

MATHEMATICS

SECTION – A

1. (d)

Given $A^2 = 3A$, $|A| \neq 0$, order of A is 3

$$\therefore |A^2| = |3A|$$

$$|A|^2 = 3^3 |A| \quad (|A^2| = |A|^2 \quad \& \quad |KA| = K^n |A|)$$

$$|A| = 27$$

2. (b)

As per question, direction cosines of the line :

$$\ell = \cos 45^\circ = \frac{1}{\sqrt{2}}, m = \cos 120^\circ = \frac{-1}{2}, n = \cos \theta$$

where θ is the angle, which line makes with positive z-axis.

We know that, $\ell^2 + m^2 + n^2 = 1$

$$\Rightarrow \frac{1}{2} + \frac{1}{4} + \cos^2 \theta = 1$$

$$\cos^2 \theta = \frac{1}{4}$$

$$\Rightarrow \cos \theta = \frac{1}{2} = \cos \frac{\pi}{3} \quad (\theta \text{ being acute})$$

$$\Rightarrow \theta = \frac{\pi}{3}$$

3. (b)

$$\vec{a} = \hat{i} + \alpha \hat{j} + 3 \hat{k}$$

$$\vec{b} = 3 \hat{i} - \alpha \hat{j} + \hat{k}$$

$$\text{area of parallelogram} = |\vec{a} \times \vec{b}| = 8\sqrt{3} .$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & \alpha & 3 \\ 3 & -\alpha & 1 \end{vmatrix} = \hat{i}(4\alpha) - \hat{j}(-8) + \hat{k}(-4\alpha)$$

$$\therefore |\vec{a} \times \vec{b}| = \sqrt{64 + 32\alpha^2} = 8\sqrt{3}$$

$$\Rightarrow 2 + \alpha^2 = 6 \Rightarrow \alpha^2 = 4$$

$$\therefore \vec{a} \cdot \vec{b} = 3 - \alpha^2 + 3 = 2$$

4. (c)

$$M = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -2 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -2 & 2 \end{bmatrix}$$

$$|M| = 6, \text{adj } M = \begin{bmatrix} 2 & -2 \\ 2 & 1 \end{bmatrix}$$

$$\text{Now } M^{-1} = \frac{\text{adj } M}{|M|}$$

$$\therefore M^{-1} = \frac{1}{6} \begin{bmatrix} 2 & -2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1/3 & -1/3 \\ 1/3 & 1/6 \end{bmatrix}$$

5. (d)

$$\text{Let } I = \int_0^{\pi} \cos x e^{\sin x} dx \quad \dots(\text{i})$$

$$I = \int_0^{\pi} -\cos x e^{\sin x} dx \quad \dots(\text{ii}) \quad (\text{By property } \int_0^a f(x)dx = \int_0^a f(a-x)dx)$$

Adding (i) and (ii)

$$2I = 0$$

$$I = 0$$

6. (b)

$$y = \sin(m \sin^{-1} x) \quad \dots(\text{i})$$

$$\Rightarrow \frac{dy}{dx} = \cos(m \sin^{-1} x) \times \left(\frac{m}{\sqrt{1-x^2}} \right) \quad (\text{Differentiating w.r.t. } x)$$

$$\Rightarrow \sqrt{1-x^2} \frac{dy}{dx} = m \cos(m \sin^{-1} x)$$

(Again differentiating w.r.t. x)

$$\therefore \sqrt{1-x^2} \frac{d^2y}{dx^2} + \left(\frac{dy}{dx} \right) \cdot \frac{1}{2\sqrt{1-x^2}} \cdot (-2x) = -m^2 \sin(m \sin^{-1} x) \times \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow (1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + m^2 y = 0 \quad (\text{from (i)})$$

7. (d)

$$\text{Given differential equation is } \left(\frac{d^2y}{dx^2} \right)^5 + 4 \frac{\left(\frac{d^2y}{dx^2} \right)^3}{\left(\frac{d^3y}{dx^3} \right)} + \frac{d^3y}{dx^3} = x^2 - 1$$

$$\Rightarrow \left(\frac{d^2y}{dx^2} \right)^5 \left(\frac{d^3y}{dx^3} \right) + 4 \left(\frac{d^2y}{dx^2} \right)^3 + \left(\frac{d^3y}{dx^3} \right)^2 = (x^2 - 1) \left(\frac{d^3y}{dx^3} \right)$$

The highest order derivative in the given equation is $\frac{d^3y}{dx^3}$ and its highest power is 2 Therefore

$m = 3$ and $n = 2$.

$$m + n = 3 + 2 = 5$$

8. (d)

$$\text{Given that } \int \frac{f(x) dx}{\log \sin x} = \log [\log(\sin x)]$$

Differentiating both sides w.r.t. x , we get

$$\frac{f(x)}{\log \sin x} = \frac{\cot x}{\log \sin x} \Rightarrow f(x) = \cot x.$$

9. (a)

$$\text{Given, } \begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0$$

$$\Rightarrow x(x^2 - 2) - 2(x - 3) + 3(2 - 3x) = 0$$

$$\Rightarrow x^3 - 2x - 2x + 6 + 6 - 9x = 0$$

$$\Rightarrow x^3 - 13x + 12 = 0$$

$$\therefore (x + 4)(x^2 - 4x + 3) = 0 \quad [\because x = -4 \text{ is a root}]$$

$$\Rightarrow (x + 4)(x - 1)(x - 3) = 0$$

Hence ; the sum of other two roots = $1 + 3 = 4$

10. (c)

The corner points of the feasible region are $(0, 0)$, $(3, 0)$, $(3, 2)$, $(2, 3)$ and $(0, 3)$

11. (c)

$$\text{Given, } x dy - y dx = 0$$

$$\Rightarrow x dy = y dx \Rightarrow \frac{1}{y} dy = \frac{1}{x} dx$$

Integrating both sides

$$\Rightarrow \int \frac{dy}{y} = \int \frac{dx}{x}$$

$$\log y = \log x + \log c$$

$y = cx \quad \therefore$ it represents a straight line passing through origin.

12. (c)

Given ; E and F are independent i.e. $P(E \cap F) = P(E).P(F)$

$$P(E \cup F) = P(E) + P(F) - P(E).P(F)$$

$$\Rightarrow 0.5 = 0.3 + P(F) - 0.3 \times P(F)$$

$$\Rightarrow 0.7 P(F) = 0.2 \quad \therefore P(F) = \frac{2}{7}$$

Now ; $P(E/F) - P(F/E)$

$$\begin{aligned} &= \frac{P(E \cap F)}{P(F)} - \frac{P(F \cap E)}{P(E)} = \frac{P(E).P(F)}{P(F)} - \frac{P(E).P(F)}{P(E)} \\ &= P(E) - P(F) = \frac{3}{10} - \frac{2}{7} = \frac{1}{70} \end{aligned}$$

13. (c)

$$\because B = A' \Rightarrow |B| = |A'| = |A|$$

$$\therefore |AB| = |A||B| = (-4)(-4) = 16$$

14. (b)

Given, $|\vec{a}| = |\vec{b}|$, $|\vec{a} \times \vec{b}| = |\vec{a}|$ and $\vec{a} \perp \vec{b}$

$$\text{Now, } |\vec{a} \times \vec{b}| = |\vec{a}| \Rightarrow |\vec{a}||\vec{b}|\sin 90^\circ = |\vec{a}| \Rightarrow |\vec{b}| = 1 = |\vec{a}|$$

$\therefore \vec{a}$ and \vec{b} are mutually perpendicular unit vectors.

Let θ be the angle between $(\vec{a} + \vec{b} + (\vec{a} \times \vec{b}))$ and \vec{a}

$$\text{Let } \vec{a} = \hat{i}, \vec{b} = \hat{j}$$

$$\therefore \vec{a} \times \vec{b} = \hat{k}$$

$$\cos \theta = \frac{(\hat{i} + \hat{j} + \hat{k}) \cdot \hat{i}}{\sqrt{3} \sqrt{1}} = \frac{1}{\sqrt{3}} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

15. (b)

Given ; $Z = px + qy$

Since the minimum value of Z occurs at $(3, 0)$ and $(1, 1)$:

$$\Rightarrow 3p = p + q \text{ or } p = \frac{q}{2}$$

16. (c)

$$\text{Given, } P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$$

$$KI - 8P = \begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix} - \begin{bmatrix} 16 & -8 \\ 40 & -24 \end{bmatrix} = \begin{bmatrix} K-16 & 8 \\ -40 & K+24 \end{bmatrix} \quad \dots(i) \quad \left(\because I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right)$$

$$P^2 = P \times P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix} \times \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -5 & 4 \end{bmatrix}$$

$$P^3 = P^2 \times P = \begin{bmatrix} -1 & 1 \\ -5 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 10 & -7 \end{bmatrix}$$

$$P^6 = P^3 \times P^3 = \begin{bmatrix} 3 & -2 \\ 10 & -7 \end{bmatrix} \times \begin{bmatrix} 3 & -2 \\ 10 & -7 \end{bmatrix} = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix} \quad \dots(\text{ii})$$

from (i) & (ii)

$$\therefore P^6 = KI - 8P$$

$$\begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix} = \begin{bmatrix} K-16 & 8 \\ -40 & K+24 \end{bmatrix} \Rightarrow -11 = K - 16 \Rightarrow K = 5$$

R

17. (a)

$$\text{Given, } \overrightarrow{OA} = \hat{i} + 2\hat{j} + 3\hat{k}, \quad \overrightarrow{OB} = -3\hat{i} - 2\hat{j} + \hat{k}$$

$$\text{Area of } \Delta OAB = \frac{1}{2} |\overrightarrow{OA} \times \overrightarrow{OB}|$$

$$\text{Now, } \overrightarrow{OA} \times \overrightarrow{OB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -3 & -2 & 1 \end{vmatrix} = 8\hat{i} - 10\hat{j} + 4\hat{k}$$

$$\Rightarrow |\overrightarrow{OA} \times \overrightarrow{OB}| = \sqrt{64+100+16} = \sqrt{180}$$

$$\text{So area of } \Delta OAB = \frac{1}{2} \sqrt{180} = 3\sqrt{5} \text{ sq. units}$$

18. (b)

$$\text{Given that } f(x) = \begin{cases} \frac{|x-1|}{1-x} + a, & x > 1 \\ a+b, & x=1 \\ \frac{|x-1|}{1-x} + b, & x < 1 \end{cases}$$

$\because f(x)$ is continuous at $x = 1$; therefore, $\lim_{h \rightarrow 0} f(1+h) = \lim_{h \rightarrow 0} f(1-h) = f(1)$

$$\therefore f(1) = a + b \text{ (given)}$$

$$\text{RHL} = \lim_{h \rightarrow 0} f(1+h) = \lim_{h \rightarrow 0} \frac{|1+h-1|}{1-(1+h)} + a = -1 + a$$

$$\Rightarrow a + b = -1 + a \Rightarrow b = -1$$

$$\text{LHL} = \lim_{h \rightarrow 0} f(1-h) = \lim_{h \rightarrow 0} \frac{|1-h-1|}{1-(1-h)} + b = 1 + b$$

$$\therefore a + b = 1 + b \Rightarrow a = 1$$

19. (a)

Given equations $3\ell + m + 5n = 0$... (i)

$$6mn - 2n\ell + 5\ell m = 0$$

$$m(6n + 5\ell) - 2n\ell = 0 \quad \dots \text{(ii)}$$

From (i) $m = -3\ell - 5n$... (iii)

Putting in (ii) $\Rightarrow -(6n + 5\ell)(3\ell + 5n) - 2n\ell = 0$

$$\Rightarrow 30n^2 + 45\ell n + 15\ell^2 = 0$$

$$\Rightarrow 2n^2 + 3n\ell + \ell^2 = 0$$

$$\Rightarrow (2n + \ell)(n + \ell) = 0$$

\therefore either $\ell = -2n$ or $\ell = -n$

$m = n$ or $m = -2n$ from (iii)

\therefore Direction numbers of the two lines are

$$<-2n, n, n> \text{ and } <-n, -2n, n>$$

i.e., $<-2, 1, 1>$ and $<-1, -2, 1>$

if θ is the acute angle between the lines, then

$$\cos \theta = \frac{|(-2) \times (-1) + 1 \times (-2) + 1 \times 1|}{\sqrt{4+1+1} \sqrt{1+4+1}} = \frac{1}{6} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{6}\right)$$

Hence, Both A and R are true and R is the correct explanation of A

20. (d)

Assertion : Domain of $\sec^{-1}x$ is $(-\infty, -1] \cup [1, \infty)$

$\therefore \sec^{-1}(2x + 1)$ is meaningful, if

$$2x + 1 \geq 1 \text{ or } 2x + 1 \leq -1$$

$$x \geq 0 \text{ or } x \leq -1$$

$$\therefore x \in (-\infty, -1] \cup [0, \infty)$$

So, Assertion is false

$$\sec^{-1}(-2) = \pi - \sec^{-1}(2) = \pi - \frac{\pi}{3} = \frac{2\pi}{3} \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$$

So, reason is true.

SECTION – B

21. Let angle between \vec{b} and \vec{c} is α and given $|\vec{b} \times \vec{c}| = \sqrt{15}$

$$\Rightarrow |\vec{b}| |\vec{c}| \sin \alpha = \sqrt{15}$$

$$\sin \alpha = \frac{\sqrt{15}}{4}; \therefore \cos \alpha = \frac{1}{4}$$

[1½]

$$\text{Now, } \vec{b} - 2\vec{c} = \lambda \vec{a}$$

$$\Rightarrow |\vec{b} - 2\vec{c}|^2 = \lambda^2 |\vec{a}|^2$$

$$\left[\because \vec{a}^2 = |\vec{a}|^2 \right]$$

$$|\vec{b}|^2 + 4|\vec{c}|^2 - 4(\vec{b} \cdot \vec{c}) = \lambda^2 |\vec{a}|^2$$

[1]

$$16 + 4 - 4\{|\vec{b}| |\vec{c}| \cos \alpha\} = \lambda^2$$

$$16 + 4 - 4 \times 4 \times 1 \times \frac{1}{4} = \lambda^2 \Rightarrow \lambda^2 = 16 \Rightarrow \lambda = \pm 4.$$

[1½]

OR

Here, $\ell = \cos \theta, m = \cos \beta, n = \cos \theta, (\because \ell = n)$

$$\text{Now, } \ell^2 + m^2 + n^2 = 1 \Rightarrow 2 \cos^2 \theta + \cos^2 \beta = 1$$

[1½]

$$\Rightarrow 2 \cos^2 \theta = \sin^2 \beta \quad [\text{Given, } \sin^2 \beta = 3 \sin^2 \theta]$$

$$\Rightarrow 2 \cos^2 \theta = 3 \sin^2 \theta$$

$$\Rightarrow 2 \cos^2 \theta - 3(1 - \cos^2 \theta) = 0$$

$$\Rightarrow 5 \cos^2 \theta = 3,$$

$$\therefore \cos^2 \theta = \frac{3}{5}$$

[1½]

22. Given, $y = x \sin y$; differentiating both sides w.r.t. x.

$$\frac{dy}{dx} = x \cos y \frac{dy}{dx} + \sin y \cdot 1$$

$$\Rightarrow \frac{dy}{dx} - x \cos y \frac{dy}{dx} = \sin y$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin y}{1 - x \cos y}$$

[1]

$$\text{or } x \frac{dy}{dx} = \frac{x \sin y}{1 - x \cos y}, \text{ (multiplying both sides by } x)$$

$$\Rightarrow x \frac{dy}{dx} = \frac{y}{1 - x \cos y} \quad (\because x \sin y = y)$$

[1]

23. Let V , S and r denote the volume, surface area and radius of the salt ball respectively at any instant t .

$$\text{Then } V = \frac{4}{3}\pi r^3 \text{ and } S = 4\pi r^2$$

It is given that the rate of decrease of the volume V is proportional to the surface area S .

i.e. $\frac{dV}{dt} \propto S$ or $\frac{dV}{dt} = -KS$, where $K > 0$ is the constant of proportionality.

$$\frac{dV}{dt} = -KS$$

[1]

$$\Rightarrow \frac{d}{dt} \left(\frac{4}{3}\pi r^3 \right) = -K(4\pi r^2)$$

$$\Rightarrow 4\pi r^2 \frac{dr}{dt} = -4\pi K r^2$$

$$\Rightarrow \frac{dr}{dt} = -K$$

So, r decrease with a constant rate

[1]

24. Let required vector $\vec{c} = \lambda(\vec{a} \times \vec{b})$

$$\Rightarrow \vec{c} = \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 3 & -1 & 2 \end{vmatrix}$$

$$\text{or } \vec{c} = \lambda(\hat{i} - 11\hat{j} - 7\hat{k}) \quad \dots(i)$$

[1]

$$\Rightarrow |\vec{c}| = |\lambda| \sqrt{1+121+49}$$

$$\Rightarrow \sqrt{171} = |\lambda| \sqrt{171} \quad (\text{given } |\vec{c}| = \sqrt{171})$$

$$\Rightarrow \lambda = \pm 1 \text{ put in equation (i), we have}$$

$$\vec{c} = \pm(\hat{i} - 11\hat{j} - 7\hat{k})$$

[1]

$$25. \sin^{-1} \left[\cos \left\{ \sin^{-1} \left(-\frac{\sqrt{3}}{2} \right) \right\} \right]$$

$$= \sin^{-1} \left\{ \cos \left(\frac{-\pi}{3} \right) \right\}$$

[½]

$$= \sin^{-1} \left(\cos \left(\frac{\pi}{3} \right) \right)$$

$[\because \cos(-\theta) = \cos \theta]$

[½]

$$= \sin^{-1} \left(\frac{1}{2} \right) = \frac{\pi}{6}$$

[1]

OR

Given; $f : R \rightarrow \{x \in R : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in R$

$$\therefore f(x) = \begin{cases} \frac{x}{1+x} & ; \quad x \geq 0 \\ \frac{x}{1-x} & ; \quad x < 0 \end{cases}$$

For one-one function: Let $f(x_1) = f(x_2)$

$$\Rightarrow \frac{x_1}{1+|x_1|} = \frac{x_2}{1+|x_2|}$$

Case I : When x_1 & x_2 are positive:

$$f(x_1) = f(x_2)$$

$$\frac{x_1}{1+x_1} = \frac{x_2}{1+x_2} \Rightarrow x_1 = x_2$$

[½]

Case II : When x_1 & x_2 are negative:

$$\frac{x_1}{1-x_1} = \frac{x_2}{1-x_2} \Rightarrow x_1 = x_2$$

[½]

Case III : When $x_1 > 0$ and $x_2 < 0$

We have $f(x_1) = f(x_2)$

$$\Rightarrow \frac{x_1}{1+x_1} = \frac{x_2}{1-x_2}$$

$$\Rightarrow x_1 - x_1 x_2 = x_2 + x_1 x_2$$

$$\Rightarrow x_1 - x_2 = 2x_1 x_2$$

This is not possible when $x_1 > 0$ and $x_2 < 0$

$$\therefore x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$$

$\therefore f$ is one-one

SECTION – C

26. Let $I = \int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$

Put $\sin x - \cos x = t \Rightarrow (\cos x + \sin x)dx = dt$

[½]

When $x = \frac{\pi}{6}$, $t = \frac{(1-\sqrt{3})}{2}$ & when $x = \frac{\pi}{3}$, $t = \frac{(\sqrt{3}-1)}{2}$

also, $\sin x - \cos x = t \Rightarrow \sin^2 x + \cos^2 x - 2\sin x \cos x = t^2$

[½]

$$\Rightarrow \sin 2x = 1 - t^2$$

$$\therefore I = \int_{\frac{(1-\sqrt{3})}{2}}^{\frac{(\sqrt{3}-1)}{2}} \frac{1}{\sqrt{1-t^2}} dt$$

$$= \left[\sin^{-1} t \right]_{(1-\sqrt{3})/2}^{(\sqrt{3}-1)/2} \quad [1]$$

$$= \sin^{-1} \left(\frac{\sqrt{3}-1}{2} \right) - \sin^{-1} \left(\frac{1-\sqrt{3}}{2} \right)$$

$$= 2 \sin^{-1} \left(\frac{\sqrt{3}-1}{2} \right) \quad [1]$$

27. Let E_1 and E_2 be the events of getting a total 10 by A and B respectively
favorable outcomes of total 10 is $\{(6, 4), (5, 5), (4, 6)\}$

$$\text{Total outcomes} = 6^2 = 36$$

$$\text{Now } P(E_1) = P(E_2) = \frac{3}{36} = \frac{1}{12}$$

$$P(\bar{E}_1) = P(\bar{E}_2) = \frac{11}{12} \quad [1]$$

$$P(A \text{ wins}) = P(E_1) + P(\bar{E}_1)P(\bar{E}_2)P(E_1) + \dots$$

$$= \frac{1}{12} + \frac{1}{12} \times \left(\frac{11}{12} \right)^2 + \frac{1}{12} \times \left(\frac{11}{12} \right)^4 + \dots$$

$$= \frac{1}{12} \left[1 + \underbrace{\left(\frac{11}{12} \right)^2 + \left(\frac{11}{12} \right)^4 + \dots}_{\text{Infinite G.P.}} \right] = \frac{1}{12} \left[\frac{1}{1 - \frac{11^2}{12^2}} \right] \quad \left(\because S_\infty = \frac{a}{1-r} \right) \quad [1]$$

$$P(A \text{ wins}) = \frac{1}{12} \times \left[\frac{12^2}{144 - 121} \right] = \frac{12}{23}$$

$$P(B \text{ wins}) = 1 - P(\text{win A}) = 1 - \frac{12}{23} = \frac{11}{23} \quad [1]$$

28. Let P be the principal at any time t. According to the given problem,

$$\frac{dP}{dt} = \left(\frac{5}{100} \right) \times P$$

$$\text{or } \frac{dP}{dt} = \frac{P}{20} \quad \dots\dots(i) \quad [1]$$

Separating the variables in equation (i), we get

$$\frac{dP}{P} = \frac{dt}{20} \quad \dots\dots(ii)$$

Integrating both sides of equation (ii), we get

$$\log P = \frac{t}{20} + C_1$$

$$\text{or } P = e^{\frac{t}{20}} \cdot e^{C_1}$$

$$\text{or } P = Ce^{\frac{t}{20}} \text{ (where } e^{C_1} = C) \quad \dots\dots(iii) \quad [1]$$

Now $P = 1000$, when $t = 0$

Substituting the values of P and t in (iii), we get $C = 1000$. Therefore, equation (iii), gives

$$P = 1000e^{\frac{t}{20}}$$

Let t years be the time required to double the principal. Then

$$2000 = 1000e^{\frac{t}{20}} \Rightarrow t = 20 \log_e 2 \quad [1]$$

OR

$$\text{The given differential equation can be written as } \frac{dy}{dx} = \frac{ycos\left(\frac{y}{x}\right) + x}{x cos\left(\frac{y}{x}\right)} \quad \dots\dots(i)$$

$$\text{Let } F(x, y) = \frac{ycos\left(\frac{y}{x}\right) + x}{x cos\left(\frac{y}{x}\right)} = \frac{y}{x} + \sec\left(\frac{y}{x}\right) = f\left(\frac{y}{x}\right) \quad [1]$$

Thus, $F(x, y)$ is a homogeneous function of degree zero.

Therefore, the given differential equation is a homogeneous differential equation.

To solve it we make the substitution

$$y = vx \quad \dots\dots(ii)$$

Differentiating equation (ii) with respect to x , we get

$$\frac{dy}{dx} = v + x \frac{dv}{dx} \quad \dots\dots(iii)$$

Substituting the value of y and $\frac{dy}{dx}$ in equation (i), we get

$$\begin{aligned} v + x \frac{dv}{dx} &= \frac{v \cos v + 1}{\cos v} \\ \Rightarrow x \frac{dv}{dx} &= \frac{v \cos v + 1}{\cos v} - v \end{aligned}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{1}{\cos v}$$

$$\Rightarrow \cos v dv = \frac{1}{x} dx \quad (\text{Separating variables})$$

[1]

Integrate both sides

$$\Rightarrow \int \cos v dv = \int \frac{1}{x} dx$$

$$\Rightarrow \sin v = \log|x| + \log|C|$$

$$\Rightarrow \sin v = \log|Cx|$$

$$\text{Replacing } v \text{ by } \frac{y}{x}, \text{ we get } \sin\left(\frac{y}{x}\right) = \log|Cx|$$

[1]

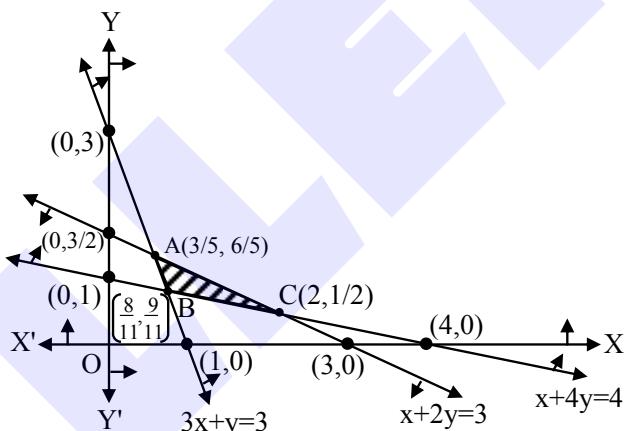
Which is the general solution of the differential equation (i).

29. Minimize $Z = 6x + 21y$

subject to constraints

$$x + 2y \leq 3, x + 4y \geq 4$$

$$3x + y \geq 3, x \geq 0, y \geq 0$$



[1]

Corner points	$Z = 6x + 21y$
$A\left(\frac{3}{5}, \frac{6}{5}\right)$	$Z_A = \frac{144}{5} = 28.8$
$B\left(\frac{8}{11}, \frac{9}{11}\right)$	$Z_B = \frac{237}{11} = 21.54$
$C\left(2, \frac{1}{2}\right)$	$Z_C = \frac{45}{2} = 22.5$

[1½]

Since, feasible region is bounded, so

Minimum value of Z is 21.54 at point $B\left(\frac{8}{11}, \frac{9}{11}\right)$

[½]

30.
$$\int_0^{\pi/2} \frac{\cos x}{1 + \cos x + \sin x} dx$$

$$= \int_0^{\pi/2} \frac{\cos^2(x/2) - \sin^2(x/2)}{2\cos^2(x/2) + 2\sin(x/2)\cos(x/2)} dx \quad [1/2]$$

$$= \frac{1}{2} \int_0^{\pi/2} \frac{1 - \tan^2(x/2)}{1 + \tan(x/2)} dx \quad \left[\text{Divide Nr & Dr by } \cos^2 \frac{x}{2} \right]$$

$$= \frac{1}{2} \int_0^{\pi/2} \left[1 - \tan\left(\frac{x}{2}\right) \right] dx \quad [1/2]$$

$$= \frac{1}{2} \left[x + 2 \log \cos \frac{x}{2} \right]_0^{\pi/2} \quad [1\frac{1}{2}]$$

$$= \frac{\pi}{4} + \log \frac{1}{\sqrt{2}}. \quad [1/2]$$

OR

Let $I = \int_0^{\pi/2} \left| \sin\left(x - \frac{\pi}{4}\right) \right| dx$

$x - \frac{\pi}{4}$ is $-ve$ when $x \leq \frac{\pi}{4}$ and $+ve$ when $x > \frac{\pi}{4}$ [1/2]

$$\therefore I = - \int_0^{\pi/4} \sin\left(x - \frac{\pi}{4}\right) dx + \int_{\pi/4}^{\pi/2} \sin\left(x - \frac{\pi}{4}\right) dx \quad [1/2]$$

$$= \left[\cos\left(x - \frac{\pi}{4}\right) \right]_0^{\pi/4} - \left[\cos\left(x - \frac{\pi}{4}\right) \right]_{\pi/4}^{\pi/2} = \left(1 - \frac{1}{\sqrt{2}} \right) - \left(\frac{1}{\sqrt{2}} - 1 \right) \quad [1\frac{1}{2}]$$

$$= 2 - \sqrt{2}. \quad [1/2]$$

31. Let $I = \int \frac{3x+5}{(x^3-x^2-x+1)} dx$

Let $\frac{3x+5}{(x^3-x^2-x+1)} = \frac{3x+5}{(x-1)^2(x+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+1)}$ [1]

$$\Rightarrow (3x+5) = A(x-1)(x+1) + B(x+1) + C(x-1)^2 \quad \dots(1)$$

Put $x = 1$ in equation (1) we get $B = 4$ Put $x = -1$ in equation (1) we get $C = \frac{1}{2}$ Comparing the coefficient of ' x^2 ' on both sides of (1); we get:

$$A + C = 0 \Rightarrow A = -C = -\frac{1}{2} \quad [1]$$

$$\Rightarrow \frac{(3x+5)}{(x^3-x^2-x+1)} = \frac{-1}{2(x-1)} + \frac{4}{(x-1)^2} + \frac{1}{2(x+1)}$$

$$\begin{aligned}
 \therefore I &= \int \frac{(3x+5)}{(x^3 - x^2 - x + 1)} dx \\
 &= \frac{-1}{2} \int \frac{dx}{x-1} + 4 \int \frac{dx}{(x-1)^2} + \frac{1}{2} \int \frac{dx}{(x+1)} \\
 &= \frac{-1}{2} \log|x-1| - \frac{4}{(x-1)} + \frac{1}{2} \log|x+1| + C
 \end{aligned} \tag{1}$$

SECTION – D

32. Given $A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6 \end{bmatrix}$

$$\begin{aligned}
 \Rightarrow |A| &= 3(12 - 6) - 4(0 + 3) + 2(0 - 2) \\
 &= 18 - 12 - 4 = 2 \neq 0
 \end{aligned} \tag{1}$$

Hence, A^{-1} exists.

Now, co-factor are given as:

$$\begin{aligned}
 C_{11} &= 6, C_{12} = -3, C_{13} = -2, \\
 C_{21} &= -28, C_{22} = 16, C_{23} = 10, \\
 C_{31} &= -16, C_{32} = 9, C_{33} = 6
 \end{aligned} \tag{1}$$

$$\text{So, } \text{adj } A = \begin{bmatrix} 6 & -3 & -2 \\ -28 & 16 & 10 \\ -16 & 9 & 6 \end{bmatrix}^T = \begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & 9 \\ -2 & 10 & 6 \end{bmatrix} \tag{1}$$

$$\therefore A^{-1} = \frac{1}{|A|} \cdot \text{adj } A \Rightarrow A^{-1} = \frac{1}{2} \begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & 9 \\ -2 & 10 & 6 \end{bmatrix} \quad \dots\dots(i)$$

The given system of linear equation are :

$$3x + 4y + 2z = 8$$

$$2y - 3z = 3$$

$$x - 2y + 6z = -2$$

It can be represented as :

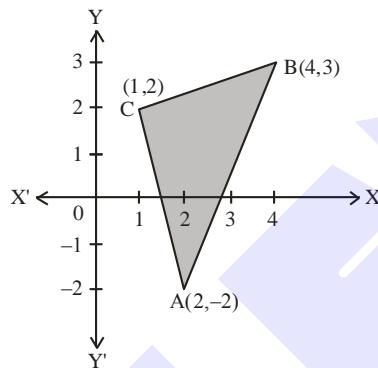
$$\Rightarrow \begin{bmatrix} 3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix}$$

$$\Rightarrow AX = B$$

$$\Rightarrow X = A^{-1} B \tag{1}$$

$$\begin{aligned}
 X &= \frac{1}{2} \begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & 9 \\ -2 & 10 & 6 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix} \quad (\text{From equation (i)}) \\
 &= \frac{1}{2} \begin{bmatrix} 48 - 84 + 32 \\ -24 + 48 - 18 \\ -16 + 30 - 12 \end{bmatrix} \\
 \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} &= \frac{1}{2} \begin{bmatrix} -4 \\ 6 \\ 2 \end{bmatrix} \quad \Rightarrow x = -2, y = 3, z = 1 \quad [1]
 \end{aligned}$$

33.



[Correct Fig. 1 Mark]

Line AB is : $y = \frac{5}{2}x - 7$; $x = \frac{2}{5}(y + 7)$, line BC is: $y = \frac{1}{3}(x + 5) \Rightarrow x = 3y - 5$

Line AC is: $y = -4x + 6$; $x = \frac{y-6}{-4}$ [1]

$$\begin{aligned}
 \text{Required area} &= \left[\int_{-2}^3 (\text{lineAB}) dy \right] - \left[\int_2^3 (\text{lineBC}) dy + \int_{-2}^2 (\text{lineAC}) dy \right] \\
 \Rightarrow & \left[\frac{2}{5} \int_{-2}^3 (y+7) dy \right] - \left[\int_2^3 (3y-5) dy - \frac{1}{4} \int_{-2}^2 (y-6) dy \right] \quad [1] \\
 &= \frac{2}{5} \left[\left(\frac{y^2}{2} + 7y \right) \Big|_{-2}^3 \right] - \left[\left(\frac{3y^2}{2} - 5y \right) \Big|_2^3 - \frac{1}{4} \left(\frac{y^2}{2} - 6y \right) \Big|_{-2}^2 \right] \\
 &= \frac{2}{5} \left[\left(\frac{9}{2} + 21 \right) - (2 - 14) \right] - \left[\left\{ \left(\frac{27}{2} - 15 \right) - (6 - 10) \right\} - \frac{1}{4} \{(2 - 12) - (2 + 12)\} \right] \\
 &= \frac{2}{5} \left[\frac{9}{2} + 33 \right] - \left[\left(\frac{27}{2} - 11 \right) - \frac{1}{4}(-24) \right] \\
 &= \left(\frac{2}{5} \times \frac{75}{2} \right) - \left(\frac{5}{2} + 6 \right) = 15 - \frac{17}{2} = \frac{13}{2} \text{ square units} \quad [2]
 \end{aligned}$$

34. Given lines are

$$\vec{r} = 3\hat{i} + 8\hat{j} + 3\hat{k} + \lambda(3\hat{i} - \hat{j} + \hat{k}) \quad \dots(i)$$

$$\text{and } \vec{r} = -3\hat{i} - 7\hat{j} + 6\hat{k} + \mu(-3\hat{i} + 2\hat{j} + 4\hat{k}) \quad \dots(ii)$$

Equation of lines (i) and (ii) respectively in cartesian form are

$$\text{Let } AB : \frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1} = \lambda \quad \dots(iii)$$

$$\text{and } CD : \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4} = \mu \quad \dots(iv)$$

[1]

\therefore Also let L & M be end points of line of shortest distance on AB & CD

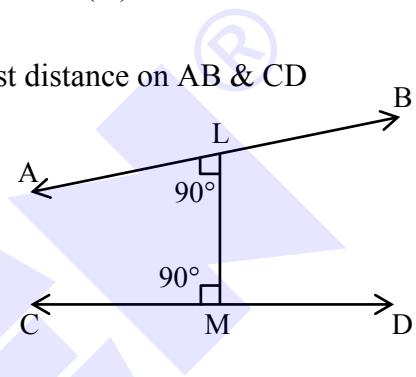
\therefore Co-ordinates of L is $(3\lambda + 3, -\lambda + 8, \lambda + 3)$

[1]

and $M(-3\mu - 3, 2\mu - 7, 4\mu + 6)$

Direction ratio of LM are

$$3\lambda + 3\mu + 6, -\lambda - 2\mu + 15, \lambda - 4\mu - 3$$



[1]

Since $LM \perp AB$

$$\therefore 3(3\lambda + 3\mu + 6) - 1(-\lambda - 2\mu + 15) + 1(\lambda - 4\mu - 3) = 0$$

$$\Rightarrow 11\lambda + 7\mu = 0 \quad \dots(v)$$

Also $LM \perp CD$

$$\therefore -3(3\lambda + 3\mu + 6) + 2(-\lambda - 2\mu + 15) + 4(\lambda - 4\mu - 3) = 0$$

$$\Rightarrow -7\lambda - 29\mu = 0 \quad \dots(vi)$$

Solving (v) and (vi) we get

$$\lambda = 0 \text{ and } \mu = 0$$

$$\therefore L(3, 8, 3) \text{ and } M(-3, -7, 6) \quad [1]$$

$$\text{Hence shortest distance } LM = \sqrt{(3+3)^2 + (8+7)^2 + (3-6)^2} = \sqrt{36+225+9}$$

$$= 3\sqrt{30} \text{ units} \quad [\frac{1}{2}]$$

Vector equation of LM is

$$\vec{r} = 3\hat{i} + 8\hat{j} + 3\hat{k} + t(6\hat{i} + 15\hat{j} - 3\hat{k}) \quad [\frac{1}{2}]$$

OR

At $t = 6$

Position of first insect is at $A(6 + 6, 8 - 6, 3 + 6) \Rightarrow A(12, 2, 9)$ [½]

Position of 2nd insect at $B(1 + 6, 2 + 6, 2(6)) \Rightarrow B(7, 8, 12)$ [½]

$$\therefore \text{Distance between insects after 6 min.} = \sqrt{(12-7)^2 + (2-8)^2 + (9-12)^2}$$

$$= \sqrt{25+36+9} = \sqrt{70} \text{ inches}$$

For the closest distance between two insects

Given equation of lines are $\ell_1 : x = 6 + t, y = 8 - t, z = 3 + t$

$$\Rightarrow \vec{r} = (6\hat{i} + 8\hat{j} + 3\hat{k}) + t(\hat{i} - \hat{j} + \hat{k}) \quad \dots(i) \quad [½]$$

and $\ell_2 : x = 1 + t, y = 2 + t, z = 2t$

$$\Rightarrow \vec{r} = (\hat{i} + \hat{j}) + t(\hat{i} + \hat{j} + 2\hat{k}) \quad [½]$$

On Comparing it with $\vec{r} = \vec{a} + \lambda \vec{b}$

$$\vec{a}_1 = 6\hat{i} + 8\hat{j} + 3\hat{k}, \vec{b}_1 = \hat{i} - \hat{j} + \hat{k}$$

$$\vec{a}_2 = \hat{i} + \hat{j}, \vec{b}_2 = \hat{i} + \hat{j} + 2\hat{k}$$

$$\text{Now } \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -3\hat{i} - \hat{j} + 2\hat{k} \quad [½]$$

$$\vec{a}_2 - \vec{a}_1 = -5\hat{i} - 7\hat{j} - 3\hat{k} \quad [½]$$

$$\begin{aligned} \text{Shortest distance} &= \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} = \frac{|(-5\hat{i} - 7\hat{j} - 3\hat{k}) \cdot (-3\hat{i} - \hat{j} + 2\hat{k})|}{|-3\hat{i} - \hat{j} + 2\hat{k}|} \\ &= \frac{|15 + 7 - 6|}{\sqrt{9+1+4}} = \frac{16}{\sqrt{14}} = \frac{8}{7}\sqrt{14} \text{ inches} \end{aligned} \quad [1] \quad [½]$$

35. Reflexive :

Let (a, b) be an arbitrary element of $N \times N$, $\forall a, b \in N$. Then

$$\Rightarrow (a, b) R (a, b)$$

$$\Rightarrow ab(b+a) = ba(a+b) \quad [\text{by commutativity of addition and multiplication on } N]$$

$$\Rightarrow \text{L.H.S.} = \text{R.H.S.}$$

Thus, $(a, b) R (a, b)$ for all $(a, b) \in N \times N$. So R is reflexive on $N \times N$.

[1]

Symmetric :

Let $(a, b), (c, d) \in N \times N$ be such that $(a, b) R (c, d)$. Then,

$$\Rightarrow ad(b+c) = bc(a+d)$$

$$\Rightarrow cb(d+a) = da(c+b)$$

$$\Rightarrow (c, d) R (a, b)$$

[1½]

Thus, $(a, b) R (c, d) \Rightarrow (c, d) R (a, b)$ for all $(a, b), (c, d) \in N \times N$

So, R is symmetric on $N \times N$.

Transitive :

Let $(a, b), (c, d), (e, f) \in N \times N$ such that $(a, b) R (c, d)$ and $(c, d) R (e, f)$. Then,

$$(a, b) R (c, d) \Rightarrow ad(b+c) = bc(a+d) \Rightarrow \frac{b+c}{bc} = \frac{a+d}{ad} \Rightarrow \frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a} \quad \dots(i)$$

$$\text{and, } (c, d) R (e, f) \Rightarrow cf(d+e) = de(c+f) \Rightarrow \frac{d+e}{de} = \frac{c+f}{cf} \Rightarrow \frac{1}{e} + \frac{1}{d} = \frac{1}{f} + \frac{1}{c} \quad \dots(ii)$$

Adding (i) and (ii), we get

$$\begin{aligned} & \left(\frac{1}{c} + \frac{1}{b} \right) + \left(\frac{1}{e} + \frac{1}{d} \right) = \left(\frac{1}{d} + \frac{1}{a} \right) + \left(\frac{1}{f} + \frac{1}{c} \right) \\ \Rightarrow & \frac{1}{b} + \frac{1}{e} = \frac{1}{a} + \frac{1}{f} \Rightarrow \frac{b+e}{be} = \frac{a+f}{af} \\ \Rightarrow & af(b+e) = be(a+f) \Rightarrow (a, b) R (e, f) \end{aligned} \quad [2]$$

So, R is transitive on $N \times N$.

Hence; R being reflexive, symmetric and transitive; is an equivalence relation on $N \times N$. [½]

OR

Given, $A = \{x \in Z : 0 \leq x \leq 12\}$ and $R = \{(a, b) : a, b \in A, |a - b| \text{ is divisible by } 4\}$

(i) For Reflexive relation :

Let $a \in A$

Now, $|a - a| = 0$, which is divisible by 4

So, $(a, a) \in R \forall a \in A$

Hence, R is reflexive.

[1]

(ii) For Symmetric relation:

Let $a, b \in A$ such that $(a, b) \in R$

i.e. $|a - b|$ is divisible by 4.

$\Rightarrow |-(b - a)| = |b - a|$ is also divisible by 4.

So, $(b, a) \in R$.

Hence, R is symmetric.

[1]

(iii) For Transitive relation:

Let $a, b, c \in A$ such that (a, b) and $(b, c) \in R$

i.e. $|a - b|$ & $|b - c|$ is divisible by 4.

$$\text{Let } |a - b| = 4k_1$$

$$\& \quad |b - c| = 4k_2$$

$$\Rightarrow (a - b) = \pm 4k_1 \quad \dots \dots \dots \text{(i)}$$

$$\& \quad (b - c) = \pm 4k_2 \quad \dots \dots \dots \text{(ii)}$$

Adding equations (i) & (ii);

$$\Rightarrow (a - b) + (b - c) = \pm 4k_1 \pm 4k_2 = \pm 4(k_1 + k_2)$$

$\Rightarrow a - c$ is divisible by 4.

$\Rightarrow |a - c|$ is divisible by 4.

So, $(a, c) \in R$

Hence, R is transitive.

[2]

R is an equivalence relation.

Further, let $(x, 1) \in R \forall x \in A$

$\Rightarrow |x - 1|$ is divisible by 4

$\Rightarrow x - 1 = 0, 4, 8$

$\Rightarrow x = 1, 5, 9$

\therefore Equivalence class of 1 = [1] = {1, 5, 9}

[½]

[\because The set of all elements related to 1 represents its equivalence class]

Now, we will find equivalence class of 2 i.e. [2]

Let $(x, 2) \in R \forall x \in A$

$\Rightarrow |x - 2| = 0, 4, 8$

$\Rightarrow x = 2, 6, 10$

Equivalence class of [2] = {2, 6, 10}.

[½]

SECTION – E

36. (i) Given volume of cylinder $V = \frac{539}{2}$ cubic units

$$\therefore V = \pi r^2 h = \frac{539}{2}$$

$$h = \frac{539}{2\pi r^2}$$

Total surface area of the tank

$$S = 2\pi rh + 2\pi r^2$$

$$S = 2\pi r \left(\frac{539}{2\pi r^2} \right) + 2\pi r^2 = \frac{539}{r} + 2\pi r^2 \text{ square units}$$

[1]

(ii) $\therefore S = \frac{539}{r} + 2\pi r^2$

$$\begin{aligned} \frac{dS}{dr} &= -\frac{539}{r^2} + 4\pi r \\ &= -\frac{539}{r^2} + \frac{4 \times 22r}{7} \\ &= 11 \left(\frac{-343 + 8r^3}{7r^2} \right) \end{aligned}$$

....(i)

For critical points

$$\frac{dS}{dr} = 11 \left(\frac{-343 + 8r^3}{7r^2} \right) = 0$$

$$8r^3 = 343$$

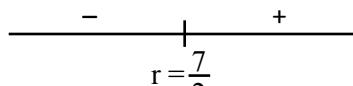
$$r^3 = \frac{343}{8}$$

$$r = \frac{7}{2} \text{ unit}$$

[1]

(iii) By first derivative test

$$\text{When } r < \frac{7}{2}; \quad \frac{dS}{dr} < 0$$



$$\text{When } r > \frac{7}{2}; \quad \frac{dS}{dr} > 0$$

$\therefore \frac{dS}{dr}$ changes its sign from negative to positive at neighborhood of $r = \frac{7}{2}$

So, $r = \frac{7}{2}$ is point of minima [1]

\therefore Surface area is minimum at $r = \frac{7}{2}$ and corresponding height

$$h = \frac{539}{2\pi r^2} = \frac{539 \times 7 \times 2 \times 2}{2 \times 22 \times 7 \times 7} = 7 \text{ unit}$$
 [1]

OR

(iii) Again differentiate equation (i) w.r.t. 'r'

$$\begin{aligned} \frac{d^2S}{dr^2} &= \frac{2 \times 539}{r^3} + 4\pi \\ \left. \frac{d^2S}{dr^2} \right|_{r=\frac{7}{2}} &> 0 \end{aligned}$$
 [1]

So, S is minimum at $r = \frac{7}{2}$

$$h = \frac{539}{2\pi r^2} = \frac{539 \times 7 \times 2 \times 2}{2 \times 22 \times 7 \times 7} = 7 \text{ unit}$$
 [1]

37. (i) Since the perimeter of the floor = 200 m

$$\begin{aligned} \text{i.e. } 2 \times \pi \left(\frac{y}{2} \right) + 2x &= 200 \\ \Rightarrow \pi y + 2x &= 200 \end{aligned}$$
 ... (i) [1]

(ii) $\because A = x \times y$

$$\begin{aligned} \Rightarrow A &= x \left(\frac{200 - 2x}{\pi} \right) && [\text{using (i)}] \\ \Rightarrow A &= \frac{2}{\pi} (100x - x^2) \end{aligned}$$
 [1]

$$(iii) \quad \because A = \frac{2}{\pi} (100x - x^2)$$

$$\begin{aligned} \Rightarrow \frac{dA}{dx} &= \frac{2}{\pi} (100 - 2x) \\ \Rightarrow \frac{d^2A}{dx^2} &= \frac{2}{\pi} \times (-2) = -\frac{4}{\pi} \end{aligned}$$
 [1]

For maximum value of A

$$\frac{dA}{dx} = 0$$

$$\Rightarrow \frac{2}{\pi} \times (100 - 2x) = 0$$

$$\Rightarrow x = 50$$

$$\text{Now, at } x = 50, \quad \frac{d^2A}{dx^2} = \frac{-4}{\pi} < 0$$

i.e., A is maximum at $x = 50$

$$\text{So, Maximum value of } A = \frac{2}{\pi} [100 \times 50 - (50)^2]$$

$$= \frac{2}{\pi} [5000 - 2500] = \frac{2}{\pi} \times 2500 = \frac{5000}{\pi} \text{ m}^2 \quad [1]$$

OR

- (iii) Let B is the area of whole floor including the semi-circular ends, Then

$$B = 2 \times \frac{1}{2} \pi \left(\frac{y}{2} \right)^2 + xy$$

$$B = \frac{\pi}{4} y^2 + xy$$

$$B = \frac{\pi}{4} \left(\frac{200 - 2x}{\pi} \right)^2 + x \left(\frac{200 - 2x}{\pi} \right) \quad [\text{using (i)}] \quad [1]$$

$$\Rightarrow B = \frac{1}{4\pi} (200 - 2x)^2 + \frac{x}{\pi} (200 - 2x)$$

$$\Rightarrow B = \frac{(200 - 2x)}{\pi} \left[\frac{200 - 2x}{4} + x \right]$$

$$\Rightarrow B = \frac{(200 - 2x)}{\pi} \frac{(200 + 2x)}{4} = \frac{40000 - 4x^2}{4\pi} \quad \dots(ii)$$

$$\Rightarrow \frac{dB}{dx} = \frac{1}{4\pi} (-8x) \text{ and } \frac{d^2B}{dx^2} = -\frac{8}{4\pi}$$

For maximum value of B

$$\frac{dB}{dx} = 0 \Rightarrow \frac{1}{4\pi} (-8x) = 0 \Rightarrow x = 0$$

$$\text{at } x = 0, \quad \frac{d^2B}{dx^2} = -\frac{8}{4\pi} < 0$$

i.e., B is maximum at $x = 0$

$$\text{So, maximum value of } B = \frac{40000 - x^2}{4\pi} = \frac{40000}{4\pi} = \frac{10000}{\pi} \text{ m}^2 \quad [\text{From equation (ii)}] \quad [1]$$

38. Let X be the random variable which represents number of tails.

Here X can be 0, 1 or 2

- (i) Probability distribution is

X	0	1	2
P(X)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

[2]

- (ii) $S = \{\text{HHH}, \text{HHT}, \text{HTH}, \text{HTT}, \text{THH}, \text{THT}, \text{TTH}, \text{TTT}\}$

Let E : at least 2 heads

$$E = \{\text{HHH}, \text{HHT}, \text{HTH}, \text{THH}\}$$

[½]

and F : at most 2 head

$$F = \{\text{HHT}, \text{HTH}, \text{HTT}, \text{THH}, \text{THT}, \text{TTH}, \text{TTT}\}$$

$$\therefore E \cap F = \{\text{HHT}, \text{HTH}, \text{THH}\}$$

[½]

Clearly $P(E \cap F) = \frac{3}{8}$ and $P(F) = \frac{7}{8}$

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{3/8}{7/8} = \frac{3}{7}$$

[1]