

MATHEMATICS

SOLUTION

SECTION – A

1. (B)

Given $A = A^T$ and $B = B^T$

$$(AB - BA)^T = (AB)^T - (BA)^T = B^T A^T - A^T B^T = BA - AB = -(AB - BA)$$

$\therefore AB - BA$ is a skew-symmetric matrix.

2. (D)

Given, $|A| = 5$, $n = 3$

Let $\text{adj } A = B$

Now, $|B| = |\text{adj } A| = |A|^{n-1} = 5^2 = 25$

Now, $|\text{adj } B| = |B|^{n-1} = 25^2 = 625$

$\therefore |\text{adj}(\text{adj } A)| = 625$

3. (C)

We have, $\vec{a} + \vec{b} = 6\hat{i} - 2\hat{j} + (7+\lambda)\hat{k}$ and $\vec{a} - \vec{b} = 4\hat{i} + (7-\lambda)\hat{k}$

Since $\vec{a} + \vec{b}$ & $\vec{a} - \vec{b}$ are orthogonal

$$\therefore (\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0 \Rightarrow [6\hat{i} - 2\hat{j} + (7+\lambda)\hat{k}] \cdot [4\hat{i} + (7-\lambda)\hat{k}] = 0$$

$$\Rightarrow 24 + (7+\lambda)(7-\lambda) = 0 \Rightarrow \lambda^2 = 73 \Rightarrow \lambda = \pm\sqrt{73}$$

4. (D)

We have, $f(0) = 1$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{h \rightarrow 0} f(0+h) = \lim_{h \rightarrow 0} \frac{|\sin h|}{h} = 1$$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{|\sin(-h)|}{-h} = -1$$

$$\therefore \lim_{x \rightarrow 0^+} f(x) \neq \lim_{x \rightarrow 0^-} f(x)$$

Hence, $f(x)$ is discontinuous at $x = 0$

Since $f(x)$ is discontinuous at $x = 0$, so it is not differentiable at $x = 0$.

5. (B) Given, $aRb \Leftrightarrow a$ is a factor of b

a is a factor of $a \Rightarrow a R a \quad \forall a \in N \Rightarrow R$ is reflexive.

Now; 2 is a factor of 4 but 4 is not a factor of 2

$\Rightarrow R$ is not symmetric

Let aRb and $bRc \Rightarrow b = ma, c = nb$

$$\Rightarrow c = n(ma) \Rightarrow c = (mn)a \Rightarrow aRc$$

$\Rightarrow R$ is transitive

6. (D)

Order = 2, degree is not defined.

7. (B) The co-ordinates of the corner points of the feasible region are

$$A\left(\frac{18}{7}, \frac{2}{7}\right), B\left(\frac{7}{2}, \frac{3}{4}\right), C\left(\frac{3}{2}, \frac{15}{4}\right), D\left(\frac{3}{13}, \frac{24}{13}\right)$$

Corner Points	$Z=5x+2y$
$A\left(\frac{18}{7}, \frac{2}{7}\right)$	$Z_A = \frac{94}{7}$
$B\left(\frac{7}{2}, \frac{3}{4}\right)$	$Z_B = 19$
$C\left(\frac{3}{2}, \frac{15}{4}\right)$	$Z_C = 15$
$D\left(\frac{3}{13}, \frac{24}{13}\right)$	$Z_D = \frac{63}{13}$

So, maximum value of $Z=19$ at $B\left(\frac{7}{2}, \frac{3}{4}\right)$

8. (D)

$$\text{Projection of } \vec{a} \text{ on } \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{(\hat{i} - \hat{j}) \cdot (-\hat{j} + \hat{k})}{|-\hat{j} + \hat{k}|} = \frac{1}{\sqrt{2}}$$

9. (B)

$$\text{Let } I = \int x^x (1 + \log x) dx$$

$$\text{Put } x^x = t \Rightarrow x^x (1 + \log x) dx = dt \quad \therefore I = \int 1 \cdot dt = t + C \Rightarrow I = x^x + C$$

10. (A)

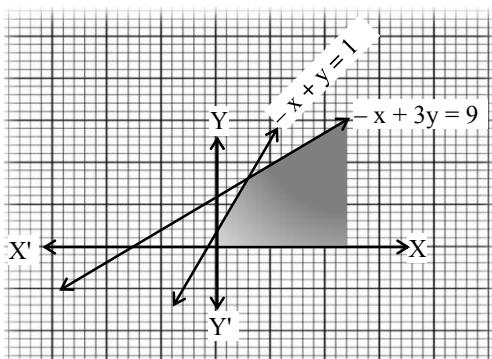
Given $B = \begin{bmatrix} 2 & a & 5 \\ -1 & 4 & b \\ c & -4 & 9 \end{bmatrix}$ is a symmetric matrix

$$\therefore B^T = B$$

$$\Rightarrow \begin{bmatrix} 2 & -1 & c \\ a & 4 & -4 \\ 5 & b & 9 \end{bmatrix} = \begin{bmatrix} 2 & a & 5 \\ -1 & 4 & b \\ c & -4 & 9 \end{bmatrix} \Rightarrow a = -1, b = -4, c = 5 \quad \therefore a + b + c = 0$$

11. (B)

Unbounded feasible region.



12. (C)

We have, $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & 2 \\ 9 & 6 & -2 \end{vmatrix} = 0$
 $\Rightarrow 5(-2x-12) - 3(14-18) - 1(-42-9x) = 0 \Rightarrow x = -6$

13. (B)

$$\begin{aligned} A^2 - 4A &= \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - 4 \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 9 & 16 \\ 8 & 17 \end{bmatrix} - \begin{bmatrix} 4 & 16 \\ 8 & 12 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 5I \end{aligned}$$

14. (D)

We have $P(A' \cup B') = \frac{1}{4} \Rightarrow P((A \cap B)') = \frac{1}{4}$

$$\Rightarrow P(A \cap B) = 1 - \frac{1}{4} = \frac{3}{4}$$

Since $P(A \cap B) \neq 0$, so A & B are not mutually exclusive

$$\text{Also } P(A) \times P(B) = \frac{1}{2} \times \frac{7}{12} = \frac{7}{24} \neq P(A \cap B)$$

So, A & B are not independent.

15. (C)

Given, $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0 \Rightarrow \frac{1}{\sqrt{1-y^2}} dy = -\frac{1}{\sqrt{1-x^2}} dx$

Integrating both side

$$\int \frac{1}{\sqrt{1-y^2}} dy = - \int \frac{1}{\sqrt{1-x^2}} dx$$

$$\sin^{-1} y = -\sin^{-1} x + C \Rightarrow \sin^{-1} y + \sin^{-1} x = C$$

16. (B)

We have, $y = \cos^{-1} \left(\frac{x^2 - 1}{x^2 + 1} \right)$

Put $x = \cot \theta \Rightarrow \theta = \cot^{-1} x$

$$\therefore y = \cos^{-1} \left(\frac{\cot^2 \theta - 1}{\cot^2 \theta + 1} \right) \Rightarrow y = \cos^{-1} \left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \right)$$

$$\Rightarrow y = \cos^{-1} (\cos 2\theta) \Rightarrow y = 2\theta \Rightarrow y = 2 \cot^{-1} x$$

differentiating w.r.t. x

$$\frac{dy}{dx} = \frac{-2}{1+x^2}$$

17. (B)

$$|\vec{a} \times \vec{b}| = 35, |\vec{a}| |\vec{b}| \sin \theta = 35$$

$$\Rightarrow \sin \theta = \frac{35}{\sqrt{26} \times 7} = \frac{5}{\sqrt{26}}$$

$$\therefore \cos \theta = \sqrt{1 - \sin^2 \theta} = \sqrt{1 - \frac{25}{26}} = \frac{1}{\sqrt{26}}$$

$$\text{Now, } \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta = \sqrt{26} \times 7 \times \frac{1}{\sqrt{26}} = 7$$

18. (C)

$$\text{We have, } \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

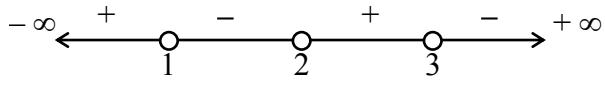
$$(1 - \sin^2 \alpha) + (1 - \sin^2 \beta) + (1 - \sin^2 \gamma) = 1$$

$$\Rightarrow \sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$$

19. (A) We have, $f(x) = 2 \log(x-2) - x^2 + 4x + 1$

$$\Rightarrow f'(x) = \frac{2}{x-2} - 2x + 4$$

$$\Rightarrow f'(x) = -2 \frac{(x-1)(x-3)}{x-2} > 0$$



$\therefore f(x)$ is strictly increasing on $(2, 3)$.

\therefore Assertion is True.

Hence, A and R both are true and R is correct explanation of A.

20. (D)

We know equation of line passes through $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is given by

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

\therefore Reason is true.

Now, equation of line passes through $A(2, -1, 4)$ & $B(1, 1, -2)$ is $\frac{x-2}{1-(2)} = \frac{y+1}{1-(-1)} = \frac{z-4}{-2-4}$

$$\Rightarrow \frac{x-2}{-1} = \frac{y+1}{2} = \frac{z-4}{-6}$$

\therefore Assertion is false.

Hence, A is false but R is true.

SECTION – B

21. Let $\sin^{-1} \frac{3}{5} = A, \sin^{-1} \frac{5}{13} = B$

Then, $A, B \in \left[-\frac{\pi}{2}, \frac{\pi}{2} \right] \Rightarrow \cos A > 0$ and $\cos B > 0$

$$\Rightarrow \sin A = \frac{3}{5} \text{ and } \sin B = \frac{5}{13}$$

$$\text{Now, } \cos A = \sqrt{1 - \sin^2 A} = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$$

$$\cos B = \sqrt{1 - \sin^2 B} = \sqrt{1 - \frac{25}{169}} = \frac{12}{13} \quad [1]$$

$$\text{Now, } \cos \left(\sin^{-1} \frac{3}{5} + \sin^{-1} \frac{5}{13} \right) = \cos(A+B)$$

$$= \cos A \cos B - \sin A \sin B = \frac{4}{5} \times \frac{12}{13} - \frac{3}{5} \times \frac{5}{13} = \frac{33}{65} \quad [1]$$

OR

Let $x_1, x_2 \in R$, such that

$$\text{If } f(x_1) = f(x_2) \Rightarrow x_1^3 = x_2^3 \Rightarrow x_1^3 - x_2^3 = 0$$

$$\Rightarrow (x_1 - x_2)(x_1^2 + x_1 x_2 + x_2^2) = 0 \Rightarrow (x_1 - x_2) \left[\left(x_1 + \frac{x_2}{2} \right)^2 + \frac{3}{4} x_2^2 \right] = 0$$

$$\Rightarrow x_1 - x_2 = 0 \quad \left[\because \left(x_1 + \frac{x_2}{2} \right)^2 + \frac{3}{4} x_2^2 \neq 0 \right]$$

$$\Rightarrow x_1 = x_2 \quad [1]$$

$\therefore f$ is one-one

Let $y \in R$

$$\text{Put } y = f(x) \Rightarrow y = x^3 \Rightarrow x = y^{1/3}$$

Thus, for each y in the co-domain R there exists $y^{1/3}$ in domain R , such that

$$f(y^{1/3}) = (y^{1/3})^3 = y$$

$\therefore f$ is onto [1]

Hence f is one-one and onto

22. At any instant t , let r be the radius, v the volume and s the surface area of the balloon.

Then, $\frac{ds}{dt} = 2 \text{ cm}^2 / \text{sec}$ (given)

Now $s = 4\pi r^2$

$$\Rightarrow \frac{ds}{dt} = 8\pi r \cdot \frac{dr}{dt}$$

$$2 = 8\pi r \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{4\pi r} \quad \dots\dots(1) \quad [1]$$

Now, $v = \frac{4}{3}\pi r^3$

$$\frac{dv}{dt} = 4\pi r^2 \cdot \frac{dr}{dt} \Rightarrow \frac{dv}{dt} = 4\pi r^2 \cdot \left(\frac{1}{4\pi r}\right) \quad [\text{From (1)}]$$

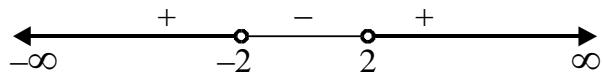
$$\frac{dv}{dt} = r$$

$$\text{at } r = 6 \text{ cm} \quad \frac{dv}{dt} = 6 \text{ cm}^3 / \text{sec} \quad [1]$$

23. We have, $f(x) = \frac{x}{2} + \frac{2}{x} \Rightarrow f'(x) = \frac{1}{2} - \frac{2}{x^2} = \frac{x^2 - 4}{2x^2}$

For strictly increasing, $f'(x) > 0$

$$\Rightarrow \frac{x^2 - 4}{2x^2} > 0 \Rightarrow \frac{(x-2)(x+2)}{x^2} > 0$$



$$\Rightarrow x \in (-\infty, -2) \cup (2, \infty)$$

So, $f(x)$ is strictly increasing on $(-\infty, -2) \cup (2, \infty)$. [1]

For strictly decreasing, $f'(x) < 0$

$$\Rightarrow \frac{x^2 - 4}{2x^2} < 0 \Rightarrow \frac{(x-2)(x+2)}{x^2} < 0 \Rightarrow x \in (-2, 2) - \{0\} \quad [1]$$

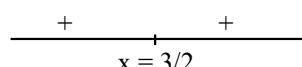
Therefore, f is strictly decreasing in $(-2, 2) - \{0\}$ and f is strictly increasing in $(-\infty, -2) \cup (2, \infty)$.

24. $f(x) = 4x^3 - 18x^2 + 27x - 7$

$$f'(x) = 12x^2 - 36x + 27 = 3(4x^2 - 12x + 9) = 3(2x - 3)^2$$

$$f'(x) = 0 \Rightarrow x = \frac{3}{2} \text{ (critical point)}$$

Since $f'(x) > 0$ for all $x < \frac{3}{2}$ and for all $x > \frac{3}{2}$.



[1]

Here, $f'(x)$ does not change its sign at neighbourhood point of $x = \frac{3}{2}$

Hence, $x = \frac{3}{2}$ is a point of inflection i.e., neither a point of maxima nor a point of minima.

$x = \frac{3}{2}$ is the only critical point, and f has neither maxima nor minima. [1]

OR

$$\text{Let } f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 1, x \in [1, 4]$$

$$\text{Then, } f'(x) = 12x^3 - 24x^2 + 24x - 48$$

$$\text{Now, } f'(x) = 0$$

$$\Rightarrow 12x^3 - 24x^2 + 24x - 48 = 0 \Rightarrow x^3 - 2x^2 + 2x - 4 = 0 \quad [1]$$

$$\Rightarrow x^2(x-2) + 2(x-2) = 0 \Rightarrow (x-2)(x^2+2) = 0$$

$$\Rightarrow x = 2 \quad [\because x^2 + 2 \neq 0]$$

$$\text{Now, } f(2) = -63,$$

$$f(1) = -40 \text{ and } f(4) = 257.$$

So, the minimum and maximum values of $f(x)$ on $[1, 4]$ are -63 and 257 respectively.

[1]

25. Let $I = \int \sqrt{2x^2 + 3x + 4} dx$

$$I = \sqrt{2} \int \sqrt{\left(x + \frac{3}{4}\right)^2 + \left(\frac{\sqrt{23}}{4}\right)^2} dx$$

$$\text{Put } x + \frac{3}{4} = t \Rightarrow dx = dt$$

$$\therefore I = \sqrt{2} \int \sqrt{t^2 + \left(\frac{\sqrt{23}}{4}\right)^2} dt$$

$$I = \sqrt{2} \left[\frac{t}{2} \sqrt{t^2 + \frac{23}{16}} + \frac{23}{32} \log \left| t + \sqrt{t^2 + \frac{23}{16}} \right| \right] + C \quad [1]$$

$$\left[\because \int \sqrt{t^2 + a^2} dt = \frac{t}{2} \sqrt{t^2 + a^2} + \frac{a^2}{2} \log \left| t + \sqrt{t^2 + a^2} \right| + C \right]$$

$$\Rightarrow I = \frac{\sqrt{2}}{2} \left(x + \frac{3}{4} \right) \cdot \sqrt{\left(x + \frac{3}{4} \right)^2 + \frac{23}{16}} + \frac{23\sqrt{2}}{32} \log \left(\left(x + \frac{3}{4} \right) + \sqrt{\left(x + \frac{3}{4} \right)^2 + \frac{23}{16}} \right) + C$$

$$I = \frac{(4x+3)\sqrt{2x^2+3x+4}}{8} + \frac{23\sqrt{2}}{32} \log \left| \frac{4x+3 + \sqrt{2x^2+3x+4}}{\sqrt{2}} \right| + C \quad [1]$$

SECTION – C

26. We have, $x = 3 \sin t - \sin 3t \Rightarrow \frac{dx}{dt} = 3 \cos t - 3 \cos 3t$ (1)

Also, $y = 3 \cos t - \cos 3t \Rightarrow \frac{dy}{dt} = -3 \sin t + 3 \sin 3t$ (2)

$$\therefore \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-3 \sin t + 3 \sin 3t}{3 \cos t - 3 \cos 3t} = \frac{\sin 3t - \sin t}{\cos t - \cos 3t} = \frac{2 \cos 2t \sin t}{2 \sin 2t \sin t} = \cot 2t \quad [1]$$

$$\therefore \frac{d^2y}{dx^2} = -2 \operatorname{cosec}^2 2t \cdot \frac{dt}{dx} = \frac{-2 \operatorname{cosec}^2 2t}{3 \cos t - 3 \cos 3t} \quad [1]$$

$$\text{at } t = \frac{\pi}{3}, \frac{d^2y}{dx^2} = \frac{-2 \operatorname{cosec}^2 \left(2 \frac{\pi}{3}\right)}{3 \left(\cos \frac{\pi}{3} - \cos \pi\right)} = -\frac{16}{27} \quad [1]$$

27. Let E_1, E_2 and E_3 be the event that the problem is solved by three students respectively. Then,

$$P(E_1) = \frac{1}{2}, P(E_2) = \frac{1}{3}, P(E_3) = \frac{1}{4}$$

$$\Rightarrow P(\bar{E}_1) = \frac{1}{2}, P(\bar{E}_2) = \frac{2}{3}, P(\bar{E}_3) = \frac{3}{4}$$

P (exactly one of them solves the problem)

$$= P[(E_1 \cap \bar{E}_2 \cap \bar{E}_3) \cup (\bar{E}_1 \cap E_2 \cap \bar{E}_3) \cup (\bar{E}_1 \cap \bar{E}_2 \cap E_3)] \quad [1]$$

$$= P(E_1 \cap \bar{E}_2 \cap \bar{E}_3) + P(\bar{E}_1 \cap E_2 \cap \bar{E}_3) + P(\bar{E}_1 \cap \bar{E}_2 \cap E_3)$$

$$= P(E_1) \times P(\bar{E}_2) \times P(\bar{E}_3) + P(\bar{E}_1) \times P(E_2) \times P(\bar{E}_3) + P(\bar{E}_1) \times P(\bar{E}_2) \times P(E_3) \quad [1]$$

$$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} + \frac{1}{2} \times \frac{1}{3} \times \frac{3}{4} + \frac{1}{2} \times \frac{2}{3} \times \frac{1}{4} = \frac{11}{24} \quad [1]$$

OR

Since X denotes the number of red balls then X can take the value 0, 1, 2 or 3

$$P(X = 0) = P(\text{getting no red ball}) = \frac{^4C_3}{^7C_3} = \frac{4}{35}$$

$$P(X = 1) = P(\text{getting 1 red and 2 white balls}) = \frac{^3C_1 \times ^4C_2}{^7C_3} = \frac{18}{35}$$

$$P(X = 2) = P(\text{getting 2 red and 1 white ball}) = \frac{^3C_2 \times ^4C_1}{^7C_3} = \frac{12}{35}$$

$$P(x = 3) = P(\text{getting 3 red balls}) = \frac{^3C_3}{^7C_3} = \frac{1}{35} \quad [1]$$

Thus, probability distribution of X is given below.

$X = x_i$	0	1	2	3
$P(x_i)$	$\frac{4}{35}$	$\frac{18}{35}$	$\frac{12}{35}$	$\frac{1}{35}$

$$\therefore \text{Mean } \mu = \sum x_i p_i = \left(0 \times \frac{4}{35}\right) + \left(1 \times \frac{18}{35}\right) + \left(2 \times \frac{12}{35}\right) + \left(3 \times \frac{1}{35}\right) = \frac{9}{7} \quad [2]$$

$$28. \text{ Let } I = \int_0^{\pi/2} \left(\frac{1}{3+2\cos x} \right) dx = \int_0^{\pi/2} \frac{1}{3+2\left(\frac{1-\tan^2 x/2}{1+\tan^2 x/2}\right)} dx = \int_0^{\pi/2} \frac{\sec^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)+5} dx \quad [1]$$

$$\text{Put } \tan \frac{x}{2} = t \Rightarrow \sec^2 \frac{x}{2} dx = 2dt$$

also, when $x = 0 \Rightarrow t = 0$

$$x = \frac{\pi}{2} \Rightarrow t = 1$$

$$\therefore I = 2 \int_0^1 \frac{1}{t^2 + (\sqrt{5})^2} dt = 2 \frac{1}{\sqrt{5}} \left[\tan^{-1} \frac{t}{\sqrt{5}} \right]_0^1 = \frac{2}{\sqrt{5}} \tan^{-1} \frac{1}{\sqrt{5}} \quad [2]$$

OR

$$\text{Let } I = \int_0^{\pi/4} \frac{\sin x + \cos x}{3 + \sin 2x} dx \Rightarrow I = \int_0^{\pi/4} \frac{\sin x + \cos x}{4 - (\sin x - \cos x)^2} \quad [1]$$

$$\text{Put } \sin x - \cos x = 1, (\cos x + \sin x)dx = dt, x = 0, t = -1, x = \frac{\pi}{4}, t = 0$$

$$I = \int_{-1}^0 \frac{dt}{(2)^2 - t^2} \Rightarrow I = \frac{1}{4} \left[\log \frac{2+t}{2-t} \right]_{-1}^0 \quad [1]$$

$$I = \frac{1}{4} \left[\log 1 - \log \frac{1}{3} \right] \Rightarrow I = -\frac{1}{4} \log \frac{1}{3} = \frac{1}{4} \log 3 \quad [1]$$

29. We have,

$$\frac{dy}{dx} = \frac{e^x (\sin^2 x + \sin 2x)}{y(2 \log y + 1)}$$

$$\Rightarrow y(2 \log y + 1) dy = e^x (\sin^2 x + \sin 2x) dx \quad [1]$$

Integrating both sides.

$$\Rightarrow \int y(2 \log y + 1) dy = \int e^x (\sin^2 x + \sin 2x) dx$$

$$\Rightarrow 2 \int y \log y dy + \int y dy = \int e^x (\sin^2 x + \sin 2x) dx$$

$$\Rightarrow 2 \left[\log y \cdot \frac{y^2}{2} - \int \frac{1}{y} \cdot \frac{y^2}{2} dy \right] + \frac{y^2}{2} = e^x \sin^2 x + C \quad [1]$$

$$\Rightarrow y^2 \log y - \int y dy + \frac{y^2}{2} = e^x \sin^2 x + C$$

$$\Rightarrow y^2 \log y = e^x \sin^2 x + C$$

This is the required solution. [1]

OR

The given differential equation may be written as $\frac{dy}{dx} = \frac{x^2 - 2y^2 + xy}{x^2}$, which is homogeneous differential equation.

$$\text{Put } y = vx \text{ and } \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$\therefore v + x \frac{dv}{dx} = \frac{x^2 - 2v^2 x^2 + vx^2}{x^2} \quad [1]$$

$$\Rightarrow v + x \frac{dv}{dx} = 1 - 2v^2 + v \Rightarrow x \frac{dv}{dx} = 1 - 2v^2 \Rightarrow \frac{1}{1 - 2v^2} dv = \frac{1}{x} dx \quad [1]$$

On integrating both sides

$$\begin{aligned} \int \frac{1}{1 - 2v^2} dv &= \int \frac{1}{x} dx & \Rightarrow \frac{1}{2} \int \frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2 - v^2} dv &= \int \frac{1}{x} dx \\ \Rightarrow \frac{1}{2\sqrt{2}} \log \left| \frac{\frac{1}{\sqrt{2}} + v}{\frac{1}{\sqrt{2}} - v} \right| &= \log|x| + C & \Rightarrow \frac{1}{2\sqrt{2}} \log \left| \frac{x + \sqrt{2}y}{x - \sqrt{2}y} \right| - \log|x| &= C \quad \left[\because v = \frac{y}{x} \right] \end{aligned} \quad [1]$$

This is the required solution.

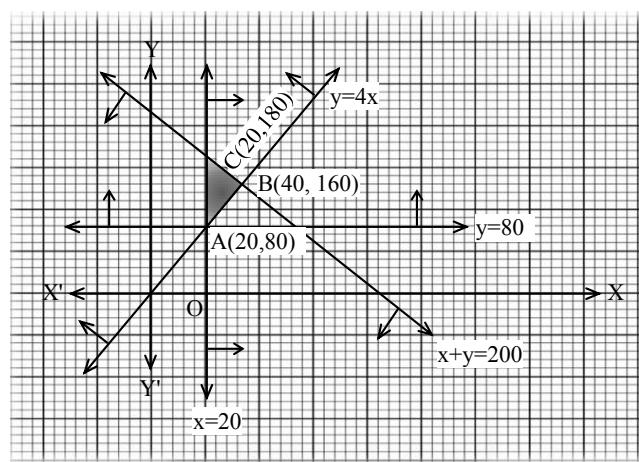
30. Maximize $Z = 400x + 300y$

Subject to constraints: $x \geq 20$, $y \geq 4x$, $y \geq 80$ and $x + y \leq 200$.

Corner points	$Z = 400x + 300y$
A (20, 80)	32000
B (40, 160)	64000
C (20, 180)	62000

\therefore maximum value of Z is 64000 at

$x = 40$ and $y = 160$



[1]

[2]

31. Let $I = \int \frac{x^3 - 1}{x^3 + x} dx \Rightarrow I = \int \left(1 - \frac{x+1}{x(x^2+1)}\right) dx \dots\dots(1)$

Let $\frac{x+1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1} \Rightarrow x+1 = A(x^2+1) + (Bx+C)x \dots\dots(2)$ [1]

Put $x=0 \Rightarrow A=1$

On comparing coefficient of x in (ii), we get $C=1$

On comparing coefficient of x^2 in (ii), we get $B=-1$

Thus, $\frac{x+1}{x(x^2+1)} = \frac{1}{x} + \frac{1-x}{x^2+1} \dots\dots(1)$

$$\therefore I = \int \left(1 - \frac{1}{x} - \frac{1-x}{x^2+1}\right) dx$$

$$\begin{aligned} \Rightarrow I &= \int 1 dx - \int \frac{1}{x} dx - \int \frac{1}{x^2+1} dx + \int \frac{x}{x^2+1} dx \\ &= x - \log|x| - \tan^{-1} x + \frac{1}{2} \log(x^2+1) + C \end{aligned} \quad [1]$$

SECTION – D

32. First we sketch the region whose area is to be found out, this region is the intersection of the regions $\{(x,y) : x^2 + y^2 \leq 4\}$ and $\{(x, y) : x + y \geq 2\}$

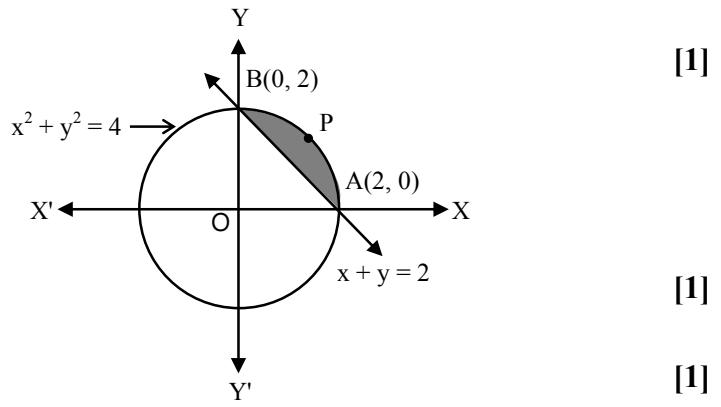
The point of intersection of $x^2 + y^2 = 4$ and $x + y = 2$ are $A(2, 0)$ and $B(0, 2)$ [1]

Required region is the shaded region APBA

Required area

$$\begin{aligned} &= \int_0^2 (y_{\text{circle}} dx) - \int_0^2 (y_{\text{line}} dx) \\ &= \int_0^2 \sqrt{4-x^2} dx - \int_0^2 (2-x) dx \\ &= \left[\frac{x}{2} \sqrt{4-x^2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2 - \left[2x - \frac{x^2}{2} \right]_0^2 \\ &= 0 + 2 \frac{\pi}{2} - 2 = (\pi - 2) \text{ sq. units} \end{aligned}$$

Correct fig.



33. Given, $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $R = \{(a, b) : \text{both } a \text{ and } b \text{ are either odd or even}\}$

(i) **Reflexive :** let $a \in A$

Then, it is clear that a and a are both odd or both even

$$\therefore (a, a) \in R, \forall a \in A$$

$\therefore R$ is reflexive

[1½]

(ii) **Symmetric :** Let $a, b \in A$

If $(a, b) \in R \Rightarrow a$ and b are both odd or both even

$\Rightarrow b$ and a are also both odd or both even

$$\Rightarrow (b, a) \in R,$$

$\therefore R$ is symmetric

[1½]

(iii) **Transitive :** Let $a, b, c \in A$

If $(a, b) \in R \Rightarrow a$ and b are both even or both odd

...(1)

and $(b, c) \in R \Rightarrow b$ and c are both even or both odd

...(2)

from (1) & (2) a and c are both even or both odd

$$\Rightarrow (a, c) \in R$$

$\therefore R$ is transitive

Since R is reflexive, symmetric and transitive on A therefore R is an equivalence relation on A .

[2]

OR

$$f : R \rightarrow R ; f(x) = \frac{x}{x^2 + 1}$$

(1) For one-one function: we have $f(x_1) = f(x_2), \forall x_1, x_2 \in R$

$$\frac{x_1}{x_1^2 + 1} = \frac{x_2}{x_2^2 + 1}$$

$$\Rightarrow x_1 x_2^2 + x_1 = x_2 x_1^2 + x_2 \quad \Rightarrow (x_1 x_2^2 - x_2 x_1^2) + (x_1 - x_2) = 0$$

$$\Rightarrow x_1 x_2 (x_2 - x_1) + (x_1 - x_2) = 0 \Rightarrow (x_1 - x_2)(-x_1 x_2 + 1) = 0$$

$$\Rightarrow x_1 = x_2 \text{ and } x_1 x_2 = 1 \text{ or } x_1 = \frac{1}{x_2}$$

Example : Let $x_1 = 2 \in R ; x_2 = 1/2 \in R$

Then, $f(x_1) = f(2) = 2/5$ and $f(x_2) = f(1/2) = 2/5$

$$\Rightarrow x_1 \neq x_2 \text{ but } f(x_1) = f(x_2)$$

$\therefore f$ is not one-one function

[2½]

(2) For onto function: Let $y = f(x)$

$$\Rightarrow y = \frac{x}{x^2 + 1} \Rightarrow x^2y - x + y = 0$$

$$\Rightarrow x = \frac{1 \pm \sqrt{1 - 4y^2}}{2y} \in R \text{ if } 1 - 4y^2 \geq 0 \text{ and } y \neq 0 \quad [1\frac{1}{2}]$$

$$\Rightarrow 4y^2 - 1 \leq 0 \text{ or } (2y + 1)(2y - 1) \leq 0$$

$$\begin{array}{c} + \\ -1/2 \\ - \\ + \\ 1/2 \end{array}$$

$$\Rightarrow x \in \left[-\frac{1}{2}, 0\right) \cup \left(0, \frac{1}{2}\right]$$

Thus, every element of Co-domain does not have its pre-images in domain.

Hence, $f : R \rightarrow R$ is not onto.

$\therefore f$ is neither one-one nor onto.

[1]

34. Given equation of line is $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11} = \lambda$ (say)(1)

Let N be the foot of perpendicular from the point P(2, -1, 5) to the line (1)

\therefore Coordinates of N are $(10\lambda + 11, -4\lambda - 2, -11\lambda - 8)$

[1]

Direction ratios of PN are :

$$10\lambda + 11 - 2, -4\lambda - 2 + 1, -11\lambda - 8 - 5 = 10\lambda + 9, -4\lambda - 1, -11\lambda - 13$$

[1]

and direction ratios of line (1) are 10, -4, -11.

Since PN \perp to line (1)

$$\Rightarrow 10(10\lambda + 9) - 4(-4\lambda - 1) - 11(-11\lambda - 13) = 0 \quad [\because a_1a_2 + b_1b_2 + c_1c_2 = 0]$$

$$\text{or } 100\lambda + 90 + 16\lambda + 4 + 121\lambda + 143 = 0$$

$$\Rightarrow 237\lambda + 237 = 0$$

$$\Rightarrow \lambda = -1$$

[2]

Hence ; foot of perpendicular is N (1,2,3)

$$\text{and } PN = \sqrt{(2-1)^2 + (-1-2)^2 + (5-3)^2}$$

$$= \sqrt{1+9+4} = \sqrt{14} \text{ units} \quad [1]$$

OR

The given lines are

$$L_1 : \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k}); \quad L_2 : \vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(\hat{i} - \hat{j} + \hat{k})$$

On comparing L_1 & L_2 with standard equation

$$\begin{aligned} \vec{r} &= \vec{a}_1 + \lambda \vec{b}_1 \quad \& \quad \vec{r} = \vec{a}_2 + \mu \vec{b}_2 \\ \vec{a}_1 &= \hat{i} + 2\hat{j} + 3\hat{k}, \quad \vec{b}_1 = \hat{i} - \hat{j} + \hat{k} \\ \vec{a}_2 &= 2\hat{i} - \hat{j} - \hat{k}, \quad \vec{b}_2 = \hat{i} - \hat{j} + \hat{k} \end{aligned} \quad [1]$$

$$\text{Now, } \vec{a}_2 - \vec{a}_1 = \hat{i} - 3\hat{j} - 4\hat{k}$$

$$\text{Clearly } \vec{b}_1 = \vec{b}_2 = \vec{b} \text{ (let)}$$

$\therefore L_1$ & L_2 are parallel. [1]

$$\text{Now, } \vec{b} \times (\vec{a}_2 - \vec{a}_1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 1 & -3 & -4 \end{vmatrix} = 7\hat{i} + 5\hat{j} - 2\hat{k} \quad [1]$$

$$\text{Also, } |\vec{b}| = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{3} \quad [1]$$

Shortest distances between 2 parallel lines

$$= \frac{|\vec{b} \times (\vec{a}_2 - \vec{a}_1)|}{|\vec{b}|} = \frac{|(7\hat{i} + 5\hat{j} - 2\hat{k})|}{\sqrt{3}} = \frac{\sqrt{49 + 25 + 4}}{\sqrt{3}} = \sqrt{26} \text{ units} \quad [1]$$

35. The given equations are $x - y + 2z = 1$; $2y - 3z = 1$; $3x - 2y + 4z = 2$

$$\text{Let } A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \quad [1]$$

Then, the given system in matrix form is $AX = B$.

$$\text{Now, } \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A \cdot \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} = I$$

$$\Rightarrow A^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} \quad [2]$$

$$\text{Now, } AX = B \Rightarrow X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -2+0+2 \\ 9+2-6 \\ 6+1-4 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix} \Rightarrow x = 0, y = 5, z = 3 \quad [2]$$

SECTION – E

36. Clearly, G be the centroid of ΔABC , therefore coordinates of G are

$$\left(\frac{3+4+2}{3}, \frac{0+3+3}{3}, \frac{1+6+2}{3} \right) = (3, 2, 3)$$

(i) Since, A $\equiv (0, 1, 2)$ and G $= (3, 2, 3)$

$$\therefore \overrightarrow{AG} = (3-0)\hat{i} + (2-1)\hat{j} + (3-2)\hat{k} = 3\hat{i} + \hat{j} + \hat{k}$$

$$\Rightarrow |\overrightarrow{AG}|^2 = 3^2 + 1^2 + 1^2 = 9 + 1 + 1 = 11 \Rightarrow |\overrightarrow{AG}| = \sqrt{11} \quad [1]$$

(ii) Here, $\overrightarrow{AB} = 3\hat{i} - \hat{j} - \hat{k}$, $\overrightarrow{AC} = 4\hat{i} + 2\hat{j} + 4\hat{k}$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & -1 \\ 4 & 2 & 4 \end{vmatrix} = -2\hat{i} - 16\hat{j} + 10\hat{k}$$

$$= |\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(-2)^2 + (-16)^2 + 10^2} = \sqrt{4 + 256 + 100} = \sqrt{360} = 6\sqrt{10}$$

$$\therefore \text{Area of } \Delta ABC = \frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{AC}|$$

$$\text{Hence, area of } \Delta ABC = \frac{1}{2} \times 6\sqrt{10} = 3\sqrt{10} \text{ sq. units} \quad [1]$$

(iii) Here, $\overrightarrow{AB} = 3\hat{i} - \hat{j} - \hat{k}$

$$\Rightarrow |\overrightarrow{AB}| = \sqrt{9 + 1 + 1} = \sqrt{11}$$

$$\text{Also, } \overrightarrow{AC} = 4\hat{i} + 2\hat{j} + 4\hat{k}$$

$$\Rightarrow |\overrightarrow{AC}| = \sqrt{16 + 4 + 16} = \sqrt{36} = 6$$

$$\text{Now, } |\overrightarrow{AB}| + |\overrightarrow{AC}| = \sqrt{11} + 6$$

$$= 3.32 + 6 = 9.32 \text{ units}$$

[2]

OR

(iii) The length of the perpendicular from the vertex D on the opposite face

$$= |\text{Projection of } \overrightarrow{AD} \text{ on } \overrightarrow{AB} \times \overrightarrow{AC}|$$

$$= \left| \frac{(2\hat{i} + 2\hat{j}) \cdot (-2\hat{i} - 16\hat{j} + 10\hat{k})}{\sqrt{(-2)^2 + (-16)^2 + 10^2}} \right|$$

$$= \left| \frac{-4 - 32}{\sqrt{360}} \right| = \frac{36}{6\sqrt{10}} = \frac{6}{\sqrt{10}} \text{ units}$$

[2]

37. (i) Since 'x' be the charge per bike per day and n be the number of bike rented per day

$$\text{Revenue } R(x) = n \times x$$

$$R(x) = (2000 - 10x)x \Rightarrow R(x) = 2000x - 10x^2$$

[1]

- (ii) Since $x = 260 > 200$

Hence, no bike will be rented ($n = 0$)

$$\therefore R(x) = 0$$

[1]

- (iii) We have, $R(x) = 2000x - 10x^2$

$$R'(x) = 2000 - 20x \text{ and } R''(x) = -20$$

for $R(x)$ to be the maximum $R'(x) = 0$ and $R''(x) < 0$

$$\Rightarrow 2000 - 20x = 0 \Rightarrow x = 100$$

Thus, $R(x)$ is maximum at $x = 100$

[2]

OR

- (iii) We have, $R(x) = 2000x - 10x^2$

$$R'(x) = 2000 - 20x \text{ and } R''(x) = -20$$

for $R(x)$ to be maximum $R'(x) = 0$ and $R''(x) < 0$

$$\Rightarrow 2000 - 20x = 0 \Rightarrow x = 100$$

$\therefore R(x)$ is maximum at $x = 100$

$$\therefore \text{Maximum revenue} = R(100) = 2000(100) - 10(100)^2 = \text{Rs.}1,00,000$$

[2]

38. (i) Let E_1 be the event that it rain on chosen day

E_2 be the event that it does not rain on chosen day

and A be the event that the weatherman predict rain.

$$\text{Then we have } P(E_1) = \frac{6}{366}, P(E_2) = \frac{360}{366}$$

$$P(A/E_1) = \frac{80}{100}, P(A/E_2) = \frac{20}{100}$$

\therefore Probability that it will not rain on the chosen day, if weatherman predict rain for that day $= P(E_2/A)$

$$= \frac{P(E_2) \cdot P(A/E_2)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)}$$

$$= \frac{\frac{360}{366} \times \frac{20}{100}}{\frac{6}{366} \times \frac{80}{100} + \frac{360}{366} \times \frac{20}{100}} = \frac{15}{16}$$

[2]

(ii) Let E_1 be the event that it rain on chosen day

E_2 be the event that it does not rain on chosen day

and A be the event that the weatherman predict rain.

Then we have

$$P(E_1) = \frac{6}{365}, P(E_2) = \frac{359}{365}$$

$$P(A/E_1) = \frac{80}{100}, P(A/E_2) = \frac{20}{100}$$

\therefore Probability that it will rain on the chosen day, if weatherman predict rain for that day = $P(E_1 / A)$

$$= \frac{P(E_1) \cdot P(A/E_1)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)}$$

$$= \frac{\frac{6}{365} \times \frac{80}{100}}{\frac{6}{365} \times \frac{80}{100} + \frac{359}{365} \times \frac{20}{100}} = \frac{24}{383}$$

[2]