MATHEMATICS

Time 3 hrs **Max. Marks : 80**

General Instructions :

1. This question paper contains **FIVE SECTIONS – A, B, C, D and E.** Each section is compulsory. However, there are internal choices in some question.
2. **SECTION A** has **18 MCQ's** and **02 Assertion-Reason** based questions of **1 mark** each.
3. **SECTION B** has **5 Very Short Answer (VSA)-type** questions of **2 marks** each.
4. **SECTION C** has **6 Short Answer (SA)-type** questions of **3 marks** each.
5. **SECTION D** has **4 Long Answer (LA)-type** questions of **5 marks** each.
6. **SECTION E** has **3 source based/case based/passage based/integrated units of assessment** (**4 marks** each) with sub parts.

SECTION – A

The following questions are multiple-choice questions with one correct answer. Each question carries 1 mark.

1. If A is a non-singular square matrix of order 3 such that $A^2 = 3A$, then value of $|A|$ is:
(a) –3 (b) 3 (c) 9 (d) 27

2. A line AB in three-dimensional space makes angle 45° and 120° with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals:
(a) 45° (b) 60° (c) 75° (d) 30°

3. Let $\vec{a} = \hat{i} + \alpha\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - \alpha\hat{j} + \hat{k}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $8\sqrt{3}$ square units, then $\vec{a}\cdot\vec{b}$ is:
(a) 3 (b) 2 (c) 4 (d) 1

4. If $A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $M = AB$, then M^{-1} is equal to-

(a) $\begin{bmatrix} 2 & -2 \\ 2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1/3 & 1/3 \\ -1/3 & 1/6 \end{bmatrix}$ (c) $\begin{bmatrix} 1/3 & -1/3 \\ 1/3 & 1/6 \end{bmatrix}$ (d) $\begin{bmatrix} 1/3 & -1/3 \\ -1/3 & 1/6 \end{bmatrix}$

5. $\int_0^{\pi} \cos x\, e^{\sin x} dx$ is equal to:
(a) e + 1 (b) e – 1 (c) 1 (d) 0

6. If y = sin (m $\sin^{-1}$x), then which one of the following equation is true?

(a) $(1-x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}+m^2y=0$

(b) $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+m^2y=0$

(c) $(1+x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-m^2y=0$

(d) $(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-m^2x=0$

7. If m and n are the order and degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^5+4\frac{\left(\frac{d^2y}{dx^2}\right)^3}{\left(\frac{d^3y}{dx^3}\right)}+\frac{d^3y}{dx^3}=x^2-1,\text{ then } m+n \text{ is}$$

(a) 8 (b) 4 (c) 6 (d) 5

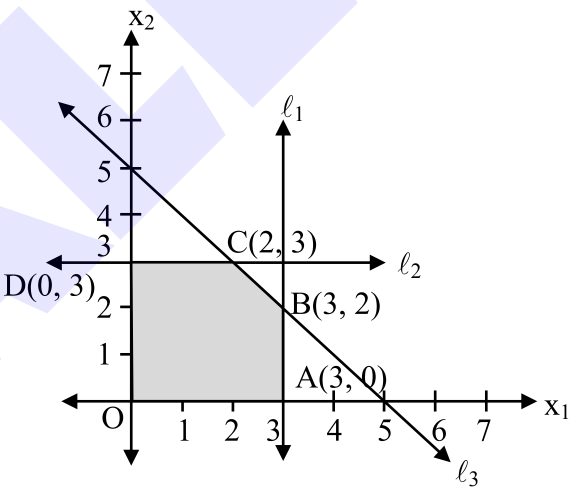
8. If $\int \frac{f(x)\,dx}{\log\sin x}=\log\left[\log(\sin x)\right]$, then $f(x)$ is

(a) sin x (b) cos x (c) log sin x (d) cot x

9. If x = – 4 is a root of $\begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix}$ = 0, then the sum of the other two roots is :

(a) 4 (b) –3 (c) 2 (d) 5

10. The corner points of the feasible region determined by the system of linear inequalities are :



(a) (0, 0), (–3, 0), (3, 2), (2, 3)

(b) (3, 0), (3, 2), (2, 3), (0, –3)

(c) (0, 0), (3, 0), (3, 2), (2, 3), (0, 3)

(d) None of these

11. The solution of differential equation xdy – ydx = 0 represents:

(a) a rectangular hyperbola

(b) parabola whose vertex is at origin

(c) straight line passing through origin

(d) a circle whose centre is at origin

12. Two events E and F are independent. If P(E) = 0.3 and P(E ∪ F) = 0.5, then P(E/F) – P(F/E) equals to :

(a) $\frac{2}{7}$ (b) $\frac{3}{35}$ (c) $\frac{1}{70}$ (d) $\frac{1}{7}$

13. If A is square matrix of order 3 and B = A' such that |A| = – 4, then |AB| is equal to :

(a) 9 (b) 12 (c) 16 (d) – 16

14. Let $\vec{a}$ and $\vec{b}$ be two non-zero vectors perpendicular to each other and $|\vec{a}| = |\vec{b}|$. If $|\vec{a} \times \vec{b}| = |\vec{a}|$, then the angle between the vectors $\left(\vec{a} + \vec{b} + \left(\vec{a} \times \vec{b}\right)\right)$ and $\vec{a}$ is :

(a) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (b) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (c) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (d) $\sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$

15. Corner points of the feasible region determined by the system of linear constraints are (0, 3), (1, 1) and (3, 0). Let Z = px + qy, where p, q > 0. Condition on p and q, so that the minimum of Z occurs at (3,0) and(l, 1) is

(a) p = 2q (b) $p = \frac{q}{2}$ (c) p = 3q (d) p = q

16. Let I be an identity matrix of order 2 × 2 and $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$. Then the value of K, for which $P^6 = KI - 8P$, where $K \in N$ is.

(a) –6 (b) –5 (c) 5 (d) 6

17. The area of a triangle formed by vertices O, A and B, where $\overrightarrow{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\overrightarrow{OB} = -3\hat{i} - 2\hat{j} + \hat{k}$ is:

(a) $3\sqrt{5}$ sq. units (b) $5\sqrt{5}$ sq. units (c) $6\sqrt{5}$ sq. units (d) 4 sq. units

18. If $f(x) = \begin{cases} \frac{|x-1|}{1-x} + a, & x > 1 \\ a+b, & x = 1 \\ \frac{|x-1|}{1-x} + b, & x < 1 \end{cases}$ is continuous at x = 1, then the values of 'a' and 'b' are respectively:

(a) 1, 1 (b) 1, –1 (c) 2, 3 (d) None of these

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A) :** The angle between the lines whose direction cosines are given by the equations $3\ell + m + 5n = 0$ and $6mn - 2n\ell + 5\ell m = 0$ is $\cos^{-1}\left(\frac{1}{6}\right)$.

Reason (R) : An angle between two lines is given by $\cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\ \sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$,

where a_1, b_1, c_1 and a_2, b_2, c_2 are direction ratio's of lines.

20. **Assertion (A) :** The domain of the function $\sec^{-1}(2x + 1)$ is $\left(-\infty, \frac{1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$

Reason (R) : $\sec^{-1}(-2) = \frac{2\pi}{3}$

SECTION – B

This section comprises of very short answer type-questions (VSA) of 2 marks each

21. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $\vec{a} \neq \vec{0}$ and $\vec{a} \times \vec{b} = 2\vec{a} \times \vec{c}$, $|\vec{a}| = |\vec{c}| = 1$, $|\vec{b}| = 4$ and $|\vec{b} \times \vec{c}| = \sqrt{15}$. If $\vec{b} - 2\vec{c} = \lambda\vec{a}$, then find the value of λ.

OR

A line makes the same angle θ, with each of the x and z–axis. If the angle β, which it makes with y-axis is such that $\sin^2\beta = 3\sin^2\theta$, then find the value of $\cos^2\theta$.

22. If $y = x \sin y$ then prove that $x\frac{dy}{dx} = \frac{y}{1 - x\cos y}$.

23. A spherical ball of salt is dissolving in water in such a manner that the rate of decrease of volume at any instant is proportional to the surface. Prove that the radius is decreasing at a constant rate.

24. Find the vector of magnitude $\sqrt{171}$ which is perpendicular to both of the vectors $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$.

25. Find the value of $\sin^{-1}\left[\cos\left\{\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right\}\right]$?

OR

Show that the function f: $R \to \{x \in R : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in R$ is one-one.

SECTION – C

This section comprises of short answer type questions (SA) of 3 marks each

26. Evaluate : $\int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$

27. A and B throw a pair of dice alternately, till one of them gets a total of 10 and wins the game. Find their respective probabilities of winning, if A starts first.

28. In a bank, principal increases continuously at the rate of 5% per year. In how many years Rs. 1000 double itself ?

OR

Show that the differential equation $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$ is homogenous and solve it.

29. Solve the linear programming problem graphically

Minimize $Z = 6x + 21y$,

subject to constraints $x + 2y \le 3$,

$x + 4y \ge 4$, $3x + y \ge 3$,

$x \ge 0$, $y \ge 0$

30. Evaluate: $\int_0^{\pi/2} \frac{\cos x}{1 + \cos x + \sin x} dx$

OR

Evaluate: $\int_0^{\pi/2} \left|\sin\left(x - \frac{\pi}{4}\right)\right| dx$

31. Evaluate: $\int \frac{(3x+5)}{(x^3 - x^2 - x + 1)} dx$

SECTION – D

This section comprises of long answer-type questions (LA) of 5 marks each

32. If $A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6 \end{bmatrix}$, find A^{-1}.

Hence, Solve the following system of equations.

$3x + 4y + 2z = 8$

$2y - 3z = 3$

$x - 2y + 6z = -2$

33. Using the method of integration, find the area of the triangular region whose vertices are (2, –2), (4, 3) and (1, 2).

34. Find the shortest distance and the vector equation of the line of shortest distance of the lines

given by $\vec{r} = 3\hat{i} + 8\hat{j} + 3\hat{k} + \lambda(3\hat{i} - \hat{j} + \hat{k})$

and $\vec{r} = -3\hat{i} - 7\hat{j} + 6\hat{k} + \mu(-3\hat{i} + 2\hat{j} + 4\hat{k})$

OR

Two insects are crawling along different lines in three-dimension. At time t (min.) the first insect is at point (x, y, z) on the line x = 6 + t, y = 8 – t and z = 3 + t. also at time t (min.) the second insect is at the point (x, y, z) on the line x = 1 + t, y = 2 + t, z = 2t. Assume that the distance are given in inches. How for apart are the insects at t = 6 min.? What is the closest the 2 insect will ever get to each other?

35. Let N denote the set of all natural numbers and R be the relation on N × N by (a, b) R (c, d) ⇔ ad(b + c) = bc(a + d). Check whether R is an equivalence relation on N × N.

OR

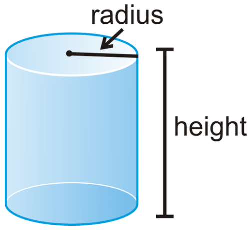
Let A = {x ∈ Z : 0 ≤ x ≤ 12}. Show that R = {(a, b) : a, b ∈ A, |a – b| is divisible by 4} is an equivalence relation. Find the set of all elements related to 1. Also find the equivalence class of 2.

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)

36. **CASE-STUDY : I**

A person has manufactured a water tank in the shape of a closed right circular cylinder. The volume of the cylinder is $\frac{539}{2}$ cubic units. If the height and radius of the cylinder be h and r, then



Based on the above information, answer the following questions :

(i) Find the total surface area function (S) of tank in terms of r.

(ii) Find the critical points of the functions.

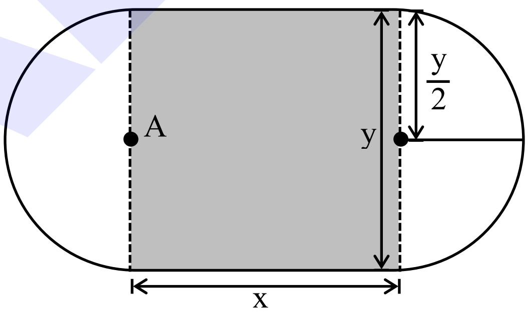
(iii) Use first order derivative test to find the value of r and h, when surface area of the tank is minimum.

OR

(iii) Use second order derivative test to find the value of r and h, when surface area of the tank is minimum.

37. **CASE-STUDY : II**

An architect designs a building for a multi-national company. The floor consists of a rectangular region with semi-circular ends having a perimeter of 200 m as shown below.



Design of floor

Based on the above information answer the following:

(i) If x and y represents the length and breadth of the rectangular region, then what is the relation between variables x and y.

(ii) Express the area of the rectangular region A as a function of x.

(iii) Find the maximum value of area A.

OR

(iii) The CEO of the multi-national company is interested in maximizing the area of the whole floor including the semi-circular ends, find the maximum area of whole floor.

38. CASE-STUDY : III

At the start of a cricket match, a coin is tossed and the team winning the toss has the opportunity to choose to bat or bowl. Such a coin is unbiased with equal probabilities of getting head and tail.

Based on the above information, answer the following questions :

(i) If such a coin is tossed 2 times, then find the probability distribution of number of tails

(ii) Find the conditional probability of getting at least two heads in three tosses of such a coin, if it is know that we get atmost two heads.