

MATHEMATICS

SOLUTION

SECTION – A

1. (C)

Given $A = \begin{bmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{bmatrix}$ is a skew-symmetric matrix

$$\Rightarrow A' = -A$$

$$\begin{bmatrix} 0 & 2 & b \\ a & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -a & 3 \\ -2 & 0 & 1 \\ -b & -1 & 0 \end{bmatrix} \Rightarrow a = -2, b = 3$$

2. (B)

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 5 & 16 \\ 9 & 12 \end{bmatrix} \Rightarrow (AB)^T = \begin{bmatrix} 5 & 9 \\ 16 & 12 \end{bmatrix}$$

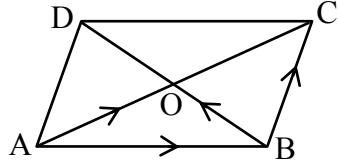
3. (D)

Given $\overrightarrow{AC} = \vec{a}$, $\overrightarrow{BD} = \vec{b}$

$$\Rightarrow \overrightarrow{AO} = \frac{\vec{a}}{2}, \overrightarrow{BO} = \frac{\vec{b}}{2}$$

In $\triangle AOB$, using triangle law

$$\overrightarrow{AB} + \overrightarrow{BO} = \overrightarrow{AO} \Rightarrow \overrightarrow{AB} = \overrightarrow{AO} - \overrightarrow{BO} \Rightarrow \overrightarrow{AB} = \frac{1}{2}(\vec{a} - \vec{b})$$



4. (B)

$$\text{Given that } f(x) = \begin{cases} \frac{|x-1|}{1-x} + a, & x > 1 \\ a+b, & x=1 \\ \frac{|x-1|}{1-x} + b, & x < 1 \end{cases}$$

$\therefore f(x)$ is continuous at $x = 1$; therefore, $\lim_{h \rightarrow 0} f(1+h) = \lim_{h \rightarrow 0} f(1-h) = f(1)$

$\therefore f(1) = a + b$ (given)

$$\text{RHL} = \lim_{h \rightarrow 0} f(1+h) = \lim_{h \rightarrow 0} \frac{|1+h-1|}{1-(1+h)} + a = -1 + a \Rightarrow a + b = -1 + a \Rightarrow b = -1$$

$$\text{LHL} = \lim_{h \rightarrow 0} f(1-h) = \lim_{h \rightarrow 0} \frac{|1-h-1|}{1-(1-h)} + b = 1 + b \Rightarrow a + b = 1 + b \Rightarrow a = 1$$

5. (C)

$$\text{Let } I = \int_2^5 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{7-x}} dx \quad \dots\dots(1)$$

$$\Rightarrow I = \int_2^5 \frac{\sqrt{7-x}}{\sqrt{7-x} + \sqrt{x}} dx \quad \dots\dots(2) \quad \left[\because \int_a^b f(x) dx = \int_a^b f(a+b-x) dx \right]$$

Adding (1) and (2); we get

$$2I = \int_2^5 \frac{\sqrt{x}}{\sqrt{7-x} + \sqrt{x}} dx + \int_2^5 \frac{\sqrt{7-x}}{\sqrt{7-x} + \sqrt{x}} dx \Rightarrow 2I = \int_2^5 \frac{\sqrt{x} + \sqrt{7-x}}{\sqrt{7-x} + \sqrt{x}} dx = \int_2^5 1 dx$$

$$\text{or } 2I = [x]_2^5 = 5 - 2 \Rightarrow I = \frac{3}{2}$$

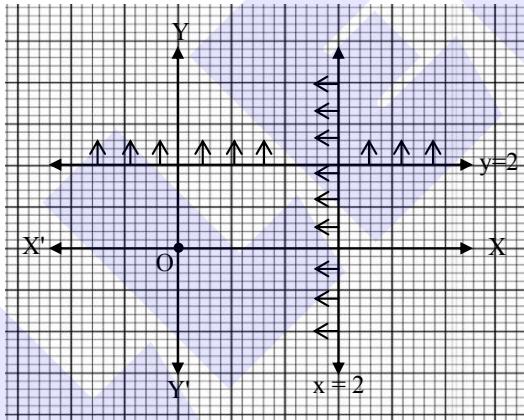
6. (A)

Given differential equation can be written as

$$(y - Px)^3 = (a^2 P^2 + b^2) \quad \left\{ \text{where } P = \frac{dy}{dx} \right\}$$

Degree = 3 and order = 1

7. (A)



Hence; as per the graph; the region lies in the first and second quadrant.

8. (C)

$$\text{Given, } \vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 2\hat{k} \text{ and } \vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$$

$$\therefore \vec{b} + \vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$$

$$\therefore \text{Projection of } (\vec{b} + \vec{c}) \text{ on } \vec{a} = (\vec{b} + \vec{c}) \cdot \hat{a} = (3\hat{i} + \hat{j} + 2\hat{k}) \cdot \frac{(2\hat{i} - 2\hat{j} + \hat{k})}{3} = \frac{6}{3} = 2 \text{ units}$$

9. (B)

$$\int_0^a \frac{1}{1+4x^2} dx = \frac{\pi}{8} \Rightarrow \int_0^a \frac{1}{1+(2x)^2} dx = \frac{\pi}{8}$$

$$\Rightarrow \frac{1}{2} [\tan^{-1}(2x)]_0^a = \frac{\pi}{8} \Rightarrow \frac{1}{2} [\tan^{-1} 2a - \tan^{-1} 0] = \frac{\pi}{8} \Rightarrow \tan^{-1} 2a = \frac{\pi}{4}$$

$$\Rightarrow 2a = \tan \frac{\pi}{4} \Rightarrow 2a = 1 \Rightarrow a = \frac{1}{2}$$

10. (C)

$$\begin{aligned}\text{Required number of possible matrices} &= (\text{Number of possible entries})^{\text{number of elements}} \\ &= (2)^{2 \times 3} = (2)^6 = 64\end{aligned}$$

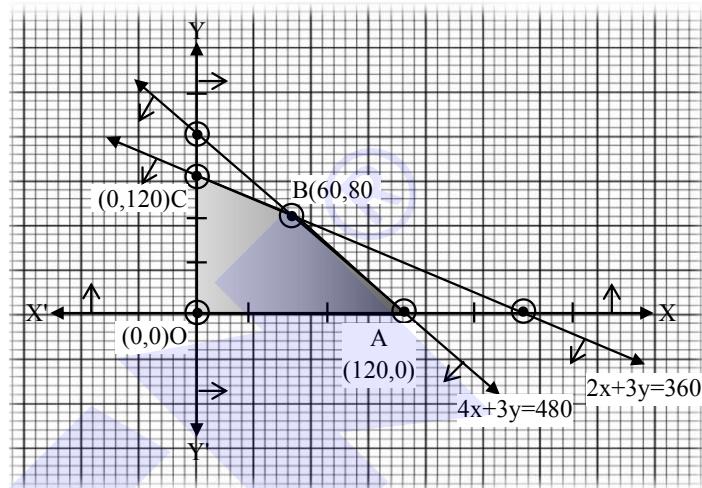
11. (B)

$$\text{We have to maximize } Z = 12x + 10y$$

$$\text{Subject to } 4x + 3y \leq 480, 2x + 3y \leq 360, x, y \geq 0$$

Corner points	$Z = 12x + 10y$
O (0, 0)	0
A (120, 0)	1440
B (60, 80)	1520
C (0, 120)	1200

Hence ; value of Z is maximum at (60, 80)



12. (D)

The system of linear equations is given as :

$$5x + ky = 5 \text{ and } 3x + 3y = 5$$

It can be written in matrix equation form as :

$$\begin{bmatrix} 5 & k \\ 3 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

$$\text{or } AX = B$$

So ; given system of linear equations is consistent if $|A| \neq 0$

$$\Rightarrow \begin{vmatrix} 5 & k \\ 3 & 3 \end{vmatrix} \neq 0 \Rightarrow 15 - 3k \neq 0 \Rightarrow k \neq 5$$

13. (D)

Given $A^2 = 3A$, $|A| \neq 0$, order of A is 3

$$\therefore |A^2| = |3A|$$

$$|A|^2 = 3^3 |A| \quad (|A^2| = |A|^2 \text{ & } |KA| = K^n |A|, \text{ where } n \text{ is order of matrix } A)$$

$$|A| = 27$$

14. (C)

Given ; E and F are independent i.e. $P(E \cap F) = P(E).P(F)$

$$P(E \cup F) = P(E) + P(F) - P(E).P(F)$$

$$\Rightarrow 0.5 = 0.3 + P(F) - 0.3 \times P(F) \Rightarrow 0.7 P(F) = 0.2$$

$$\therefore P(F) = \frac{2}{7}$$

Now ; $P(E/F) - P(F/E)$

$$\begin{aligned} &= \frac{P(E \cap F)}{P(F)} - \frac{P(F \cap E)}{P(E)} = \frac{P(E) \cdot P(F)}{P(F)} - \frac{P(E) \cdot P(F)}{P(E)} = P(E) - P(F) \\ &= \frac{3}{10} - \frac{2}{7} = \frac{1}{70} \end{aligned}$$

15. (C)

Method-1 :

Given, $x dy - y dx = 0$

$$\Rightarrow x dy = y dx \Rightarrow \frac{1}{y} dy = \frac{1}{x} dx$$

Integrating both sides

$$\Rightarrow \int \frac{dy}{y} = \int \frac{dx}{x}$$

$$\log y = \log x + \log c$$

$$y = cx$$

\therefore it represents a straight line passing through origin.

Method-2 :

Given, $x dy - y dx = 0$

$$\Rightarrow \frac{x dy - y dx}{x^2} = 0 \Rightarrow d\left(\frac{y}{x}\right) = 0$$

Integrating both sides

$$\Rightarrow \int d\left(\frac{y}{x}\right) = \int 0$$

$$\frac{y}{x} = c \Rightarrow y = cx$$

\therefore it represents a straight line passing through origin.

.....(1)

(Differentiating w.r.t. x)

(Again differentiating w.r.t. x)

16. (B)

$$y = \sin(m \sin^{-1} x)$$

$$\Rightarrow \frac{dy}{dx} = \cos(m \sin^{-1} x) \times \left(\frac{m}{\sqrt{1-x^2}} \right)$$

$$\Rightarrow \sqrt{1-x^2} \frac{dy}{dx} = m \cos(m \sin^{-1} x)$$

$$\therefore \sqrt{1-x^2} \frac{d^2y}{dx^2} + \left(\frac{dy}{dx} \right) \cdot \frac{1}{2\sqrt{1-x^2}} \cdot (-2x) = -m^2 \sin(m \sin^{-1} x) \times \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow (1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + m^2 y = 0 \quad [\text{from (1)}]$$

17. (B)

$$\text{Let } \vec{a} = \cos \alpha \cos \beta \hat{i} + \cos \alpha \sin \beta \hat{j} + \sin \alpha \hat{k}$$

$$|\vec{a}| = \sqrt{\cos^2 \alpha \cos^2 \beta + \cos^2 \alpha \sin^2 \beta + \sin^2 \alpha}$$

$$|\vec{a}| = \sqrt{\cos^2 \alpha (\cos^2 \beta + \sin^2 \beta) + \sin^2 \alpha} = \sqrt{\cos^2 \alpha + \sin^2 \alpha}$$

$$|\vec{a}| = 1 \text{ i.e., unit vector.}$$

18. (D)

Direction cosines of $(2\hat{i} + 2\hat{j} - \hat{k})$ are $\frac{2}{\sqrt{4+4+1}}, \frac{2}{\sqrt{4+4+1}}, \frac{-1}{\sqrt{4+4+1}}$

$$\text{i.e., } \frac{2}{3}, \frac{2}{3}, \frac{-1}{3}.$$

19. (C)

$$\begin{aligned}\text{Assertion : } \tan^{-1} \left[2 \sin \left(2 \cos^{-1} \frac{\sqrt{3}}{2} \right) \right] &= \tan^{-1} \left[2 \sin \left(2 \times \frac{\pi}{6} \right) \right] = \tan^{-1} \left[2 \sin \left(\frac{\pi}{3} \right) \right] \\ &= \tan^{-1} \left[2 \times \frac{\sqrt{3}}{2} \right] = \tan^{-1} (\sqrt{3}) = \frac{\pi}{3}\end{aligned}$$

\therefore Assertion is true

Reason : Reason is false because given that $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$ while $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$ for $\tan^{-1}(\tan x) = x$.

Hence, Assertion is true but Reason is false.

20. (B)

Assertion : Given lines are

$$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k}) \quad \dots\dots(1)$$

$$\text{and } \vec{r} = (2\hat{i} - \hat{j} - 3\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k}) \quad \dots\dots(2)$$

Where

$$\vec{a}_1 = \hat{i} + \hat{j} - \hat{k}, \quad \vec{b}_1 = 2\hat{i} - 2\hat{j} + \hat{k}$$

$$\vec{a}_2 = 2\hat{i} - \hat{j} - 3\hat{k}, \quad \vec{b}_2 = \hat{i} + 2\hat{j} + 2\hat{k}$$

The required line is perpendicular to (1) as well as (2)

So, this line must be parallel to $\vec{b}_1 \times \vec{b}_2$

$$\text{Now, } \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 1 \\ 1 & 2 & 2 \end{vmatrix} = -6\hat{i} - 3\hat{j} + 6\hat{k}$$

\therefore Equation of line passing through the point $(2, -1, 3)$ and parallel to $\vec{b}_1 \times \vec{b}_2$ is

$$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + t(-6\hat{i} - 3\hat{j} + 6\hat{k})$$

So, assertion is true.

Reason : Reason is true as it is a condition.

Hence, Assertion (A) and Reason (R) both are true but R is not the correct explanation of A.

SECTION – B

21. $\operatorname{cosec}^2(\tan^{-1} 2) + \sec^2(\cot^{-1} 3)$

$$= 1 + \cot^2(\tan^{-1} 2) + 1 + \tan^2(\cot^{-1} 3)$$

$$= 1 + \cot^2 \left(\cot^{-1} \frac{1}{2} \right) + 1 + \tan^2 \left(\tan^{-1} \frac{1}{3} \right) \quad \left[\because \cot^{-1} x = \tan^{-1} \frac{1}{x} \right] \quad [1]$$

$$= 1 + \frac{1}{4} + 1 + \frac{1}{9} = \frac{85}{36} \quad [1]$$

OR

Yes, f is one-one and onto.

Let $x_1, x_2 \in A$ be such that

If $f(x_1) = f(x_2)$

$$\Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$

$$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$$

$$\Rightarrow x_1x_2 - 2x_2 - 3x_1 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$$

$$\Rightarrow x_1 = x_2$$

$\therefore f$ is one-one.

[1]

Let $y \in B = R - \{1\}$, then $f(x) = y$

$$\Rightarrow y = \frac{x - 2}{x - 3}, x \neq 3$$

$$\Rightarrow x - 2 = yx - 3y$$

$$\Rightarrow x - xy = 2 - 3y$$

$$\Rightarrow x(1 - y) = 2 - 3y$$

$$\Rightarrow x = \frac{2 - 3y}{1 - y}$$

$$\therefore 1 - y \neq 0 \Rightarrow y \neq 1 \quad \forall x \in A$$

$\therefore \text{Range} = R - \{1\}$

So, Co-domain = Range

Hence, f is onto.

[1]

22. Let V , S and r denote the volume, surface area and radius of the salt ball respectively at any instant t .

$$\text{Then } V = \frac{4}{3}\pi r^3 \text{ and } S = 4\pi r^2$$

It is given that the rate of decrease of the volume V is proportional to the surface area S .

i.e. $\frac{dV}{dt} \propto S$ or $\frac{dV}{dt} = -KS$, where $K > 0$ is the constant of proportionality.

$$\frac{dV}{dt} = -KS$$

[1]

$$\Rightarrow \frac{d}{dt} \left(\frac{4}{3}\pi r^3 \right) = -K(4\pi r^2)$$

$$\Rightarrow 4\pi r^2 \frac{dr}{dt} = -4\pi Kr^2 \quad \Rightarrow \frac{dr}{dt} = -K$$

So, r decreases with a constant rate.

[1]

23. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = -\hat{i} + 3\hat{j} + 4\hat{k}$. Then

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ -1 & 3 & 4 \end{vmatrix} = \hat{i}(8-3) - \hat{j}(4+1) + \hat{k}(3+2) = 5\hat{i} - 5\hat{j} + 5\hat{k}$$

[½]

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{(5)^2 + (-5)^2 + (5)^2} = 5\sqrt{3}.$$

[½]

Therefore, unit vector perpendicular to the vectors \vec{a} and \vec{b} is given by

$$\pm \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \pm \frac{5\hat{i} - 5\hat{j} + 5\hat{k}}{5\sqrt{3}}.$$

[½]

Hence required vectors are $\pm 10\sqrt{3} \left(\frac{5\hat{i} - 5\hat{j} + 5\hat{k}}{5\sqrt{3}} \right)$ or $\pm 10(\hat{i} - \hat{j} + \hat{k})$



OR

Let $\vec{a} = -2\hat{i} + 3\hat{j} + 5\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{c} = 7\hat{i} - \hat{k}$, are position vector of points A, B and C respectively.

$$\therefore \overrightarrow{AB} = \vec{b} - \vec{a} = 3\hat{i} - \hat{j} - 2\hat{k},$$

$$\overrightarrow{AC} = \vec{c} - \vec{a} = 9\hat{i} - 3\hat{j} - 6\hat{k} = 3(3\hat{i} - \hat{j} - 2\hat{k})$$

[1]

$$\Rightarrow \overrightarrow{AC} = 3 \cdot \overrightarrow{AB}$$

$\therefore \overrightarrow{AC} \parallel \overrightarrow{AB}$ but A is common in \overrightarrow{AC} and \overrightarrow{AB}

\therefore A, B and C are collinear

[1]

24. Given $(x^2 + y^2)^2 = xy$

$$x^4 + y^4 + 2x^2y^2 = xy$$

Differentiating both sides w.r.t. x

$$4x^3 + 4y^3 \frac{dy}{dx} + 2 \left[2xy^2 + x^2 \cdot 2y \frac{dy}{dx} \right] = x \frac{dy}{dx} + y$$

[1]

$$\frac{dy}{dx} (4y^3 + 4x^2y - x) = (y - 4x^3 - 4xy^2)$$

$$\Rightarrow \frac{dy}{dx} = \frac{y - 4x^3 - 4xy^2}{4y^3 + 4x^2y - x}$$

[1]

25. Given $(\hat{i} + 3\hat{j} + 9\hat{k}) \times (3\hat{i} - \lambda\hat{j} + \mu\hat{k}) = \vec{0}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 9 \\ 3 & -\lambda & \mu \end{vmatrix} = \vec{0} \Rightarrow \hat{i}(3\mu + 9\lambda) - \hat{j}(\mu - 27) + \hat{k}(-\lambda - 9) = \vec{0}$$

[1]

$$\therefore \mu - 27 = 0 \quad \dots\dots(1), \quad -\lambda - 9 = 0 \quad \dots\dots(2) \text{ and } 3\mu + 9\lambda = 0 \quad \dots\dots(3)$$

$$\Rightarrow \mu = 27, \lambda = -9 \text{ which are satisfy to eqn. (3)}$$

[1]

SECTION - C

26. Let $I = \int e^x \left(\frac{\sin 4x - 4}{1 - \cos 4x} \right) dx = \int e^x \left(\frac{2 \sin 2x \cos 2x - 4}{2 \sin^2 2x} \right) dx$

$$= \int e^x \left(\frac{2 \sin 2x \cos 2x}{2 \sin^2 2x} - \frac{4}{2 \sin^2 2x} \right) dx \quad [1]$$

$$= \int e^x (\cot 2x - 2 \operatorname{cosec}^2 2x) dx$$

$$= \int e^x (f(x) + f'(x)) dx \quad [\text{where } f(x) = \cot 2x \Rightarrow f'(x) = -2 \operatorname{cosec}^2 2x]$$

$$= e^x f(x) + C = e^x \cot 2x + C \quad [2]$$

27. Let E : event that a total of 6 is obtained and F : event that a total of 7 is obtained.

$$\Rightarrow E = \{(1, 5), (5, 1), (2, 4), (4, 2), (3, 3)\} \text{ and } F = \{(1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)\}$$

$$\Rightarrow P(E) = \frac{5}{36} \text{ and } P(F) = \frac{6}{36} = \frac{1}{6} \quad [1]$$

Since A starts the game ;

$$\Rightarrow P(\text{A wins in the third throw of pair of dice}) = P(\bar{E}).P(\bar{F}).P(E) = \frac{31}{36} \times \frac{5}{6} \times \frac{5}{36} = \frac{775}{7776} \quad [2]$$

OR

We are given $E(X) = \sum_{i=1}^n x_i p_i = 2.94$

$$\Rightarrow 1 \times \frac{1}{2} + 2 \times \frac{1}{5} + 4 \times \frac{3}{25} + (2k) \times \frac{1}{10} + (3k) \times \frac{1}{25} + (5k) \times \frac{1}{25} = 2.94 \quad [1]$$

$$\Rightarrow \frac{1}{2} + \frac{2}{5} + \frac{12}{25} + k \left\{ \frac{2}{10} + \frac{3}{25} + \frac{1}{5} \right\} = 2.94 \Rightarrow k \left\{ \frac{2}{10} + \frac{3}{25} + \frac{1}{5} \right\} = 2.94 - \frac{1}{2} - \frac{2}{5} - \frac{12}{25} \quad [1]$$

$$\Rightarrow k \left(\frac{10+6+10}{50} \right) = \frac{294 - 50 - 40 - 48}{100} \Rightarrow k = \frac{50}{26} \times \frac{156}{100} = 3 \quad [1]$$

28. $I = \int_0^\pi \frac{4x \sin x}{1 + \cos^2 x} dx \quad \dots\dots(1)$

$$I = \int_0^\pi \frac{4(\pi-x) \sin(\pi-x)}{1 + \cos^2(\pi-x)} dx \quad [\text{Applying } \int_0^a f(x) dx = \int_0^a f(a-x) dx]$$

$$I = \int_0^\pi \frac{4\pi \sin x}{1 + \cos^2 x} dx - \int_0^\pi \frac{4x \sin x}{1 + \cos^2 x} dx$$

$$I = \int_0^\pi \frac{4\pi \sin x}{1 + \cos^2 x} dx - I \quad [\text{From equation (1)}] \quad [1]$$

$$2I = 4\pi \int_0^\pi \frac{\sin x}{1 + \cos^2 x} dx$$

$$2I = 4\pi \cdot 2 \times \int_0^{\pi/2} \frac{\sin x}{1 + \cos^2 x} dx \quad \{\text{Applying } \int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx \quad \text{if } f(2a-x) = f(x)\}$$

$$\text{put } \cos x = t \Rightarrow -\sin x \, dx = dt$$

when $x = 0$ and when $x = \pi/2$

then $t = 1$, then $t = 0$

$$\therefore I = 4\pi \int_1^0 \frac{-dt}{1+t^2} = 4\pi \int_0^1 \frac{dt}{1+t^2} \quad \left[\because \int_a^b f(x)dx = - \int_b^a f(x)dx \right] \quad [1]$$

$$I = 4\pi \left[\tan^{-1} t \right]_0^1$$

$$I = 4\pi [\tan^{-1} 1 - \tan^{-1} 0]$$

$$I = 4\pi \times \frac{\pi}{4} = \pi^2 \quad [1]$$

OR

$$\text{Let } I = \int_1^4 \{ |x-1| + |x-2| + |x-4| \} dx$$

$$I = \int_1^2 [(x-1) - (x-2) - (x-4)] dx + \int_2^4 [(x-1) + (x-2) - (x-4)] dx \quad [1]$$

$$I = \int_1^2 (5-x) dx + \int_2^4 (x+1) dx$$

$$I = \left(5x - \frac{x^2}{2} \right)_1^2 + \left(\frac{x^2}{2} + x \right)_2^4 \quad [1]$$

$$I = \left[(10-2) - \left(5 - \frac{1}{2} \right) \right] + \left[(8+4) - (2+2) \right]$$

$$I = \frac{7}{2} + 8 = \frac{23}{2} \quad [1]$$

$$29. (1+x^2) \frac{dy}{dx} + y = e^{\tan^{-1} x}$$

$$\frac{dy}{dx} + \frac{y}{1+x^2} = \frac{e^{\tan^{-1} x}}{1+x^2}$$

This is linear differential equation of the form $\frac{dy}{dx} + Py = Q$

$$\text{Where } P = \frac{1}{1+x^2} \text{ and } Q = \frac{e^{\tan^{-1} x}}{1+x^2}$$

$$\text{IF} = e^{\int P dx} = e^{\int \frac{1}{1+x^2} dx} = e^{\tan^{-1} x} \quad [1]$$

Solution of D.E. is given by

$$y(I.F.) = \int Q(I.F.) dx$$

$$y \times e^{\tan^{-1} x} = \int \frac{e^{\tan^{-1} x}}{1+x^2} \times e^{\tan^{-1} x} dx \quad [1]$$

$$\Rightarrow ye^{\tan^{-1} x} = \int \frac{(e^{\tan^{-1} x})^2}{1+x^2} dx$$

$$\Rightarrow ye^{\tan^{-1} x} = \int e^{2t} dt = \frac{e^{2t}}{2} + C \quad [\because \tan^{-1} x = t \Rightarrow \frac{1}{1+x^2} dx = dt]$$

$$\Rightarrow ye^{\tan^{-1} x} = \frac{e^{2\tan^{-1} x}}{2} + C \quad [1]$$

OR

Given differential equation is $2xy dy = (x^2 + y^2)dx \Rightarrow \frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$ (1)

This is homogeneous differential equation

Put $y = vx$ and $\frac{dy}{dx} = v + x \frac{dv}{dx}$ in eq. (1)

$$v + x \frac{dv}{dx} = \frac{x^2 + v^2 x^2}{2x^2 v} \Rightarrow v + x \frac{dv}{dx} = \frac{1+v^2}{2v} \quad [1/2]$$

$$\Rightarrow x \frac{dv}{dx} = \frac{1+v^2}{2v} - v$$

$$\Rightarrow x \frac{dv}{dx} = \frac{1+v^2-2v^2}{2v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{1-v^2}{2v} \Rightarrow \frac{2v}{1-v^2} dv = \frac{dx}{x} \quad [1/2]$$

On integrating both sides

$$\Rightarrow \int \frac{2v dv}{1-v^2} = \int \frac{dx}{x} \quad [1]$$

$$\Rightarrow -\log(1-v^2) = \log x + \log C$$

$$\Rightarrow -[\log(1-v^2) + \log x] = \log C \Rightarrow -\log x(1-v^2) = \log C$$

$$\Rightarrow \log \frac{1}{x(1-v^2)} = \log C$$

$$\Rightarrow \frac{1}{x(1-v^2)} = C \Rightarrow \frac{1}{x \left(1 - \frac{y^2}{x^2}\right)} = C \quad \left(\because v = \frac{y}{x}\right)$$

$$\Rightarrow \frac{x}{x^2 - y^2} = C \quad [1]$$

30. Minimize $Z = 6x + 21y$

subject to constraints

$$x + 2y \leq 3, x + 4y \geq 4$$

$$3x + y \geq 3, x \geq 0, y \geq 0$$

Corner points	$Z = 6x + 21y$
$A\left(\frac{3}{5}, \frac{6}{5}\right)$	$Z_A = \frac{144}{5} = 28.8$
$B\left(\frac{8}{11}, \frac{9}{11}\right)$	$Z_B = \frac{237}{11} = 21.54$
$C(2, \frac{1}{2})$	$Z_C = \frac{45}{2} = 22.5$

Since, feasible region is bounded,
so Minimum value of Z is 21.54

at point $B\left(\frac{8}{11}, \frac{9}{11}\right)$

31. Let $I = \int \frac{x^3}{(x-1)(x^2+1)} dx$

$$\frac{x^3}{(x-1)(x^2+1)} = 1 + \frac{x^2 - x + 1}{(x-1)(x^2+1)}$$

$$\text{Let } \frac{x^2 - x + 1}{(x-1)(x^2+1)} = \frac{A}{(x-1)} + \frac{Bx + C}{x^2 + 1}$$

$$x^2 - x + 1 = A(x^2 + 1) + (x - 1)(Bx + C) \quad \dots\dots(1)$$

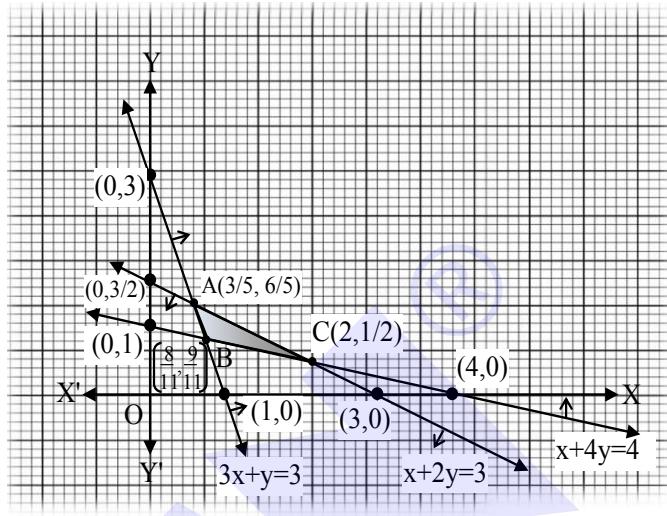
Using (1); we have :

$$\text{When } x = 1, \quad A = \frac{1}{2}$$

$$\text{and } x = 0, \quad 1 = A - C \Rightarrow C = A - 1 = -\frac{1}{2}$$

$$\text{On equating Coefficient of } x^2; \quad 1 = A + B \Rightarrow B = 1 - A = \frac{1}{2}$$

[2½]



[½]



$$\therefore \frac{x^2 - x + 1}{(x-1)(x^2+1)} = \frac{1}{2}\left(\frac{1}{x-1}\right) + \frac{1}{2}\left(\frac{x-1}{x^2+1}\right)$$

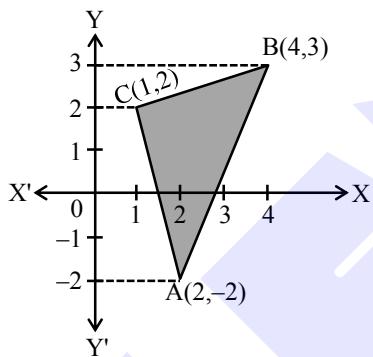
$$\text{Now, } \frac{x^3}{(x-1)(x^2+1)} = 1 + \frac{1}{2}\left(\frac{1}{x-1}\right) + \frac{1}{2}\left(\frac{x-1}{x^2+1}\right)$$

[1]

$$\begin{aligned}
 I &= \int \left[1 + \frac{1}{2} \left(\frac{1}{x-1} \right) + \frac{1}{2} \left(\frac{x-1}{x^2+1} \right) \right] dx \\
 &= \int \left[1 + \frac{1}{2} \left(\frac{1}{x-1} \right) + \frac{1}{2} \times \frac{1}{2} \left(\frac{2x}{x^2+1} \right) - \frac{1}{2} \left(\frac{1}{x^2+1} \right) \right] dx \\
 &= x + \frac{1}{2} \log(x-1) + \frac{1}{4} \log(x^2+1) - \frac{1}{2} \tan^{-1} x + C \\
 &= x + \frac{1}{4} \log(x-1)^2(x^2+1) - \frac{1}{2} \tan^{-1} x + C
 \end{aligned} \tag{1}$$

SECTION – D

32.

**[Correct Fig. 1 Mark]**

Line AB is : $y = \frac{5}{2}x - 7$; $x = \frac{2}{5}(y+7)$, line BC is: $y = \frac{1}{3}(x+5) \Rightarrow x = 3y - 5$

Line AC is: $y = -4x + 6$; $x = \frac{y-6}{-4}$ [1]

$$\begin{aligned}
 \text{Required area} &= \left[\int_{-2}^3 (\text{lineAB}) dy \right] - \left[\int_2^3 (\text{lineBC}) dy + \int_{-2}^2 (\text{lineAC}) dy \right] \\
 \Rightarrow & \left[\frac{2}{5} \int_{-2}^3 (y+7) dy \right] - \left[\int_2^3 (3y-5) dy - \frac{1}{4} \int_{-2}^2 (y-6) dy \right] \tag{1} \\
 &= \frac{2}{5} \left[\left(\frac{y^2}{2} + 7y \right) \Big|_{-2}^3 \right] - \left[\left(\frac{3y^2}{2} - 5y \right) \Big|_2^3 - \frac{1}{4} \left(\frac{y^2}{2} - 6y \right) \Big|_{-2}^2 \right] \\
 &= \frac{2}{5} \left[\left(\frac{9}{2} + 21 \right) - (2 - 14) \right] - \left[\left\{ \left(\frac{27}{2} - 15 \right) - (6 - 10) \right\} - \frac{1}{4} \{(2 - 12) - (2 + 12)\} \right] \\
 &= \frac{2}{5} \left[\frac{9}{2} + 33 \right] - \left[\left(\frac{27}{2} - 11 \right) - \frac{1}{4}(-24) \right] \\
 &= \left(\frac{2}{5} \times \frac{75}{2} \right) - \left(\frac{5}{2} + 6 \right) = 15 - \frac{17}{2} = \frac{13}{2} \text{ square units} \tag{2}
 \end{aligned}$$

33. Given, $R = \{(a, b) : a, b \in A, |a - b| \text{ is divisible by } 4\}$

(i) For Reflexive relation:

Let $a \in A$

Now, $|a - a| = 0$, which is divisible by 4

So, $(a, a) \in R \forall a \in A$

Hence, R is reflexive.

[1]

(ii) For Symmetric relation:

Let $a, b \in A$ such that $(a, b) \in R$

i.e. $|a - b|$ is divisible by 4.

$\Rightarrow |-(b - a)| = |b - a|$ is also divisible by 4.

Hence; $(b, a) \in R$.

So, R is symmetric.

[1]

(iii) For Transitive relation:

Let $a, b, c \in A$ such that $(a, b), (b, c) \in R$

i.e. $|a - b| \& |b - c|$ is divisible by 4.

Let $|a - b| = 4k_1$

& $|b - c| = 4k_2$

$\Rightarrow (a - b) = \pm 4k_1$

.....(1)

& $(b - c) = \pm 4k_2$

.....(2)

Adding equations (1) & (2);

$\Rightarrow (a - b) + (b - c) = \pm 4k_1 \pm 4k_2 = \pm 4(k_1 + k_2)$

$\Rightarrow a - c$ is divisible by 4.

$\Rightarrow |a - c|$ is divisible by 4.

Hence; $(a, c) \in R$

So, R is transitive.

Hence; R is an equivalence relation.

[1]

Further, let $(x, 1) \in R \forall x \in A$

$\Rightarrow |x - 1|$ is divisible by 4

$\Rightarrow x - 1 = 0, 4, 8, 12$

$\Rightarrow x = 1, 5, 9$ [$\because x = 13 \notin A$]

\therefore Equivalence class of [1] = {1, 5, 9}

[1]

[\because The set of all elements related to 1 represents its equivalence class]

Now, we will find equivalence class of [2]

Let $(x, 2) \in R \forall x \in A$

$\Rightarrow |x - 2| = 0, 4, 8, 12$

$\Rightarrow x - 2 = 0, 4, 8, 12$ [$\because x = 14 \notin A$]

\therefore Equivalence class of [2] = {2, 6, 10}.

[1]

OR

$f : W \rightarrow W$, such that

$$f(n) = \begin{cases} n - 1, & \text{if } n \text{ is odd} \\ n + 1, & \text{if } n \text{ is even} \end{cases}$$

For one-one :

Case-I : Let n_1 and n_2 both are odd, $\forall n_1, n_2 \in W$.

$$\text{if } f(n_1) = f(n_2)$$

$$\Rightarrow n_1 - 1 = n_2 - 1 \Rightarrow n_1 = n_2$$

[½]

Case-II : Let n_1 and n_2 both are even, $\forall n_1, n_2 \in W$

$$\text{if } f(n_1) = f(n_2)$$

$$\Rightarrow n_1 + 1 = n_2 + 1 \Rightarrow n_1 = n_2$$

[½]

Case-III : Let $n_1 = \text{odd}$ and $n_2 = \text{even } \forall n_1, n_2 \in W$

$$\text{if } f(n_1) = f(n_2)$$

$$n_1 - 1 = n_2 + 1$$

$$n_1 - n_2 = 2$$

This is contradiction,

Since the difference between an odd and even whole number can never be 2.

Thus, in this case $f(n_1) \neq f(n_2)$

$$n_1 \neq n_2 \Rightarrow f(n_1) \neq f(n_2)$$

$\therefore f$ is one-one.

[2]

For onto :

Case-I : When n is odd; in this case $n - 1$ is even.

$$f(n - 1) = n - 1 + 1 = n$$

Case-II : When n is even; in this case $n + 1$ is odd.

$$f(n + 1) = n + 1 - 1 = n$$

Thus every $n \in W$ has its pre image in W .

So range = codomain

Hence f is onto.

$\Rightarrow f$ is one-one onto

$\Rightarrow f$ is bijective function.

[2]

34. Given lines are $\ell_1 : \frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7} = t$ (say)(1)

And $\ell_2 : \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5} = s$ (say)(2)

Any point on line (1) is M (8 + 3t, -9 - 16t, 10 + 7t) and any point on line (2) is N (15 + 3s, 29 + 8s, 5 - 5s).

Direction ratios of MN are <15 + 3s - 8 - 3t, 29 + 8s + 9 + 16t, 5 - 5s - 10 - 7t>,

i.e., <7 + 3s - 3t, 38 + 8s + 16t, -5 - 5s - 7t>. [1½]

Let M and N are end points of lines of shortest distance on ℓ_1 and ℓ_2

So, MN is perpendicular to both line (1) & (2).

$$\text{i.e., } 3(7 + 3s - 3t) + (-16)(38 + 8s + 16t) + 7(-5 - 5s - 7t) = 0$$

$$\text{and } 3(7 + 3s - 3t) + 8(38 + 8s + 16t) + (-5)(-5 - 5s - 7t) = 0$$

$$\Rightarrow -154s - 314t - 622 = 0, \text{ i.e., } 77s + 157t + 311 = 0 \quad \dots\dots(3)$$

$$\text{and } 98s + 154t + 350 = 0, \text{ i.e. } 7s + 11t + 25 = 0 \quad \dots\dots(4)$$

On solving (3) and (4) simultaneously, we get

$$t = -1 \text{ and } s = -2$$

$$t = -1, \text{ gives } M(5, 7, 3) \text{ and } s = -2 \text{ gives } N(9, 13, 15) \quad [1\frac{1}{2}]$$

∴ The shortest distance between the given lines

$$= \sqrt{(9-5)^2 + (13-7)^2 + (15-3)^2}$$

$$= \sqrt{16 + 36 + 144} = \sqrt{196} = 14 \text{ units} \quad [1]$$

Also, the equations of the line MN is $\frac{x-5}{9-5} = \frac{y-7}{13-7} = \frac{z-3}{15-3}$,

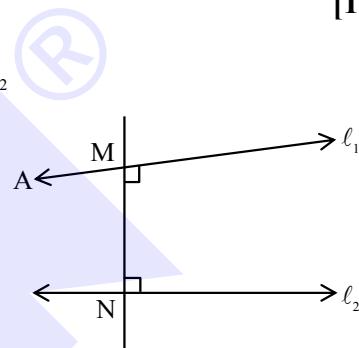
$$\text{i.e., } \frac{x-5}{2} = \frac{y-7}{3} = \frac{z-3}{6}.$$

OR

$$\text{Given line } \frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$$

Vector equation of the given line is $\vec{r} = \vec{a}_1 + \lambda \vec{v}$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$,

$$\vec{a}_1 = -5\hat{i} - 3\hat{j} + 6\hat{k} \text{ and } \vec{v} = \hat{i} + 4\hat{j} - 9\hat{k}.$$



Position vector of the given point is $\vec{a}_2 = 2\hat{i} + 4\hat{j} - \hat{k}$.

$$\text{Hence, the required distance} = \left| \frac{\vec{v} \times (\vec{a}_2 - \vec{a}_1)}{|\vec{v}|} \right| = \frac{|(\hat{i} + 4\hat{j} - 9\hat{k}) \times (7\hat{i} + 7\hat{j} - 7\hat{k})|}{|\hat{i} + 4\hat{j} - 9\hat{k}|} \quad [1]$$

$$= \frac{7}{\sqrt{1^2 + 4^2 + 9^2}} |(\hat{i} + 4\hat{j} - 9\hat{k}) \times (\hat{i} + \hat{j} - \hat{k})| \quad \dots(1) \quad [1]$$

$$\text{Now, } (\hat{i} + 4\hat{j} - 9\hat{k}) \times (\hat{i} + \hat{j} - \hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & -9 \\ 1 & 1 & -1 \end{vmatrix} = 5\hat{i} - 8\hat{j} - 3\hat{k} \quad [1\frac{1}{2}]$$

From equation (1)

$$= \frac{7}{\sqrt{98}} |5\hat{i} - 8\hat{j} - 3\hat{k}| = \frac{7\sqrt{5^2 + 8^2 + 3^2}}{\sqrt{98}} = \frac{7\sqrt{98}}{\sqrt{98}} = 7 \text{ units} \quad [1\frac{1}{2}]$$

35. Given $A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6 \end{bmatrix}$

$$\Rightarrow |A| = 3(12 - 6) - 4(0 + 3) + 2(0 - 2) = 18 - 12 - 4 = 2 \neq 0 \quad [1]$$

Hence, A^{-1} exists.

Now, co-factor are given as:

$$\begin{aligned} C_{11} &= 6, C_{12} = -3, C_{13} = -2, \\ C_{21} &= -28, C_{22} = 16, C_{23} = 10, \\ C_{31} &= -16, C_{32} = 9, C_{33} = 6 \end{aligned} \quad [1]$$

$$\text{So, adj } A = \begin{bmatrix} 6 & -3 & -2 \\ -28 & 16 & 10 \\ -16 & 9 & 6 \end{bmatrix}^T = \begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & 9 \\ -2 & 10 & 6 \end{bmatrix} \quad [1]$$

$$\therefore A^{-1} = \frac{1}{|A|} \cdot \text{adj } A$$

$$\Rightarrow A^{-1} = \frac{1}{2} \begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & 9 \\ -2 & 10 & 6 \end{bmatrix} \quad \dots(1)$$

The given system of linear equations are :

$$3x + 4y + 2z = 8$$

$$2y - 3z = 3$$

$$x - 2y + 6z = -2$$

It can be represented as :

$$\Rightarrow \begin{bmatrix} 3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix}$$

$$\Rightarrow AX = B \Rightarrow X = A^{-1}B \quad [1]$$

$$X = \frac{1}{2} \begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & 9 \\ -2 & 10 & 6 \end{bmatrix} \cdot \begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix} \quad [\text{From equation (1)}]$$

$$= \frac{1}{2} \begin{bmatrix} 48 - 84 + 32 \\ -24 + 48 - 18 \\ -16 + 30 - 12 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -4 \\ 6 \\ 2 \end{bmatrix}$$

$$\Rightarrow x = -2, y = 3, z = 1$$

[1]

SECTION – E

36. (i) Let V be the volume of cylinder

Given volume of cylinder $V = \frac{539}{2}$ cubic units

$$\therefore V = \pi r^2 h \Rightarrow h = \frac{539}{2\pi r^2}$$

Total surface area of the tank, $S = 2\pi rh + 2\pi r^2$

$$\Rightarrow S = 2\pi r \left(\frac{539}{2\pi r^2} \right) + 2\pi r^2 = \frac{539}{r} + 2\pi r^2 \text{ square units} \quad [1]$$

$$(ii) \quad \because S = \frac{539}{r} + 2\pi r^2$$

$$\frac{dS}{dr} = -\frac{539}{r^2} + 4\pi r = -\frac{539}{r^2} + \frac{4 \times 22r}{7} = 11 \left(\frac{-343 + 8r^3}{7r^2} \right)$$

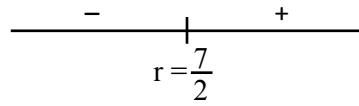
$$\text{For critical points } \frac{dS}{dr} = 11 \left(\frac{-343 + 8r^3}{7r^2} \right) = 0$$

$$8r^3 = 343 \Rightarrow r^3 = \frac{343}{8} \Rightarrow r = \frac{7}{2} \text{ units}$$

[1]

(iii) By first derivative test

$$\text{When } r < \frac{7}{2}; \quad \frac{dS}{dr} < 0$$



$$\text{When } r > \frac{7}{2}; \quad \frac{dS}{dr} > 0$$

$\therefore \frac{dS}{dr}$ changes its sign from negative to positive at neighborhood of $r = \frac{7}{2}$

So, $r = \frac{7}{2}$ is point of minima

[1]

\therefore Surface area is minimum at $r = \frac{7}{2}$ and corresponding height

$$h = \frac{539}{2\pi r^2} = \frac{539 \times 7 \times 2 \times 2}{2 \times 22 \times 7 \times 7} = 7 \text{ units}$$

[1]

OR

(iii) Again differentiate equation (1) w.r.t. 'r'

$$\frac{d^2S}{dr^2} = \frac{2 \times 539}{r^3} + 4\pi$$

$$\left. \frac{d^2S}{dr^2} \right|_{r=\frac{7}{2}} > 0$$

[1]

So, S is minimum at $r = \frac{7}{2}$

$$h = \frac{539}{2\pi r^2} = \frac{539 \times 7 \times 2 \times 2}{2 \times 22 \times 7 \times 7} = 7 \text{ units}$$

[1]

37. (i) From figure it is clear that

$$\text{Length of box} = 24 - 2x$$

$$\text{Breadth of box} = 24 - 2x$$

$$\text{Height of box} = x$$

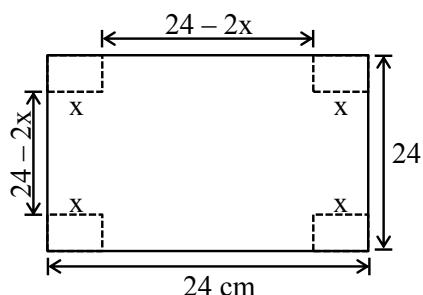
$$\text{Volume of box,}$$

$$V = \text{length} \times \text{breadth} \times \text{height}$$

$$= (24 - 2x)(24 - 2x)(x) = x(24 - 2x)^2 \text{ cm}^3$$

[1]

$$(ii) V = x(24 - 2x)^2 = 4x(12 - x)^2$$

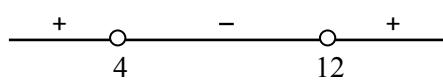


$$\frac{dV}{dx} = 4 \left[x^2(12 - x)(-1) + (12 - x)^2 \cdot 1 \right]$$

$$= 4[(12 - x)(-2x + 12 - x)] = 4[(12 - x)(12 - 3x)]$$

$$\frac{dV}{dx} = 12[(12 - x)(4 - x)] \quad \dots\dots(1)$$

For strictly increasing $\frac{dV}{dx} > 0$



$$12(12-x)(4-x) > 0$$

$$x \in (-\infty, 4) \cup (12, \infty)$$

[1]

(iii) We have,

$$V = x(24 - 2x)^2$$

$$\frac{dV}{dx} = 12[(12-x)(4-x)] \quad [\text{from equation (1)}]$$

For maxima or minima $\frac{dV}{dx} = 0$

$$\Rightarrow 12(12-x)(4-x) = 0$$

$$\Rightarrow x = 4, 12 \quad [\because x = 12 \text{ is not possible}]$$

$$\therefore x = 4$$

[1]

$$\text{Again, } \frac{d^2V}{dx^2} = 12[(12-x)(-1) + (4-x)(-1)]$$

$$= 12(x - 12 + x - 4)$$

$$= 12(2x - 16) = 24(x - 8)$$

$$\therefore \left. \frac{d^2V}{dx^2} \right|_{x=4} < 0$$

So, V is maximum at x = 4

$$V_{\max} = x(24 - 2x)^2$$

$$= 4(24 - 8)^2 = 4 \times 16^2$$

$$= 1024 \text{ cm}^3$$

[1]

OR

(iii) Length of box = $24 - 2x = 24 - 2 \times 4 = 16 \text{ cm}$

Breadth of box = $24 - 2x = 24 - 2 \times 4 = 16 \text{ cm}$

Height of box = 4 cm

$$\therefore \text{Surface area of box} = 2(\ell + b)h + 2 \times \ell b$$

$$= 2(16 + 16) \times 4 + 2 \times 16 \times 16$$

[1]

$$= 256 + 512 = 768$$

$$\therefore \text{Total cost of making the box} = 768 \times 5 = ₹ 3840$$

[1]

38. (i) Probability of hitting a target by A, B and C will be $P(A) = \frac{4}{5}$ then $P(\bar{A}) = \frac{1}{5}$

$$P(B) = \frac{3}{4} \text{ then } P(\bar{B}) = \frac{1}{4} \text{ and } P(C) = \frac{2}{3} \text{ then } P(\bar{C}) = \frac{1}{3}$$

Now, $P(\text{all hitting the target}) = P(A \cap B \cap C)$

$$= P(A).P(B).P(C)$$

$$= \frac{4}{5} \times \frac{3}{4} \times \frac{2}{3} = \frac{2}{5}$$

[1]

Hence the probability that A, B and C all hit the target is $\frac{2}{5}$

The probability that none of them will hit the target

$$P(\bar{A} \cap \bar{B} \cap \bar{C}) = P(\bar{A}) \times P(\bar{B}) \times P(\bar{C})$$

$$= \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} = \frac{1}{60}$$

[1]

(ii) The probability that any two of A, B and C will hit

$$= P(\bar{A}BC) + P(A\bar{B}C) + P(AB\bar{C})$$

$$= \frac{1}{5} \times \frac{3}{4} \times \frac{2}{3} + \frac{4}{5} \times \frac{1}{4} \times \frac{2}{3} + \frac{4}{5} \times \frac{3}{4} \times \frac{1}{3}$$

[1]

$$= \frac{1}{10} + \frac{2}{15} + \frac{1}{5} = \frac{3+4+6}{30} = \frac{13}{30}$$

[1]