Series : 1

TEST PAPER CBSE 2022-23

SET - 1

Question Paper Code No. 58/3/1

Roll No.

Candidates Must write the Code on the title page of the answer-book.

- Please check that this question paper contains 10 printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 38 questions.
- Please write down the Serial Number of the question in the answer-book before attempting it.
- 15 minutes time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.

APPLIED MATHEMATICS

Time allowed : 3 hours ***Maximum Marks : 80***

GENERAL INSTRUCTIONS:

Read the following instructions very carefully and strictly follow them:

(i) *This question paper contains* ***38*** *questions.* ***All*** *questions are* ***compulsory****.*

(ii) *This question paper is divided into* ***five*** *Sections –* ***A, B, C, D*** *and* ***E****.*

(iii) *In* ***Section A,*** *Questions no.* ***1*** *to* ***18*** *are multiple choice questions* ***(MCQs)*** *and questions number* ***19*** *and* ***20*** *are Assertion – Reason based questions of* ***1 mark*** *each.*

(iv) *In* ***Section B,*** *Questions no.* ***21*** *to* ***25*** *are very short answer* ***(VSA)*** *type questions, carrying* ***2 marks*** *each.*

(v) *In* ***Section C****, Questions no.* ***26*** *to* ***31*** *are short answer* ***(SA)*** *type questions, carrying* ***3 marks*** *each.*

(vi) *In* ***Section D****, Questions no.* ***32*** *to* ***35*** *are long answer* ***(LA)*** *type question carrying* ***5 marks*** *each.*

(vii) *In* ***Section E,*** *Questions no.* ***36*** *to* ***38*** *are case study based questions carrying* ***4 marks*** *each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and 3 questions in Section E.*

(ix) *Use of calculators is* ***not*** *allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. The last (unit) digit of $(22)^{12}$ is :

(a) 2 (b) 4 (c) 6 (d) 8

2. The least non-negative remainder, when 3^{15} is divided by 7 is :

(a) 1 (b) 5 (c) 6 (d) 7

3. If $A=\begin{bmatrix}1 & 0\\ -2 & 1\end{bmatrix}$ and $B=\begin{bmatrix}-5 & 10\\ -10 & -5\end{bmatrix}$, then AB is :

(a) $\begin{bmatrix}-5 & 10\\ 0 & -5\end{bmatrix}$ (b) $\begin{bmatrix}0 & -5\\ 25 & 10\end{bmatrix}$ (c) $\begin{bmatrix}10 & -25\\ -5 & 0\end{bmatrix}$ (d) $\begin{bmatrix}-5 & 10\\ 0 & -25\end{bmatrix}$

4. If $\begin{bmatrix}x+y & x+2\\ 2x-y & 16\end{bmatrix}=\begin{bmatrix}8 & 5\\ 1 & 3y+1\end{bmatrix}$, then the values of x and y are :

(a) x = 3, y = 5 (b) x = 5, y = 3 (c) x = 2, y = 7 (d) x = 7, y = 2

5. The ratio in which a grocer mixes two varieties of pulses costing ₹ 85 per kg and ₹ 100 per kg respectively so as to get a mixture worth ₹ 92 per kg, is:

(a) 7 : 8 (b) 8 : 7 (c) 5 : 7 (d) 7 : 5

6. If $\frac{|x+1|}{x+1}>0, x\in\mathbb{R}$, then :

(a) $x\in[-1,\infty]$ (b) $x\in(-1,\infty)$ (c) $x\in(-\infty,-1)$ (d) $x\in(-\infty,-1]$

7. A and B are square matrices each of order 3 such that |A| = –1 and |B| = 3. What is the value of |3AB| ?

(a) –9 (b) –18 (c) –27 (d) –81

8. If $\begin{vmatrix}2 & 3 & 2\\ x & x & x\\ 4 & 9 & 1\end{vmatrix}+3=0$, then the value of x is:

(a) –1 (b) 0 (c) 1 (d) 3

9. The relation between 'Marginal cost' and 'Average cost' of production 'x' units of a product is:

(a) $\frac{d(AC)}{dx}=x(MC-AC)$ (b) $\frac{d(AC)}{dx}=x(AC-MC)$

(c) $\frac{d(AC)}{dx}=\frac{1}{x}(AC-MC)$ (d) $\frac{d(AC)}{dx}=\frac{1}{x}(MC-AC)$

10. $\int(x-1)e^{-x}\,dx$ is equal to :

(a) $(x-2)e^{-x}+C$ (b) $xe^{-x}+C$ (c) $-xe^{-x}+C$ (d) $(x+1)e^{-x}+C$

11. The solution of the differential equation $\frac{dx}{x}+\frac{dy}{y}=0$ is :

(a) $\frac{1}{x}+\frac{1}{y}=C$ (b) $xy = C$

(c) $\log x \log y = C$ (d) $x + y = C$

12. If X is a Poisson variable such that P(X = 1) = 2P(X = 2), then P(X = 0) is :

(a) e (b) $\frac{1}{e}$ (c) 1 (d) e^2

13. If the calculated value of $|t|<t_v(\alpha)$, then the null hypothesis is :

(a) rejected (b) accepted

(c) cannot be determined (d) neither accepted nor rejected

14. For testing the significance of difference between the means of two independent samples, the degree of freedom (v) is taken as:

(a) n_1-n_2+2 (b) n_1-n_2-2 (c) n_1+n_2-2 (d) n_1+n_2-1

15. The straight line trend is represented by the equation :

(a) $y_c=a+bx$ (b) $y_c=a-bx$ (c) $y_c=na+b\Sigma x$ (d) $y_c=na-b\Sigma x$

16. The present value of a perpetuity of ₹ R payable at the end of each payment period, when the money is worth i per period, is given by :

(a) Ri (b) $R+\frac{R}{i}$ (c) $\frac{R}{i}$ (d) R – Ri

17. The effective rate which is equivalent to nominal rate of 10% p.a. compounded quarterly is:

(a) 10.25% (b) 10.38% (c) 10.47% (d) 10.53%

18. Region represented by $x \geq 0, y \geq 0$ lies in

(a) I quadrant (b) II quadrant (c) III quadrant (d) IV quadrant

Questions number **19 and 20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

19. **Assertion (A) :** The function $f(x) = (x + 2)e^{-x}$ is increasing in the interval $(-1, \infty)$.

Reason (R) : A function f(x) is increasing, if $f'(x) > 0$.

20. **Assertion (A):** The differential equation , representing the family of parabolas $y^2 = 4ax$, where 'a' is a parameter, is $x\frac{dy}{dx} - 2y = 0$.

Reason (R) : If the given family of curves has n parameters, then it is to be differentiated n times to eliminate the parameter and obtain the n^{th} order differential equation.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. Two pipes A and B can fill a tank in 24 minutes and 32 minutes respectively. If both the pipes are opened simultaneously, after how much time should B be closed so that the tank is full in 18 minutes ?

OR

In a one-kilometre race, A beats B by 30 seconds and B beats C by 15 seconds. If A beats C by 180 meters, then find the time taken by A to run 1 kilometre.

22. Solve for $x : \frac{x+3}{x-2} \leq 2$.

23. Solve the following system of equations by Cramer's rule :

$2x - y = 17, 3x + 5y = 6$

OR

Determine the integral value(s) of x for which the matrix A is singular:

$$A=\begin{bmatrix} x+1 & -3 & 4 \\ -5 & x+2 & 2 \\ 4 & 1 & x-6 \end{bmatrix}$$

24. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the ordinate is changing 8 times as fast as abscissa.

25. Suppose 2% of the items made by a factory are defective. Find the probability that there are 3 defective items in a sample of 100 items selected at random. (Given $e^{-2} = 0.135$)

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. A bottle is full of dettol. One-third of its dettol is taken away and an equal amount of water is poured into the bottle to fill it again. This operation is repeated three times. Find the final ratio of dettol to water in the bottle.

OR

A pipe A can fill a tank in 3 hours. There are two outlet pipes B and C from the tank which can empty it in 7 and 10 hours respectively. It all the three pipes are opened simultaneously, how long will it take to fill the tank ?

27. Find all the points of local maxima and local minima for the function $f(x) = x^3 - 6x^2 + 9x - 8$.

28. An unbiased die is thrown again and again until three sixes are obtained. Find the robability of obtaining a third six in the sixth throw of the die.

29. The mean weekly sales of a four-wheeler were 50 units per agency in 20 agencies. After an advertising campaign, the mean weekly sales increased to 55 units per agency with standard deviation of 10 units. Test whether the advertising campaign was successful.

(Use $t_{0.005} = 1.729$ for 19 d.f.)

30. An asset costs ₹ 4,50,000 with an estimated useful life of 5 years and a scrap value of ₹ 1,00,000. Using linear. depreciation method, find the annual depreciation of the asset and construct a yearly depreciation schedule.

OR

Amrita bought a car worth ₹ 12,50,000 and makes a down payment of ₹ 3,00,000. The balance amount is to be paid in 4 years by equal monthly instalments at an interest rate of 15% p.a. Find the EMI that Amrita has to pay for the car. {(Given $(1.0125)^{-48} = 0.5508565$)}

31. Maximise $z = 300x + 190y$

subject to constraints:

$$x + y \leq 24,$$
$$2x + y \leq 32,$$
$$x \geq 0, y \geq 0.$$

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. Find the inverse of the matrix :

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

and hence show that $AA^{-1} = I$.

OR

Using matrix method, solve the following system of equations for x, y and z :

$$x - y + z = 4$$
$$2x + y - 3z = 0$$
$$x + y + z = 2$$

33. Divide a number 15 into two parts such that the square of one part multiplied with the cube of the other part is maximum.

OR

Find a point on the curve $y^2 = 2x$ which is nearest to the point (1, 4).

34. Fit a straight line trend by method of least squares to the following data and find the trend values :

Year:	2010	2012	2013	2014	2015	2016	2019
Sales (in lakh ₹)	65	68	70	72	75	67	73

35. Define Compound Annual Growth Rate (CAGR) and give the formula for calculating CAGR. Using the formula, calculate CAGR of Vikas's investment given below :

Vikas invested ₹ 10,000 in a stock of a company for 6 years. The value of his investment at the end of each year is given below :

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
₹ 11,000	₹ 11,500	₹ 11,650	₹ 11,800	₹ 12,200	₹ 14,000

[Use $(1.4)^{1/6} = 1.058$]

SECTION E

This section comprises 3 case study based questions, of 4 marks each.

Case Study – 1

36. A factory produces bulbs, of which 6% are defective bulbs in a large bulk of bulbs.

Based on the above information, answer the following questions :

(i) Find the probability that in a sample of 100 bulbs selected at random, none of the bulbs is defective. (Use : $e^{-6} = 0.0024$)

(ii) Find the probability that the sample of 100 bulbs has exactly two defective bulbs.

(iii) (a) Find the probability that the sample of 100 bulbs will include not more than one defective bulb.

OR

(iii) (b) Find the mean and the variance of the distribution of number of defective bulbs in a sample of 100 bulbs.

Case Study – 2

37. A factory manufactures tennis rackets and cricket bats. A tennis racket takes $1\frac{1}{2}$ hours of machine time and 3 hours of craftsmanship in its making; while a cricket bat takes 3 hours of machine time and 1 hour of craftsmanship. In a day, the factory has availability of not more than 42 hours of machine time and 24 hours of craftsmanship. Profit on a racket and on a bat are ₹ 20 and ₹ 10 respectively.

Based on the above information, answer the following questions :

(i) If x and y are the numbers of bats and rackets manufactured by the factory, then write the expression of total profit.

(ii) Write the constraint that relates the number of craftsmanship hours.

(iii) (a) Determine the maximum profit (in ₹) earned by the factory.

OR

(iii) (b) How many bats and rackets respectively, are to manufactured to earn maximum profit ?

Case Study – 3

38. In the year 2010, Mr. Aggarwal took a home loan of ₹ 30,00,000 from State Bank of India at 7.5% p.a. compounded monthly for 20 years.

Based on the above information, answer the following questions :

(i) Determine the EMI.

(ii) Find the principal paid by Mr. Aggarwal in the 150^{th} instalment.

(iii) (a) Find the total interest paid by Mr. Aggarwal.

OR

(iii) (b) How much was paid by Mr. Aggarwal to repay the entire amount of home loan?

[Use $(1.00625)^{240} = 4.4608$; $(1.00625)^{91} = 1.7629$]