JKBOSE PATTERN
TEST PAPER
CLASS - XII
SUBJECT MATHEMATICS
ANSWER & SOLUTIONS

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Q.1:

(a) $f: R \to R$ is defined as f (x) = 3x

Let x,y εR such that f (x)= f (y)

=>3x=3y

=>x=y

$\therefore$ f is one-one

Also, for any real number (y) in co-domain r, there exists $\frac{y}{3}$ in R such that

$f\left(\frac{y}{3}\right)=3\left(\frac{y}{3}\right)=y$

$\therefore f$ is onto.

Hence, funstion f is one-one and onto.

(b) Let $\tan^{-1}\sqrt{3}=x$. Then, $\tan x=\sqrt{3}=\tan\frac{\pi}{3}$.

We know that the range of the principal value branch of $\tan^{-1}$ is $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$.

$\therefore \tan^{-1}\sqrt{3}=\frac{\pi}{3}$

Let $\sec^{-1}(-2)=y$. Then, $\sec y=-2=-\sec\left(\frac{\pi}{3}\right)=\sec\left(\pi-\frac{\pi}{3}\right)=\sec\frac{2\pi}{3}$.

We know that the range of the principal value branch of $\sec^{-1}$ is $[0,\pi]-\left\{\frac{\pi}{2}\right\}$.

$\therefore \sec^{-1}(-2)=\frac{2\pi}{3}$

Hence, $\tan^{-1}\left(\sqrt{3}\right)-\sec^{-1}(-2)=\frac{\pi}{3}-\frac{2\pi}{3}=-\frac{\pi}{3}$

(c) A and B are symmetric matrices, therefore, we have:

$A' = A$ and $B' = B$ ...(1)

Consider $(AB - BA)' = (AB)' - (BA)'$ $\quad [(A-B)' = A' - B']$

$= B'A' - A'B'$ $\quad [(AB)' = B'A']$

$= BA - AB$ $\quad$ [by (1)]

$= -(AB - BA)$

$\therefore (AB - BA)' = -(AB - BA)$

Thus, $(AB - BA)$ is a skew-symmetric matrix.

A^{-1} exists and $A^{-1} = \frac{1}{|A|} adjA.$

(d) Since A is an invertible matrix,

As matrix A is of order 2, let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

Then, $|A| = ad - bc$ and $adjA = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Now,

$$A^{-1} = \frac{1}{|A|} adjA = \begin{bmatrix} \frac{d}{|A|} & \frac{-b}{|A|} \\ \frac{-c}{|A|} & \frac{a}{|A|} \end{bmatrix}$$

$$\therefore |A^{-1}| = \begin{vmatrix} \frac{d}{|A|} & \frac{-b}{|A|} \\ \frac{-c}{|A|} & \frac{a}{|A|} \end{vmatrix} = \frac{1}{|A|^2} \begin{vmatrix} d & -b \\ -c & a \end{vmatrix} = \frac{1}{|A|^2}(ad - bc) = \frac{1}{|A|^2} . |A| = \frac{1}{|A|}$$

$$\therefore \det(A^{-1}) = \frac{1}{\det(A)}$$

(e) Let $y = x^2 + 3x + 2$

Again Differentiating $w.r.t.x$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d\,(2x+3)}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{d(2x)}{dx} + \frac{d(3)}{dx}$$

$$\frac{d^2y}{dx^2} = 2 + 0$$

$$\frac{d^2y}{dx^2} = 2$$

(f) $$\int \frac{dx}{\sin^2 x \cos^2 x}$$

$$= \int \frac{1}{\sin^2 x \cos^2 x}\,.dx$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}\,.dx \qquad (\text{Using } \sin^2 x + \cos^2 x = 1)$$

$$= \int \frac{\sin^2 x}{\sin^2 x \cos^2 x}\,.dx + \int \frac{\cos^2 x}{\sin^2 x \cos^2 x}\,.dx$$

$$= \int \frac{1}{\cos^2 x}\,.dx + \int \frac{1}{\sin^2 x}\,.dx$$

$$= \int \sec^2 x\,.\,dx + \int cosec^2 x\,.dx$$

$$= \tan x - \cot x + C$$

$$\boxed{\text{using } \int \sec^2 x.\,dx = \tan x \text{ and } \int cosec^2 x.dx = -\cot x}$$

(g) $\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{dy}{dx}\right)^2+\sin\left(\frac{dy}{dx}\right)+1=0$

The given differential equation is not a polynomial equation in its derivatives. Therefore, its degree is not defined.

(h) **vector** $\lambda\vec{a}$ is a unit vector if $|\lambda\vec{a}|$ =1
Now,

$|\lambda\vec{a}|=1$

$=>|\lambda||\vec{a}|=1$

$=>|\vec{a}|=\frac{1}{|\lambda|}$ $[\lambda\neq 0]$

$=> a=\frac{1}{|\lambda|}$ $[|\vec{a}|=a]$

Hence, vector$^{\lambda\vec{a}}$ *is* a unit vector if a=$\frac{1}{|\lambda|}$

(i) It is given that $|\vec{a}|=3$ and $|\vec{b}|=\frac{\sqrt{2}}{3}$

We know that $\vec{a}\times\vec{b}=|\vec{a}||\vec{b}|\sin\theta\,\hat{n}$, where $\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ and between $\vec{a}$ and $\vec{b}$.

Now, $\vec{a}\times\vec{b}$ is a unit vector if $|\vec{a}\times\vec{b}|=1$, so

$|\vec{a}\times\vec{b}|=1\Rightarrow\left||\vec{a}||\vec{b}|\sin\theta\,\hat{n}\right|=1\quad\Rightarrow\left||\vec{a}||\vec{b}|\sin\theta\right|=1$

$\Rightarrow 3\times\frac{\sqrt{2}}{3}\times\sin\theta=1\Rightarrow\sin\theta=\frac{1}{\sqrt{2}}\quad\Rightarrow\theta=\frac{\pi}{4}$

Hence $\vec{a}\times\vec{b}$ is a unit vector if the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$

The correct answer is (B).

(j) Let

E : A speaks truth

F : A Lies

H : head appears on the toss of a coin

We need to find the Probability that head actually appears, if A reports that a head appears

i.e. **P(E|H)**

$$P(E|H) = \frac{P(E) \cdot P(H|E)}{P(F) \cdot P(H|F) + P(E) \cdot P(H|E)}$$

P(E) = Probability that A speaks truth

$= \frac{4}{5}$

P(H|E) = Probability that head appears, if A speaks truth

$= \frac{1}{2}$

P(F) = Probability that A lies

$= 1 - P(E)$

$= 1 - \frac{4}{5} = \frac{1}{5}$

P(H|F) = Probability that head appears, if A lies

$= \frac{1}{2}$

Putting values in formula,

Putting values in formula,

$$P(E|H) = \frac{\frac{4}{5} \times \frac{1}{2}}{\frac{1}{5} \times \frac{1}{2} + \frac{4}{5} \times \frac{1}{2}}$$

$$= \frac{\frac{1}{5} \times \frac{1}{2} \times 4}{\frac{1}{5} \times \frac{1}{2}\, [1 + 4]}$$

$$= \frac{4}{5}$$

Section B

Q2: It is given that f: $\mathbf{R}_* \to \mathbf{R}_*$ is defined by $f(x) = \frac{1}{x}$.

One-one:

$$f(x) = f(y)$$
$$\Rightarrow \frac{1}{x} = \frac{1}{y}$$
$$\Rightarrow x = y$$

$\therefore f$ is one-one.

Onto:

It is clear that for $y \in \mathbf{R}_*$, there exists $x = \frac{1}{y} \in \mathrm{R}_*$ (Exists as $y \neq 0$) such that

$$f(x) = \frac{1}{\left(\frac{1}{y}\right)} = y.$$

$\therefore f$ is onto.

Thus, the given function (f) is one-one and onto.

Q3: Let $\sin^{-1}\left(-\frac{1}{2}\right) = y$. Then $\sin y = -\frac{1}{2} = -\sin\left(\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right)$.

We know that the range of the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$.

Therefore, the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ is $-\frac{\pi}{6}$.

Q4

(i) Let $A = \begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$.

It can be observed that in the second row, two entries are zero. Thus, we expand along the second row for easier calculation.

$$|A| = -0\begin{vmatrix} -1 & -2 \\ -5 & 0 \end{vmatrix} + 0\begin{vmatrix} 3 & -2 \\ 3 & 0 \end{vmatrix} - (-1)\begin{vmatrix} 3 & -1 \\ 3 & -5 \end{vmatrix} = (-15+3) = -12$$

OR

$$\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$$

$= 3\begin{vmatrix} 0 & -1 \\ -5 & 0 \end{vmatrix} - (-1)\begin{vmatrix} 0 & -1 \\ 3 & 0 \end{vmatrix} + (-2)\begin{vmatrix} 0 & 0 \\ 3 & -5 \end{vmatrix}$

$= 3\,(0(0) - (-5)\,(-1)\,) + 1\,(0(0) - 3(-1)) - 2\,(0(-5) - 3(0))$

$= 3(0 - 5) + 1\,(0 + 3) - 2\,(0 - 0)$

$= 3(-5) + 1\,(3) + 0$

$= -15 + 3$

$\mathbf{= -12}$

Q5 At $x = 5$

$f(x)$ is continuous at x = 5 if

$$\lim_{x \to 5} f(x) = f(5)$$

L.H.S	R.H.S
$\lim_{x \to 5} f(x)$	$f(5)$
$= \lim_{x \to 5} (5x - 3)$	$= 5(5) - 3$
Putting x = 5	$= 25 - 3$
$= 5(5) - 3$	$= \mathbf{22}$
$= \mathbf{22}$	

Since, L.H.S = R.H.S

Hence, f **is continuous at x = 5**

Q.6

$$2x + 3y = \sin y$$

Differentiating both sides $w.r.t.x$

$$\frac{d(2x + 3y)}{dx} = \frac{d(\sin y)}{dx}$$

$$\frac{d(2x)}{dx} + \frac{d(3y)}{dx} = \frac{d(\sin y)}{dx}$$ *(Derivative of $\sin x$ is $\cos x$)*

$$2\frac{dx}{dx} + \frac{3d(y)}{dx} = \frac{d(\sin y)}{dy} \times \frac{dy}{dx}$$

$$2 + 3\frac{dy}{dx} = \cos y \times \frac{dy}{dx}$$

$\cos y \times \frac{dy}{dx} - 3\frac{dy}{dx} = 2$

$\frac{dy}{dx}(\cos y - 3) = 2$

$\boldsymbol{\frac{dy}{dx} = \frac{2}{(\cos y - 3)}}$

Q7: Let Radius of circle = r

& Area of circle = A

We need to find rate of change of Area w. r. t Radius

i.e. we need to calculate $\frac{dA}{dr}$

We know that

Area of Circle = A = πr^2

Finding $\frac{dA}{dr}$

$\frac{dA}{dr} = \frac{d(\pi r^2)}{dr}$

$\frac{dA}{dr} = \pi \frac{d(r^2)}{dr}$

$\frac{dA}{dr} = \pi(2r)$

$\boldsymbol{\frac{dA}{dr} = 2\pi r}$

When r=3cm

$$\frac{dA}{dr} = 2\pi r$$

Putting r = 3 cm

$$\left.\frac{dA}{dr}\right|_{r=3} = 2\pi \times 3$$

$$\left.\frac{dA}{dr}\right|_{r=3} = 6\pi$$

Since Area is in cm^2 & radius is in cm

$$\frac{dA}{dr} = \mathbf{6\pi\ cm^2/cm}$$

Q8: *Consider* $I = \int (ax+b)^2 dx$

$$= \int (a^2x^2 + b^2 + 2abx) dx$$

$$= a^2 \int x^2 dx + b^2 \int dx + 2ab \int x dx$$

$$= a^2 \frac{x^3}{3} + b^2 x + 2ab \frac{x^2}{2} + C$$

$$I = \frac{a^2x^3}{3} + b^2 x + abx^2 + C$$

Q9: $\int x\sin x\ dx = \boldsymbol{x\int \sin x\, dx - \int\left(\frac{d(x)}{dx}\int \sin x\, dx\right)dx}$

$$= -x\cos x - \int -\cos x\, dx$$

$$= -x\cos x + \int \cos x + C$$

$$\boldsymbol{= -x\cos x + \sin x + C}$$

Q10: $\vec{a} = \hat{\imath} + \hat{\jmath} + 2\hat{k},$

$= 1\hat{\imath} + 1\hat{\jmath} + 2\hat{k},$

Magnitude of $\vec{a} = \sqrt{1^2 + 1^2 + 2^2}$

$$|\vec{a}| = \sqrt{1+1+4} = \sqrt{6}$$

Unit vector in direction of $\vec{a} = \frac{\mathbf{1}}{|\vec{a}|}\,.\,\boldsymbol{\vec{a}}$

$$\hat{a} = \frac{1}{\sqrt{6}}\left[1\hat{\imath} + 1\hat{\jmath} + 2\hat{k}\right]$$

Q 11: Direction cosines of a line making angle α with x – axis, β with y – axis and γ with z – axis are l, m, n

$\boldsymbol{l = \cos\alpha,\ m = \cos\beta,\ n = \cos\gamma}$

Here, α = **90°**, β = **135°**, γ = **45°,** So, direction cosines are

$l = \cos 90° = \mathbf{0}$

$m = \cos 135° = \mathbf{\cos (180 - 45°)} = -\cos 45° = \frac{-1}{\sqrt{2}}$

$n = \cos 45° = \frac{1}{\sqrt{2}}$

Section C

Q12: A relation R on a set A is said to be an **equivalence relation** if and only if the relation R is reflexive, symmetric and transitive. The equivalence relation is a relationship on the set which is generally represented by the symbol "∼".

Reflexive: A relation is said to be reflexive, if (a, a) ∈ R, for every a ∈ A.

Symmetric: A relation is said to be symmetric, if (a, b) ∈ R, then (b, a) ∈ R.

Transitive: A relation is said to be transitive if (a, b) ∈ R and (b, c) ∈ R, then (a, c) ∈ R.

$R = \{(a, b): a \leq b^2\}$

It can be observed that $\left(\frac{1}{2}, \frac{1}{2}\right) \notin \mathbf{R}$, since $\frac{1}{2} > \left(\frac{1}{2}\right)^2 = \frac{1}{4}$.

∴R is not reflexive.

Now, $(1, 4) \in R$ as $1 < 4^2$

But, 4 is not less than 1^2.

$\therefore (4, 1) \notin R$

∴R is not symmetric.

Now,

$(3, 2), (2, 1.5) \in R$

(as $3 < 2^2 = 4$ and $2 < (1.5)^2 = 2.25$)

But, $3 > (1.5)^2 = 2.25$

$\therefore (3, 1.5) \notin R$

∴ R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

Q.13: $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$

$\therefore A'=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

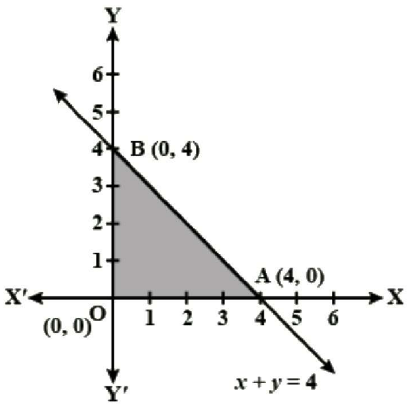
$$A'A=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$$

$$=\begin{bmatrix}(\cos\alpha)(\cos\alpha)+(-\sin\alpha)(-\sin\alpha) & (\cos\alpha)(\sin\alpha)+(-\sin\alpha)(\cos\alpha)\\ (\sin\alpha)(\cos\alpha)+(\cos\alpha)(-\sin\alpha) & (\sin\alpha)(\sin\alpha)+(\cos\alpha)(\cos\alpha)\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\alpha+\sin^2\alpha & \sin\alpha\cos\alpha-\sin\alpha\cos\alpha\\ \sin\alpha\cos\alpha-\sin\alpha\cos\alpha & \sin^2\alpha+\cos^2\alpha\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

Q.14:The feasible region determined by the constraints, $x + y \le 4$, $x \ge 0$, $y \ge 0$, is as follows.



The corner points of the feasible region are O (0, 0), A (4, 0), and B (0, 4). The values of Z at these points are as follows.

Corner point	Z = 3x + 4y	
O(0, 0)	0	
A(4, 0)	12	
B(0, 4)	16	→ Maximum

Therefore, the maximum value of Z is 16 at the point B (0,4).

Q:15(a) $f(x) = \underbrace{(2x-1)^2}_{\text{Square of number cant be negative}} + 3$

Square of number cant be negative

It can be 0 or greater than 0

Hence,

Minimum value of $(2x-1)^2 = 0$

Minimum value of $(2x - 1^2) + 3 = 0 + 3 = \mathbf{3}$

Also,

there is no maximum value of x

∴ There is **no maximum value** of f(x)

(b) $f(x) = x^3 + 1$

Finding f'(x)

$$f'(x) = \frac{d(x^3+1)}{dx}$$

$$= 3x^2$$

Putting $f'(x) = 0$

$$3x^2 = 0$$

$$x^2 = 0$$

$$x = 0$$

Finding $f''(x)$

$f'(x) = 3x^2$

$f''(x) = 6x$

Finding $f''(x)$ at $x = 0$

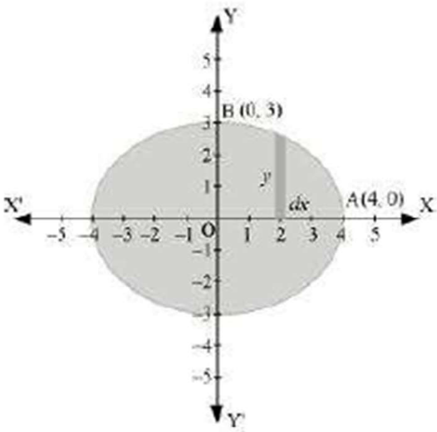
$f''(0) = 6 \times 0 = 0$

Since $f''(x) = 0$ at $x = 0$

$\therefore$ The point $x = 0$ is neither a point of local maxima nor a point of local Minima

Hence x = 0 is point of inflexion
Hence, there is no minimum or maximum value

Q. 16:The given equation of the ellipse, $\frac{x^2}{16}+\frac{y^2}{9}=1$ can be represented as



It can be observed that the ellipse is symmetrical about x-axis and y-axis.
$\therefore$ Area bounded by ellipse = 4 $\times$ Area of OAB

Area of OAB=$\int_0^4 y\,dx$

$$= \int_0^4 3\sqrt{1-\frac{x^2}{16}}dx$$

$$= \frac{3}{4}\int_0^4 \sqrt{16-x^2}\,dx$$

$$= \frac{3}{4}\left[\frac{x}{2}\sqrt{16-x^2}+\frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4$$

$$\frac{3}{4}\left[2\sqrt{16-16}+8\sin^{-1}(1)-0-8\sin^{-1}(0)\right]$$

$$= \frac{3}{4}\left[\frac{8\pi}{2}\right]$$

$$= \frac{3}{4}[4\pi]$$

$$= 3\pi$$

Therefore, area bounded by the ellipse = 4 Ч 3n = 12n units

Q.17: The given vectors are $\vec{a} = \hat{i}-2\hat{j}+3\hat{k}$ and $\vec{b} = 3\hat{i}-2\hat{j}+\hat{k}$

$$|\vec{a}| = \sqrt{1^2+(-2)^2+3^2} = \sqrt{1+4+9} = \sqrt{14}$$

$$|\vec{b}| = \sqrt{3^2+(-2)^2+1^2} = \sqrt{9+4+1} = \sqrt{14}$$

Now, $\vec{a}\cdot\vec{b} = \left(\hat{i}-2\hat{j}+3\hat{k}\right)\left(3\hat{i}-2\hat{j}+\hat{k}\right)$

$$= 1.3+(-2)(-2)+3.1$$

$$= 3+4+3$$

$$= 10$$

Also, we know that $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$.

$\therefore 10 = \sqrt{14}\sqrt{14}\cos\theta$

$\Rightarrow \cos\theta = \dfrac{10}{14}$

$\Rightarrow \theta = \cos^{-1}\left(\dfrac{5}{7}\right)$

Q.18: Angle between two vectors

$\vec{r} = \vec{a_1} + \lambda \vec{b_1}$

& $\vec{r} = \vec{a_2} + \mu \vec{b_2}$ is given by

$$\cos\theta = \left|\frac{\vec{b_1}.\vec{b_2}}{|\vec{b_1}||\vec{b_2}|}\right|$$

Given, the pair of lines is

$\vec{r} = (2\hat{\imath} - 5\hat{\jmath} + \hat{k}) + \lambda(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$	$\vec{r} = (7\hat{\imath} - 6\hat{k}) + \mu(\hat{\imath} + 2\hat{\jmath}$
So, $\vec{a_1} = 2\hat{\imath} - 5\hat{\jmath} + 1\hat{k}$	So, $\vec{a_2} = 7\hat{\imath} + 0\hat{\jmath} - 6\hat{k}$
$\vec{b_1} = 3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$	$\vec{b_2} = 1\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$

Now,

$\vec{b_1}.\vec{b_2} = (3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}) . (1\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$

$= (3 \times 1) + (2 \times 2) + (6 \times 2)$

$= 3 + 4 + 12$

= 19

Magnitude of $\overrightarrow{b_1} = \sqrt{3^2 + 2^2 + 6^2}$

$$|\overrightarrow{\boldsymbol{b_1}}| = \sqrt{9 + 4 + 36} = \sqrt{49} = \mathbf{7}$$

Magnitude of $\overrightarrow{b_2} = \sqrt{1^2 + 2^2 + 2^2}$

$$|\overrightarrow{\boldsymbol{b_2}}| = \sqrt{1 + 4 + 4} = \sqrt{9} = \mathbf{3}$$

Now:

$$\cos\theta = \left|\frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{|\overrightarrow{b_1}||\overrightarrow{b_2}|}\right|$$

$$\boldsymbol{\cos\theta = \left|\frac{19}{7 \times 3}\right|}$$

$$\cos\theta = \frac{19}{21}$$

$$\therefore \boldsymbol{\theta = \cos^{-1}\left(\frac{19}{21}\right)}$$

Therefore, the angle between the given vectors is $\cos^{-1}\left(\frac{19}{21}\right)$

Q.19: Let $y = (\log x)^{\cos x}$

Taking log both sides

$$\log y = \log\ (\log x)^{\cos x}$$

$$\log y = \cos x\ .\log(\log x) \qquad (As\ \log(a^b) = b\log a)$$

Differentiating both sides $w.r.t.x.$

$$\frac{d(\log y)}{dx} = \frac{d(\cos x \,.\, \log(\log x))}{dx}$$

$$\frac{d(\log y)}{dx}\left(\frac{dy}{dy}\right) = \frac{d(\cos x \,.\, \log(\log x))}{dx}$$

$$\frac{d(\log y)}{dx}\left(\frac{dy}{dx}\right) = \frac{d(\cos x \,.\, \log(\log x))}{dx}$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{d(\cos x \,.\, \log(\log x))}{dx}$$

> *Using product rule in* $\cos x \,.\, \log(\log x)$
>
> $(uv)' = u'v + v'u$

$$\frac{1}{y}\frac{dy}{dx} = \frac{d(\cos x)}{dx} \,.\, \log(\log x) + \frac{d(\log(\log x))}{dx} \,.\, \cos x$$

$$\frac{1}{y}\frac{dy}{dx} = -\sin x \,.\, \log(\log x) + \frac{1}{\log x} \,.\, \frac{d(\log x)}{dx} \,.\, \cos x$$

$$\frac{1}{y}\frac{dy}{dx} = -\sin x \,.\, \log(\log x) + \frac{1}{\log x} \times \frac{1}{x} \,.\, \cos x$$

$$\frac{1}{y}\frac{dy}{dx} = -\sin x \,.\, \log(\log x) + \frac{\cos x}{x \log x}$$

$$\frac{dy}{dx} = y\left(-\sin x \,.\, \log(\log x) + \frac{\cos x}{x \log x}\right)$$

Putting values of y

$$\frac{dy}{dx} = (\log x)^{\cos x}\left(-\sin x \cdot \log(\log x) + \frac{\cos x}{x \log x}\right)$$

$$\frac{dy}{dx} = (\log x)^{\cos x}\left(\frac{\cos x}{x \log x} - \sin x \cdot \log(\log x)\right)$$

Section D

(a) Let $1+x^2=t$

$\therefore 2x\,dx = dt$

$$\Rightarrow \int \frac{2x}{1+x^2}dx = \int \frac{1}{t}dt$$

$= \log|t| + C$

$= \log|1+x^2| + c$

$= \log(1+x^2) + C$

(b) Let $1 + \log x = t$

$\therefore \frac{1}{x}dx = dt$

$$\Rightarrow \int \frac{(1+\log x)^2}{x}dx = \int t^2 dt$$

$$= \frac{t^3}{3} + C$$

$$= \frac{(1+\log x)^3}{3} + C$$

one part

(a) *Let* $I=\int_0^{\frac{\pi}{2}} \cos^2 x\, dx$

$$\int \cos^2 x\, dx = \int\left(\frac{1+\cos 2x}{2}\right)dx = \frac{x}{2}+\frac{\sin 2x}{4} = \frac{1}{2}\left(x+\frac{\sin 2x}{2}\right) = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \left[F\left(\frac{\pi}{2}\right)-F(0)\right]$$

$$=\frac{1}{2}\left[\left(\frac{\pi}{2}-\frac{\sin\pi}{2}\right)-\left(0+\frac{\sin 0}{2}\right)\right]$$

$$=\frac{1}{2}\left[\frac{\pi}{2}+0-0-0\right]$$

$$=\frac{\pi}{4}$$

(b) *Let* $I=\int_{-5}^{5}|x+2|\,dx$

It can be seen that (x + 2) ≤ 0 on [-5, -2] and (x + 2) $\geq$ 0 on [-2, 5].

$$\therefore I=\int_{-5}^{-2}-(x+2)dx+\int_{2}^{5}(x+2)dx \qquad \left(\int_a^b f(x)=\int_a^e f(x)+\int_c^b f(x)\right)$$

$$I=-\left[\frac{x^2}{2}+2x\right]_{-5}^{-2}+\left[\frac{x^2}{2}+2x\right]_{-2}^{5}$$

$$=-\left[\frac{(-2)^2}{2}+2(-2)-\frac{(-5)^2}{2}-2(-5)\right]+\left[\frac{(5)^2}{2}+2(5)-\frac{(-2)^2}{2}-2(-2)\right]$$

$$=-\left[2-4-\frac{25}{2}+10\right]+\left[\frac{25}{2}+10-2+4\right]$$

$$=-2+4+\frac{25}{2}-10+\frac{25}{2}+10-2+4$$

$$=29$$

Q.21:(a) Given

$$P(A) = 0.8 \ , P(B) = 0.5 \ \& \ P(B|A) = 0.4$$

Now,

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$\mathbf{0.4} = \frac{\boldsymbol{P(A \cap B)}}{\mathbf{0.8}}$$

$$P(A \cap B) = 0.4 \times 0.8$$

$$\mathbf{P(A \cap B) = 0.32}$$

(b) $\mathbf{P(A|B)} = \frac{P(A \cap B)}{P(B)}$

$$= \frac{\mathbf{0.32}}{\mathbf{0.5}}$$

$$= \frac{32}{50}$$

$$\mathbf{= 0.64}$$

(c) $\mathbf{P(A \cup B)} = P(A) + P(B) - P(A \cap B)$

$$= 0.8 + 0.5 - \mathbf{0.32}$$

$$= 1.3 - 0.32$$

$$\mathbf{= 0.98}$$

OR

Bag I	Bag II
4 Red (R) 4 Black (B)	2 Red (R) 6 Black (B)

Let B_1 : ball is drawn from Bag I

B_2 : ball is drawn from Bag II

R : ball is drawn is red

We need to find

Probability that ball is drawn from Bag I, if ball is red

$= \mathbf{P(B_1 | R)}$

So, $P(B_1 | R) = \dfrac{P(B_1) \,.\, P(R|B_1)}{P(B_1) \,.\, P(R|B_1) + P(B_2) \,.\, P(R|B_2)}$

$P(B_1)$ = Probability that ball is drawn from Bag I

$= \frac{1}{2}$

$P(R|B_1)$ = Probability that ball is red, if drawn from Bag I

$= \dfrac{4}{4+4} = \dfrac{4}{8} = \dfrac{1}{2}$

$P(B_2)$ = Probability that ball is drawn from Bag II

$= \frac{1}{2}$

$P(R|B_2)$ = Probability that ball is red, if drawn from Bag II

$= \dfrac{2}{2+6} = \dfrac{2}{8} = \dfrac{1}{4}$

Putting values in formula,

$$P(B_1 | R) = \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{1}{2} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{2}}$$

$$= \frac{\frac{1}{4}}{\frac{1}{8}+\frac{1}{4}}$$

$$= \frac{\frac{2}{8}}{\frac{3}{8}}$$

$$= \frac{2}{3}$$

(a) The given relationship is $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$\Rightarrow \sin y = \frac{2x}{1+x^2}$$

Differentiating this relationship with respect to x, we obtain

$$\frac{d}{dx}(\sin y) = \frac{d}{dx}\left(\frac{2x}{1+x^2}\right)$$

$$\Rightarrow \cos y \frac{dy}{dx} = \frac{d}{dx}\left(\frac{2x}{1+x^2}\right) \qquad ...(1)$$

The function, $\frac{2x}{1+x^2}$, is of the form of $\frac{u}{v}$.

Therefore, by quotient rule, we obtain

$$\frac{d}{dx}\left(\frac{2x}{1+x^2}\right)=\frac{\left(1+x^2\right)\cdot\frac{d}{dx}(2x)-2x\cdot\frac{d}{dx}\left(1+x^2\right)}{\left(1+x^2\right)^2}$$

$$=\frac{\left(1+x^2\right).2-2x\cdot[0+2x]}{\left(1+x^2\right)^2}=\frac{2+2x^2-4x^2}{\left(1+x^2\right)^2}=\frac{2\left(1-x^2\right)}{\left(1+x^2\right)^2} \qquad \ldots(2)$$

Also, $\sin y=\frac{2x}{1+x^2}$

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$$\Rightarrow \cos y=\sqrt{1-\sin^2 y}=\sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}=\sqrt{\frac{\left(1+x^2\right)^2-4x^2}{\left(1+x^2\right)^2}}$$

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$$=\sqrt{\frac{\left(1-x^2\right)^2}{\left(1+x^2\right)^2}}=\frac{1-x^2}{1+x^2} \qquad \ldots(3)$$

From (1), (2), and (3), we obtain

$$\frac{1-x^2}{1+x^2}\times\frac{dy}{dx}=\frac{2\left(1-x^2\right)}{\left(1+x^2\right)^2}$$

$$\Rightarrow \frac{dy}{dx}=\frac{2}{1+x^2}$$

(b) $ax + by^2 = \cos y$

Differentiating both sides $w.r.t.x$

$$\frac{d(ax + by^2)}{dx} = \frac{d\,(\cos y)}{dx}$$

$$\frac{d(ax)}{dx} + \frac{d(by^2)}{dx} = \frac{d\,(\cos y)}{dx}$$

$$\text{a}\ \frac{dx}{dx} + b\,\frac{d(y^2)}{dx} = \frac{d}{dx}\cos \text{y}$$

$$\text{a + b}\ \frac{d\,(y^2)}{dx} \times \frac{dy}{dy} = \frac{d\,(\cos y)}{dx} \times \frac{dy}{dy}$$

$$\text{a + b}\ \frac{d\,(y^2)}{dy} \times \frac{dy}{dx} = \frac{d\,(\cos y)}{dy} \times \frac{dy}{dx}$$

$$a + b\,.2y \times \frac{dy}{dx} = -\sin y\,\frac{dy}{dx}$$

$$a + 2by\,.\frac{dy}{dx} = -\sin y\,\frac{dy}{dx}$$

$$2by\,.\frac{dy}{dx} + \sin y\,\frac{dy}{dx} = 0 - a$$

$$\frac{dy}{dx}(2by + \sin y) = -a$$

$$\boldsymbol{\frac{dy}{dx} = \frac{-a}{2by + \sin y}}$$

OR

Step 1: Put in form $\frac{dy}{dx}$ + Py = Q

$$\frac{dy}{dx} + 2y = \sin x \qquad \ldots(1)$$

Step 2: Find P and Q

Comparing (1) with $\frac{dy}{dx}$ + Py = Q

∴ **P = 2** and **Q = sin x**

Step 3: Find integrating factor, IF

$$\text{IF} = e^{\int P dx}$$

$$\text{IF} = e^{\int 2dx}$$

$$\mathbf{IF} = \boldsymbol{e^{2x}}$$

Step 4 : Solution of the equation

$$y \times \text{I.F} = \int Q \times I.F.dx + c$$

Putting values,

$$\boldsymbol{y \times e^{2x} = \int \sin x\, e^{2x} dx + c} \quad ...(2)$$

$$Let\ I = \int \sin x\, e^{2x} dx \quad ...(3)$$

$$I = \int \sin x . e^{2x} . dx$$

Integrating by parts with

$$\int f(x)\, g(x)\, dx = f(x) \int g(x)\, dx - \int [f'(x) \int g(x)\, dx]\, dx$$

Take f (x) = sin x & g (x) = e^{2x}

$$I = \sin x \int \boldsymbol{e^{2x}.dx} - \int \left[\frac{d}{dx} \sin x \int e^{2x} dx\right]$$

$$I = \sin x \frac{e^{2x}}{2} - \int \cos x \frac{e^{2x}}{2} dx$$

Again using by parts with

$\int f(x)\, g(x)\, dx = f(x) \int g(x)\, dx - \int [f'(x) \int g(x)\, dx]\, dx$

Take f (x) = cos x & g(x) = e^{2x}

$$I = \frac{1}{2} \sin x\, e^{2x} - \frac{1}{2}\left[\cos x \int e^{2x}\, dx - \int \frac{d}{dx}\cos x \int e^{2x}\, dx\right] dx$$

$$I = \frac{1}{2} \sin x\, e^{2x} - \frac{1}{2}\left[\cos x \int \frac{e^{2x}}{2} - \int (-\sin x) \int \frac{e^{2x}}{2}\, dx\right]$$

$$I = \frac{1}{2} \sin x\, e^{2x} - \frac{1}{2}\left[\cos x \frac{e^{2x}}{2} + \frac{1}{2} \int \sin x\, e^{2x}\, dx\right]$$

$$I = \frac{1}{2} \sin x\, e^{2x} - \frac{1}{2}\left[\frac{\cos x\, e^{2x}}{2} + \frac{1}{2} I\right] + C \qquad \text{(From 3)}$$

$$I = \frac{1}{2} \sin x\, e^{2x} - \frac{1}{4} \cos x\, e^{2x} - \frac{1}{4} I + C$$

$$I + \frac{1}{4} I = \frac{1}{4}\left[2 \sin x\, e^{2x} - \cos x\, e^{2x}\right] + C$$

$$\frac{5I}{4} = \frac{e^{2x}}{4}\left[2 \sin x - \cos x\right] + C$$

$$I = \frac{e^{2x}}{5}\left[2 \sin x - \cos x\right] + C$$

Now,

Putting value of I in (2)

y $e^{2x} = \frac{e^{2x}}{5}[2\sin x - \cos x] + C$

Dividing by e^{2x}

y = $\frac{1}{5}[2\sin x - \cos x] + Ce^{-2x}$