B$, where $A=\begin{bmatrix}2 & -3 & 5\\3 & 2 & -4\\1 & 1 & -2\end{bmatrix}$,

$\chi=\begin{bmatrix}x\\y\\z\end{bmatrix}$ and $B=\begin{bmatrix}11\\-5\\-3\end{bmatrix}$

$$\Rightarrow X=A^{-1}B=\begin{bmatrix}0 & 1 & -2\\-2 & 9 & -23\\-1 & 5 & -13\end{bmatrix}\begin{bmatrix}11\\-5\\-3\end{bmatrix}[\because \textit{from Eq. (i)}]$$

(1)

$$=\begin{bmatrix}0-5+6\\-22-45+69\\-11-25+39\end{bmatrix} \qquad \Rightarrow \begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$$

On comparing the corresponding elements, we get $x=1,\ y=2$ and $z=3$. (1)

33. Any line through the point (1, 1, 1) is given by

$$\frac{x-1}{a}=\frac{y-1}{b}=\frac{z-1}{c} \quad \ldots(i)$$

where a, b and c are the direction ratios of line (i),

Now, the line (i) is perpendicular to the lines

$$\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4} \quad (1)$$

and $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$, where DR's of these two lines are (1, 2, 4) and (2, 3, 4), respectively.

$\therefore$ a + 2b + 4c = 0 …(ii)

and 2a + 35 + 4c = 0 ...(iii) (1)

[$\because$ if two lines having DR's (a_1, b_1, c_1) and (a_2, b_2, c_2) are perpendicular, then $a_1a_2 + b_1b_2 + c_1c_2 = 0$]

By cross-multiplication method, we get

$$\frac{a}{8-12}=\frac{b}{8-4}=\frac{c}{3-4}\Rightarrow\frac{a}{-4}=\frac{b}{4}=\frac{c}{-1}$$

$\therefore$ DR's of line (i) are - 4, 4,-1

$\therefore$ The required cartesian equation of line (i) is

$$\frac{x-1}{-4}=\frac{y-1}{4}=\frac{z-1}{-1} \quad (1)$$

and vector equation is

$$\vec{r}=\hat{i}+\hat{j}+\hat{k}+\lambda(-4\hat{i}+4\hat{j}-\hat{k}) \quad (1)$$

Again, let θ be the angle between the given lines. Then,

$$\cos\theta=\frac{|1\times2+2\times3+4\times4|}{\sqrt{1+4+16}\sqrt{4+9+16}}=\frac{24}{\sqrt{21}\sqrt{29}}=\frac{24}{\sqrt{609}}$$

$$\therefore\theta=\cos^{-1}\left(\frac{24}{\sqrt{609}}\right) \quad (1)$$

OR

Given, equation of lines can be rewritten as

$$\vec{r}=(2\hat{i}-3\hat{j}+5\hat{k})+\lambda(\hat{i}-\hat{j}+\hat{k})$$

and $\vec{r}=(-\hat{i}-\hat{j}+5\hat{k})+\mu(2\hat{i}+4\hat{j}-3\hat{k})$

On comparing the above equations with standard vector form of equation of line, $\vec{r}=\vec{a}+\lambda\vec{b}$, we get

$$\overrightarrow{a_1}=2\hat{i}-3\hat{j}+5\hat{k},\overrightarrow{b_1}=\hat{i}-\hat{j}+\hat{k} \text{ and}$$

$$\overrightarrow{a_2}=-\hat{j}-\hat{j}+5\hat{k},\overrightarrow{b_2}=2\hat{i}+4\hat{j}-3\hat{k}$$

Now consider, $\overrightarrow{b_1}\times\overrightarrow{b_2}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&-1&1\\2&4&-3\end{vmatrix}$

$$=\hat{i}(3-4)-\hat{j}(-3-2)+\hat{k}(4+2)$$

$$\Rightarrow\overrightarrow{b_1}\times\overrightarrow{b_2}=-\hat{i}+5\hat{j}+6\hat{k} \quad (1)$$

$$\Rightarrow|\overrightarrow{b_1}\times\overrightarrow{b_2}|=\sqrt{(-1)^2+(5)^2+(6)^2}$$

$$=\sqrt{1+25+36}=\sqrt{62}$$

$$Also,\vec{a}_2-\vec{a}_1=(-\hat{i}-\hat{j}+5\hat{k})-(2\hat{i}-3\hat{j}+5\hat{k})$$

$$=-3\hat{i}+2\hat{j} \quad (1)$$

We know that shortest distance between two lines is given by $d=\left|\frac{(\vec{b}_1\times\vec{b}_2)\cdot(\overrightarrow{a_2}-\overrightarrow{a_1})}{|\vec{b}_1\times\vec{b}_2|}\right|$

On putting above values, we get

$$d=\left|\frac{(-\hat{i}+5\hat{j}+6\hat{k})\cdot(-3\hat{i}+2\hat{j})}{\sqrt{62}}\right|$$

$$=\left|\frac{3+10+0}{\sqrt{62}}\right|=\frac{13}{\sqrt{62}}=\frac{13\sqrt{62}}{62}$$

Hence, required shortest distance is $\frac{13\sqrt{62}}{62}$ units. (2)

34. Given, R = {(a, b): 5 divides (a-b)}

and Z = Set of integers

Reflexive Let a $\in$ Z be any arbitrary element.

Now, if (a,a) $\in$ R, then 5 divides a-a, which is true.

So, R is reflexive. (1)

Symmetric Let a, b $\in$ Z, such that

(a, b) $\in$ R $\in$ 5 divides (a - b)

$\Rightarrow$ 5 divides [-(a - b)]

$\Rightarrow$ 5 divides (b - a) =+ (b, a) $\in$ R

So, R is symmetric. (2)

Transitive Let a, b, c $\in$ Z, such that (a, b) $\in$ R and (b,c) $\in$ R

$\Rightarrow$ a -b and b -c both are divisible by 5.

$\Rightarrow$ a -5 + b-c is divisible by 5.

$\Rightarrow$ (a -c)is divisible by 5

$\Rightarrow$ (a,c) $\in$ R

So, R is transitive.

Thus, R is reflexive, symmetric and transitive.

Hence, R is an equivalence relation. (2)

OR

Here, R = {(P, Q): distance of point P from the origin is same as the distance of point Q from the origin}. Clearly, (P, P) $\in$ R, since the distance of point P from the origin is always the same as the distance of the same point P from the origin.

Therefore, R is reflexive. (1)

Now, let (P,Q) $\in$ R

$\Rightarrow$ The distance of point P from the origin is same as the distance of point 0 from the origin,

$\Rightarrow$ The distance of point Q from the origin is same as the distance of point P from the origin.

$\Rightarrow$ (0, P) $\in$ R

Therefore, R is symmetric. (1)

Now, let (P, Q), (Q.S) $\in$ R

The distance of points P and Q from the origin is same and also the distance of points Q and S from the origin is same.

$\Rightarrow$ The distance of points P and S from the origin is same.

$\Rightarrow$ (P,S) $\in$ R

Therefore, R is transitive. Therefore, R is an equivalence relation. (1)

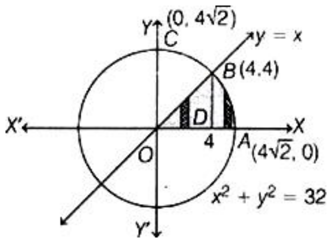
The set of all points related to P $\neq$ (0, 0) will be those points whose distance from the origin is the same as the distance of point P from the origin, in other words, if 0(0, 0) is the origin and OP = k, then the set of ail points related to P is at a distance of k from the origin. .

Hence, this set of points forms a circle with the centre as the origin and this circle passes through point P. (2)

35. We have, circle $x^2 + y^2 = 32$ ...(i)

having centre (0, 0)and radius $4\sqrt{2}$ units

and the line, y = x ...(ii)



(1)

It is clear from the figure that, required region is OABO.

On putting the value of y from Eq. (ii) in Eq. .(i), we get

$x^2 + x^2 = 32$

$\Rightarrow 2x^2 = 32$

$\Rightarrow x^2 = \dfrac{32}{2} = 16$

$\Rightarrow x = \pm 4$

From Eq. (ii), we get

y = +4 (1)

Thus, line and circle intersect at two points (4, 4) and (- 4, - 4). So, the coordinates of B are (4, 4) [since, it is in I quadrant]. Also, circle cuts the X-axis at A (4 , 0) and Y-axis at C (0, $4\sqrt{2}$) in I quadrant.

[$\because$ $4\sqrt{2}$ is radius of a circle]

Here, we have to draw two vertical strips, as perpendicular line drawn from intersection point to the X-axis, divides the region into two parts. Now, first strip Is drawn in region ODBO and then limit is taken from 0 to 4. Second strip is drawn in region DABD and then limit is taken from 4 to $4\sqrt{2}$.

Now, area of region ODBO = $\int_0^4 y\,dx$, where y is the height of vertical strip

$$=\int_0^4 x dx=\left[\frac{x^2}{2}\right]_0^4$$

$$=\frac{(4)^2}{2}-0=8\text{sq}\,units \qquad (1)$$

and area of region DABD = $\int_0^{4\sqrt{2}} y dx$, where y is the height of vertical strip in this region

$$=\int_4^{4\sqrt{2}}\sqrt{32-x^2}\,dx=\int_4^{2\sqrt{2}}\sqrt{(4\sqrt{2})^2-x^2}\,dx$$

$$=\left[\frac{x}{2}\sqrt{(4\sqrt{2})^2-x^2}+\frac{(4\sqrt{2})^2}{2}\times\sin^{-1}\left(\frac{x}{4\sqrt{2}}\right)\right]_4^{4\sqrt{2}}$$

$$\left[\left\{\frac{4\sqrt{2}}{2}\sqrt{(4\sqrt{2})^2-(4\sqrt{2})^2}+\frac{32}{2}\sin^{-1}\left(\frac{4\sqrt{2}}{4\sqrt{2}}\right)\right\}-\right.$$

$$\left.\left\{\frac{4}{2}\sqrt{(4\sqrt{2})^2-(4)^2}+\frac{32}{2}\sin^{-1}\left(\frac{4}{4\sqrt{2}}\right)\right\}\right]$$

$$=2\sqrt{2}\times 0+16\sin^{-1}(1)-2\sqrt{32-16}-16\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$=16\cdot\left(\frac{\pi}{2}\right)-2\sqrt{16}-16\cdot\left(\frac{\pi}{4}\right)$$

$= 8\pi - 8 - 4\pi = 4\pi - 8$ (1)

$\therefore$ Required area = Area of region ODBO + Area of region DABD

$= 8 + 4\pi - 8 = 4\pi$ sq units

Hence, the area of required region is 4π sq units.

(1)

36. (i) At x = 3,

P (3) = -6 $(3)^2$ +120 (3) + 25000

= -54+ 360 + 25000 = Rs. 25306

(ii) P'(x) = -12x +120

P'(5) = -12 × 5+ 120 = -60 + 120 = 60

(iii) For strictly increasing, we must put P'(x) > 0

$\Rightarrow$ -12x + 120 > 0

$\Rightarrow$ 120 > 12x

$\Rightarrow$ x < 10

$\Rightarrow$ x $\in$ (0,10)

OR

P(x) = -6x^2 + 120x + 25000

$\Rightarrow$ P'(x) = -12x + 120

For maximum profit, put P'(x) = 0

$\Rightarrow$ x = 10

Now P''(x) = -12 < 0

$\Rightarrow$ At x = 10, profit function is maximum.

37. (i) Perimeter of rectangular floor

= 2 (length + breadth)

$\Rightarrow P = 2(l + b)$

(ii) Area, A = length × breadth

A = l × b ...(i)

$$\Rightarrow P = 2(l+b) \Rightarrow \frac{P}{2}=l+b$$

From Eq. (i),

$$A=l\left(\frac{P-2l}{2}\right)$$

(iii) We have, $A=\dfrac{Pl-2l^2}{2}$

On differentiating w.r.t. l, we get

$$\frac{dA}{dl} = \frac{1}{2}(P - 4l)$$

For maximum area of floor, put $\frac{dA}{dl} = 0$

$$\therefore \frac{1}{2}(P - 4l) = 0$$

$\Rightarrow P - 4l = 0 \qquad \Rightarrow l = \frac{P}{4}$

Clearly at $l = \frac{P}{4}, \frac{d^2A}{dl^2} = -2 < 0$

$\therefore$ Area is maximum at $l = \frac{P}{4}$.

OR

We have $A = l \times b$

For maximum area $= \frac{P}{4}$

Now, $b = \frac{P - 2l}{2}$ [from part (ii)]

$$= \frac{P}{2} - l = \frac{P}{2} - \frac{P}{4} = \frac{P}{4}$$

$\therefore (A)_{max} = l \times b = \frac{P}{4} \times \frac{P}{4} = \frac{P^2}{16}$ sq units.

38. (i) Required probability $= P(A)$

$$= P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)$$

$= 0.5 \times 0.06 + 0.2 \times 0.04 + 0.3 \times 0.03$

$= 0.030 + 0.008 + 0.009 = 0.047$

(ii) Required probability

$$= P\left(\frac{\bar{E}_1}{A}\right) = 1 - P\left(\frac{E_1}{A}\right)$$

$$= 1 - \left[\frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}\right]$$

$$= 1 - \left[\frac{0.5 \times 0.06}{0.5 \times 0.06 + 0.2 \times 0.04 + 0.3 \times 0.03}\right]$$

$$= 1 - \left[\frac{0.030}{0.030 + 0.008 + 0.009}\right]$$

$$= 1 - \frac{0.030}{0.047} = 1 - \frac{30}{47} = \frac{17}{47}$$