MATHEMATICS

Time : 3 hrs **Max. Marks : 80**

General Instructions :

1. This Question paper contains - five sections **A, B, C, D** and **E**. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has **18 MCQ's** and **02** Assertion-Reason based questions of **1 mark** each.
3. **Section B** has **5 Very Short Answer (VSA)-type** questions of **2 marks** each.
4. **Section C** has **6 Short Answer (SA)-type** questions of **3 marks** each.
5. **Section D** has **4 Long Answer (LA)-type** questions of **5 marks** each.
6. **Section E** has **3 source based/case based/passage based/integrated units of assessment** of **4 marks** each with sub-parts.

SECTION–A

1. If the matrix $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is skew-symmetric, then the values of 'a' and 'b' are respectively:

(A) 2 and 3 (B) 2 and – 3 (C) – 2 and 3 (D) – 2 and – 3

2. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 \\ 1 & 6 \end{bmatrix}$ then $(AB)^T$ equals :

(A) $\begin{bmatrix} 5 & 9 \\ 4 & 3 \end{bmatrix}$ (B) $\begin{bmatrix} 5 & 9 \\ 16 & 12 \end{bmatrix}$ (C) $\begin{bmatrix} 5 & 16 \\ 9 & 16 \end{bmatrix}$ (D) $\begin{bmatrix} 5 & 9 \\ 12 & 16 \end{bmatrix}$

3. If ABCD is a parallelogram with $\overrightarrow{AC} = \vec{a}$ and $\overrightarrow{BD} = \vec{b}$, then $\overrightarrow{AB}$ is :

(A) $\vec{a} + \vec{b}$ (B) $\vec{a} - \vec{b}$ (C) $\frac{1}{2}(\vec{b} - \vec{a})$ (D) $\frac{1}{2}(\vec{a} - \vec{b})$

4. If $f(x) = \begin{cases} \frac{|x-1|}{1-x} + a, & x > 1 \\ a + b, & x = 1 \\ \frac{|x-1|}{1-x} + b, & x < 1 \end{cases}$ is continuous at x = 1, then the values of 'a' and 'b' are respectively:

(A) 1, 1 (B) 1, –1 (C) 2, 3 (D) None of these

5. The value of $\int_2^5 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{7-x}} dx$ is:

(A) 3 (B) 2 (C) 3/2 (D) 1/2

6. The degree and order of the differential equation $y = Px + \sqrt[3]{a^2P^2 + b^2}, P = \frac{dy}{dx}$ are respectively :

(A) 3 and 1 (B) 1 and 3 (C) 1 and 1 (D) 3 and 3

7. The graph of $x \le 2$ and $y \ge 2$ will be situated in the

(A) first and second quadrant (B) second and third quadrant

(C) first and third quadrant (D) third and fourth quadrant

8. Write the projection of the vector $(\vec{b} + \vec{c})$ on the vector $\vec{a}$, where $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$.

(A) 3 units (B) $\frac{1}{2}$ units (C) 2 units (D) 4 units

9. If $\int_0^a \frac{1}{1+4x^2} dx = \frac{\pi}{8}$, then value of 'a' is :

(A) 2 (B) $\frac{1}{2}$ (C) 8 (D) $\frac{1}{8}$

10. The number of all possible matrices of order 2×3 with each entry 1 or 2 is :

(A) 16 (B) 6 (C) 64 (D) 24

11. Consider the following LPP :

Maximize : $Z = 12x + 10y$

Subject to constraints: $4x + 3y \le 480$, $2x + 3y \le 360$, $x, y \ge 0$

Value of Z will be maximum at

(A) (120, 0) (B) (60, 80) (C) (100, 80) (D) (60, 100)

12. The system of linear equations $5x + ky = 5$, $3x + 3y = 5$; will be consistent if

(A) $k \ne -3$ (B) $k = -5$ (C) $k = 5$ (D) $k \ne 5$

13. If A is a non-singular square matrix of order 3 such that $A^2 = 3A$, then value of $|A|$ is:

(A) –3 (B) 3 (C) 9 (D) 27

14. Two events E and F are independent. If $P(E) = 0.3$ and $P(E \cup F) = 0.5$, then $P(E/F) - P(F/E)$ equals to :

(A) $\frac{2}{7}$ (B) $\frac{3}{35}$ (C) $\frac{1}{70}$ (D) $\frac{1}{7}$

15. The solution of differential equation $xdy - ydx = 0$ represents :

(A) a rectangular hyperbola (B) parabola whose vertex is at origin

(C) straight line passing through origin (D) a circle whose centre is at origin

16. If y = sin (m $\sin^{-1}$x), then which one of the following equations is true?

(A) $(1-x^2)\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+m^2y=0$ (B) $(1-x^2)\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}+m^2y=0$

(C) $(1+x^2)\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}-m^2y=0$ (D) $(1+x^2)\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}-m^2x=0$

17 The vector $\cos\alpha\cos\beta\,\hat{i}+\cos\alpha\sin\beta\,\hat{j}+\sin\alpha\,\hat{k}$ is a:

(A) null vector (B) unit vector

(C) constant vector (D) None of these

18. The direction cosines of the vector $(2\hat{i}+2\hat{j}-\hat{k})$ are:

(A) $-\dfrac{2}{3},\dfrac{2}{3},\dfrac{-1}{3}$ (B) $-\dfrac{2}{3},-\dfrac{2}{3},\dfrac{-1}{3}$

(C) $-\dfrac{2}{3},-\dfrac{2}{3},\dfrac{1}{3}$ (D) $\dfrac{2}{3},\dfrac{2}{3},\dfrac{-1}{3}$

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct answer out of the following choices.

(A) Both **A** and **R** are true and **R** is the correct explanation of **A**.

(B) Both **A** and **R** are true but **R** is not the correct explanation of **A**.

(C) **A** is true but **R** is false.

(D) **A** is false but **R** is true.

19. **Assertion (A) :** The principal value of $\tan^{-1}\left[2\sin\left(2\cos^{-1}\dfrac{\sqrt{3}}{2}\right)\right]$ is $\dfrac{\pi}{3}$.

Reason (R) : $\tan^{-1}(\tan x) = x,\ x\in\left[-\dfrac{\pi}{2},\dfrac{\pi}{2}\right]$

20. **Assertion (A) :** Equation of a line passing through the point P(2, –1, 3) and perpendicular to the lines $\vec{r}=(\hat{i}+\hat{j}-\hat{k})+\lambda(2\hat{i}-2\hat{j}+\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j}-3\hat{k})+\mu(\hat{i}+2\hat{j}+2\hat{k})$ is $\vec{r}=(2\hat{i}-\hat{j}+3\hat{k})+t(-6\hat{i}-3\hat{j}+6\hat{k})$

Reason (R) : Two lines $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\mu\vec{b}_2$ are perpendicular if $\vec{b}_1.\vec{b}_2=0$.

SECTION–B

[This section comprises of very short answer type questions (VSA) of 2 marks each]

21. Find the value of $\operatorname{cosec}^2(\tan^{-1}2) + \sec^2(\cot^{-1}3)$

OR

Let A = R – {3} and B = R – {1}. Consider the function f : A → B defined by $f(x) = \left(\frac{x-2}{x-3}\right)$.

Is f one-one and onto ? Justify your answer.

22. A spherical ball of salt is dissolving in water in such a manner that the rate of decrease of volume at any instant is proportional to the surface. Prove that the radius is decreasing at a constant rate.

23. Find all vectors of magnitude $10\sqrt{3}$ that are perpendicular to the vectors of $\hat{i}+2\hat{j}+\hat{k}$ and $-\hat{i}+3\hat{j}+4\hat{k}$.

OR

Show that the points with position vectors $-2\hat{i}+3\hat{j}+5\hat{k}$, $\hat{i}+2\hat{j}+3\hat{k}$ and $7\hat{i}-\hat{k}$ are collinear.

24. If $(x^2 + y^2)^2 = xy$, find $\frac{dy}{dx}$.

25. Find λ and μ if $(\hat{i}+3\hat{j}+9\hat{k})\times(3\hat{i}-\lambda\hat{j}+\mu\hat{k}) = \vec{0}$.

SECTION–C

[This section comprises of short answer type questions (SA) of 3 marks each]

26. Evaluate: $\int e^x\left(\frac{\sin 4x - 4}{1-\cos 4x}\right)dx$

27. A and B throw a pair of dice alternatively. A wins the game, if he gets a total of 6 and B wins, if he gets a total of 7. If A starts the game, then find the probability of winning the game by A in third throw of the pair of dice.

OR

The probability distribution of a random variable X is given as under :

X	1	2	4	2k	3k	5k
P(X)	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{3}{25}$	$\frac{1}{10}$	$\frac{1}{25}$	$\frac{1}{25}$

Calculate the value of k if E(X) = 2.94

28. Evaluate : $\int_0^{\pi} \frac{4x\sin x}{1+\cos^2 x}\,dx$

OR

Evaluate : $\int_1^4 \{|x-1|+|x-2|+|x-4|\}\,dx$

29. Solve the differential equation $(1 + x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$

OR

Solve the differential equation $2xy\,dy = (x^2 + y^2)\,dx$.

30. Solve the linear programming problem graphically

Minimize $Z = 6x + 21y$,

subject to constraints $x + 2y \le 3$, $x + 4y \ge 4$, $3x + y \ge 3$, $x \ge 0$, $y \ge 0$

31. Evaluate : $\int \frac{x^3}{(x-1)(x^2+1)}\,dx$

SECTION–D

[This section comprises of long answer type questions (LA) of 5 marks each]

32. Using the method of integration, find the area of the triangular region whose vertices are (2, –2), (4, 3) and (1, 2).

33. Let $A = \{x \in Z : 0 \le x \le 12\}$. Show that $R = \{(a, b) : a, b \in A, |a - b| \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of all elements related to 1. Also; write the equivalence class [2].

OR

$f : W \to W$ be defined as $f(n) = n - 1$ if n is odd and $f(n) = n + 1$, if n is even. Show that f is bijective. Here W = set of whole numbers.

34. Find the shortest distance between the lines $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$ and

$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$. Also find the equation of the line of shortest distance.

OR

Find the distance of the point (2, 4, –1) from the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$.

35. If $A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6 \end{bmatrix}$, find A^{-1}.Hence, Solve the following system of equations.

$3x + 4y + 2z = 8$

$2y - 3z = 3$

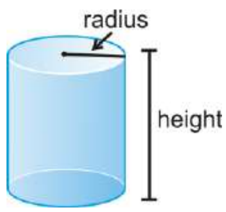
$x - 2y + 6z = -2$

SECTION–E

[This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub parts of 2 marks each.)

36. Read the following passage and answer the questions given below:

A person has manufactured a water tank in the shape of a closed right circular cylinder. The volume of the cylinder is $\frac{539}{2}$ cubic units. If the height and radius of the cylinder be h and r, then



Based on the above information, answer the following questions :

(i) Find the total surface area function (S) of tank in terms of r.

(ii) Find the critical points of the function (S).

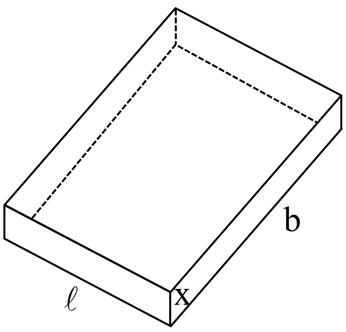
(iii) Use first order derivative test to find the value of r and h, when surface area of the tank is minimum.

OR

(iii) Use second order derivative test to find the value of r and h, when surface area of the tank is minimum.

37. Read the following passage and answer the questions given below:

A square piece of tin of side 24 cm is to be made into a box without top by cutting a square of length x cm from each corner and folding up the flaps to form a box.



Based on the above information, answer the following questions :

(i) Find the volume of the box in terms of x.

(ii) Find the interval in which the function of volume is strictly increasing.

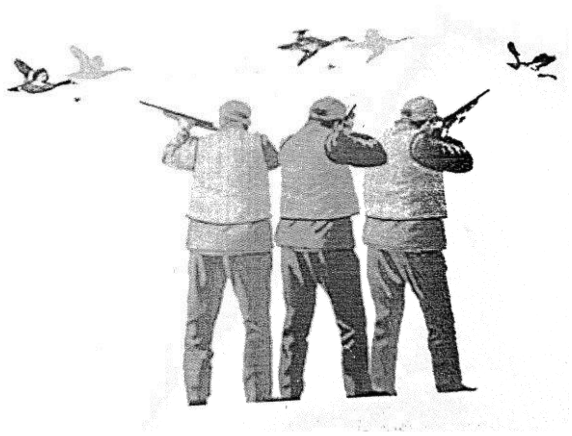
(iii) Find the maximum value of volume of box.

OR

(iii) Find the cost of box if rate of making the box is ₹ 5 per cm^2, when volume is maximum.

38. **Read the following passage and answer the questions given below:**

A coach is training 3 players. He observes that the player A can hit a target 4 times in 5 shots, player B can hit 3 times in 4 shots and the player C can hit 2 times in 3 shots.



Based on the above information, answer the following questions :

(i) Find the probability that A, B and C all will hit the target and also find the probability that 'none of them will hit the target'.

(ii) Find the probability that any two of A, B and C will hit.