CLASS - XII
TEST PAPER CBSE 2022-23
SUBJECT: APPLIED MATHEMATICS
Answer key

Question	1	2	3	4	5	6	7	8	9	10
Answer	**c**	**c**	**d**	**a**	**b**	**b**	**b**	**a**	**d**	**c**
Question	11	12	13	14	15	16	17	18	19	20
Answer	**b**	**b**	**b**	**c**	**a**	**c**	**b**	**a**	**d**	**d**

21. Let B be closed after n minutes. Then, pipe A runs for 18 minutes and B runs for n minutes to fill the tank.

$$\therefore \frac{18}{24}+\frac{n}{32}=1$$

$$\Rightarrow \frac{3}{4}+\frac{n}{32}=1 \Rightarrow n = 8.$$

Hence, pipe B must be closed after 8 min

OR

Suppose A takes 't' seconds to run 1 km race. Then, B takes (t + 30) seconds and C takes (t + 30 + 15) seconds, i.e. (t + 45) seconds.

We find A beats C by (30 + 15) seconds= 45 seconds and it is given that A beats C by 180 metres.

$\therefore$ C runs 180 min 45 seconds

$\Rightarrow$ C runs 1000 min $\left(\frac{45}{180}\times 1000\right)$ seconds= 250 seconds.

$\therefore$ t + 45 = 250 $\Rightarrow$ t = 205

Hence, A takes 205 seconds to run 1 km

22. $\frac{x+3}{x-2}-2\le 0 \Rightarrow \frac{-x+7}{x-2}\le 0$ or $\frac{x-7}{x-2}\ge 0$

Thus, the solution set is $(-\infty, 2) \cup [7, \infty)$

23. Here, $D=\begin{vmatrix} 2 & -1 \\ 3 & 5 \end{vmatrix}=13$

$$D_1=\begin{vmatrix} 17 & -1 \\ 6 & 5 \end{vmatrix}=91$$

$$D_3 = \begin{vmatrix} 2 & 17 \\ 3 & 6 \end{vmatrix} = -39$$

Thus, $x = \frac{D_1}{D} = 7; y = \frac{D_2}{D} = -3$

OR

A is singular gives

$$\begin{vmatrix} x+1 & -3 & 4 \\ -5 & x+2 & 2 \\ 4 & 1 & x-6 \end{vmatrix} = 0$$

i.e. $(x + 1) [(x + 2) (x - 6) - 2] + 3[-5x + 30 - 8] + 4[- 5 - 4x - 8] = 0$

i.e. $(x + 1) (x^2 - 4x - 14) - 15x + 66 - 52 - 16x = 0$

i.e. $x^3 - 3x^2 - 49x = 0$

$x = 0, \frac{3 \pm \sqrt{205}}{2}$

Hence, x = 0 is the only integral value.

24. Here, $6y = x^3 + 2$

$\Rightarrow 6\frac{dy}{dt} = 3x^2\frac{dx}{dt}$

As $\frac{dy}{dt} = 8\frac{dx}{dt}$, we have

$$48\cancel{\frac{dx}{dt}} = 3x^2\cancel{\frac{dx}{dt}} \Rightarrow x = 4, -4$$

when x = 4, y = 11 ; when x = –4, y = $\frac{-31}{3}$.

$\therefore$ Points on the curve are (4, 11), $\left(-4, \frac{-31}{3}\right)$

25. Let p be the probability that an item is defective so, p = $\frac{2}{100}$ = 0.02.

Here n = 100 $\therefore$ m = np = 2

$$P(X = r) = \frac{m^r}{r!}e^{-m} = \frac{2^r e^{-2}}{r!}$$

$$\Rightarrow P(X = 3) = \frac{2^3 e^{-2}}{3!} = \frac{4}{3} \times 0.135 = 0.18$$

26. Let the original quantity of dettol be x litres and the quantity of Dettol replaced by water bey litres.

So, $y = \frac{x}{3}$. After 3 operations the quantity of dettol left = $x\left(1-\frac{y}{x}\right)^3$.

After 3 operations the quantity of water in the bottle = x − $x\left(1-\frac{x}{3x}\right)^3$

Hence, the required ratio is $x\left(1-\frac{x}{3x}\right)^3 : \left[x - x\left(1-\frac{x}{3x}\right)^3\right]$

$$= \left(1-\frac{1}{3}\right)^3 : \left[1-\left(1-\frac{1}{3}\right)^3\right]$$

$$= \frac{8}{27} : \frac{19}{27}$$

= 8: 19

OR

Hence, $n_A = 3$, $n_B = 7$ and $n_C = 10$.

$$\frac{1}{n} = \frac{1}{n_A} - \frac{1}{n_B} - \frac{1}{n_C}$$

$$\Rightarrow \quad \frac{1}{n} = \frac{1}{3} - \frac{1}{7} - \frac{1}{10}$$

$$\Rightarrow \quad \frac{1}{n} = \frac{19}{210} \Rightarrow n = 11\frac{1}{19}$$

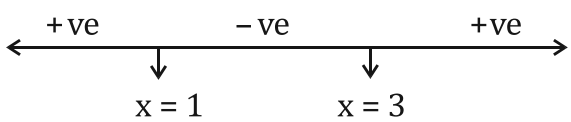
Hence, the tank is filled in $11\frac{1}{19}$ hours.

27. $y = x^3 - 6x^2 + 9x - 8$

$$\Rightarrow \frac{dy}{dx} = 3x^2 - 12x + 9$$

$$\Rightarrow \frac{dy}{dx} = 3(x-1)(x-3)$$

Critical points are 1, 3



Sign from left to right side on number line at x = 1 is changing from ⊕ve to Θve point of therefore at x = 1 is point of local maxima.

Sign from left to right side on number line at x = 3 is changing from Θve to ⊕ve point of therefore at x = 3 is point of local minima.

28. Let A be the event of obtaining two sixes in the first five throws of a die. Let B be the event of obtaining a six in the sixth throw of a die.

Then required probability = P(AB) = P(A) P(B)

$$\text{Here, } P(B) = \frac{1}{6} \text{ and } P(A) = 5_{C_2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3 = \frac{625}{3888}$$

$$\text{Thus, Required probability } = \frac{625}{3888} \times \frac{1}{6} = \frac{625}{23328}$$

29. We are given

$\mu = 50$, $\bar{x} = 55$, SD = 10, n = 20

$H_0 : \mu = 50$

$H_1 : \mu > 50$

$$t = \frac{\bar{x} - \mu}{\frac{SD}{\sqrt{n}}} = \frac{55 - 50}{\frac{10}{\sqrt{20}}} = 2.236$$

$t_{\text{cal value}} > t_{\text{tab value}}$

Hence H_0 is rejected.

So, Advertising Campaign was successful.

30. Here C = ₹ 4,50,000

S = ₹ 1,00,000

and n = 5 years.

Annual depreciation $D = \frac{C - S}{n}$ = ₹ 70,000

Thus, yearly depreciation schedule is as follows:

Years	Book value at the beginning of the year (in ₹)	Depreciation (in ₹)	Book value at the end of the year (in ₹)
1	4,50,000	70,000	3,80,000
2	3,80,000	70,000	3,10,000
3	3,10,000	70,000	2,40,000
4	2,40,000	70,000	1,70,000
5	1,70,000	70,000	1,00,000

OR

Here P = ₹ 9,50,000, i = $\frac{15}{1200}$ = 0.0125

n = 48 months

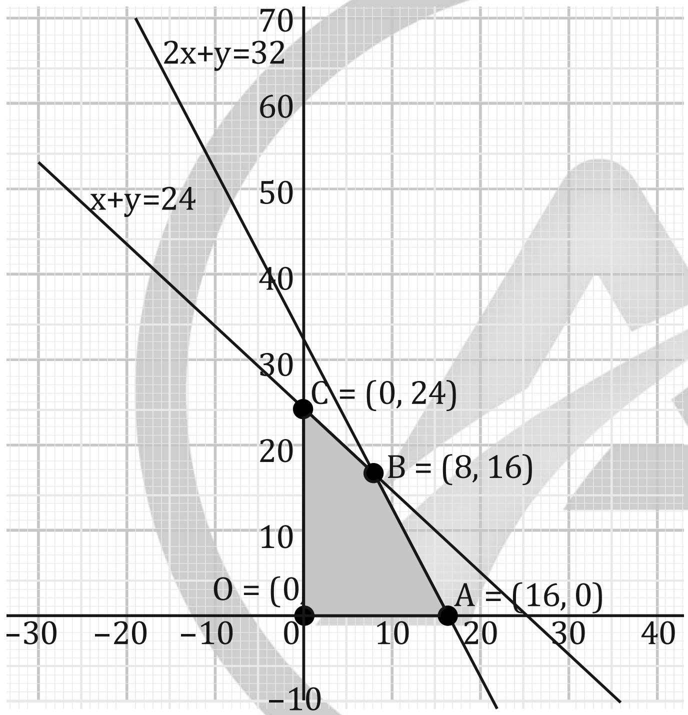
Using the reducing balancing method,

$$E = \frac{pi}{1-(1+i)^{-n}} = \frac{9{,}5{,}0000 \times 0.0125}{1-(1+0.0125)^{-48}}$$

$$= \frac{11875}{1-(1.0125)^{-48}} = \frac{11875}{1-0.5508565}$$

= ₹ 26,439.21

31.



Corner Points	Value of Z
O (0, 0)	0
A (16, 0)	4800
B (8, 16)	5440 $\rightarrow$ Max Value
C (0, 24)	4560

So Z is maximum at B (8, 16)

Max Value of Z = 5440

32. Here, |A| = –(–4 –3) – (12 + 1) +2(9 – 1)

= 7 – 13 + 16 = 10 $\neq$ 0

$$\Rightarrow \operatorname{adj}(A) = \begin{bmatrix} -7 & -13 & 8 \\ 2 & -2 & 2 \\ 3 & 7 & -2 \end{bmatrix}^T = \begin{bmatrix} -7 & 2 & 3 \\ -13 & -2 & 7 \\ 8 & 2 & -2 \end{bmatrix}$$

Hence $A^{-1} = \dfrac{1}{10}\begin{bmatrix} -7 & 2 & 3 \\ -13 & -2 & 7 \\ 8 & 2 & -2 \end{bmatrix}$

$$AA^{-1} = \frac{1}{10}\begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}\begin{bmatrix} -7 & 2 & 3 \\ -13 & -2 & 7 \\ 8 & 2 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

OR

The matrix equation AX = B is

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$|A| = 10$

$$\operatorname{adj} A = \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}' = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

Here $A^{-1} = \dfrac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$

So, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \dfrac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

Thus, $x = 2$, $y = -1$, $z = 1$

33. Let the two parts be x and 15 – x. Then, let $y = x^2(15 - x)^3$

$\Rightarrow \dfrac{dy}{dx} = x(15 - x)^2(-5x + 30)$

$\dfrac{dy}{dx} = 0$ gives $x = 0, 15, 6$

Rejecting $x = 0, 15$. Hence $x = 6$

$$\frac{d^2y}{dx^2} = (15-x)^2(-5x+30) - 5x(15-x)^2 - 2(15-x)\times x(-5x+30)$$

At $x = 6$.

$$\frac{d^2y}{dx^2} = -2430 < 0$$

Thus, y is maximum when x = 6 and y = 9.

So, the required two parts into which 15 should be divided are 6 and 9.

OR

Let P (x, y) be the required point which is nearest to Q (1, 4). Then distance PQ should be minimum and hence $(PQ)^2$ should be minimum.

Now, $(PQ)^2 = (x-1)^2 + (y-4)^2 = \left(\frac{y^2}{2} - 1\right)^2 + (y-4)^2$

$$= \frac{y^4 - 32y + 68}{4}$$

Let $D = \frac{y^4 - 32y + 68}{4}$

$$\frac{dD}{dy} = y^3 - 8$$

$$\frac{dD}{dy} = 0 \Rightarrow y = 2$$

Showing, y = 2 is a point of minima

Thus, the point is (2, 2)

34. Consider year 2014 as the year of origin. Calculation of trend values by method of least squares.

Year	Sales (in lakh ₹) y	Deviations from 2014 (x)	Squares of Deviations (x^2)	Sales deviation (xy)
2010	65	–4	16	–260
2012	68	–2	4	–136
2013	70	–1	1	–70
2014	72	0	0	0
2015	75	1	1	75
2016	67	2	4	134
2019	73	5	25	365
n = 7	$\sum y = 490$	$\sum x = 1$	$\sum x^2 = 51$	$\sum xy = 108$

The equation of the straight-line trend is

$$y_c = a + bx$$

Two normal equations are

$$\sum y = na + b\sum x$$

$$\sum xy = a\sum x + b\sum x^2$$

$\Rightarrow$ 490 = 7a + b and 108 = a + 51b

$\Rightarrow$ a = 69.9 and b = 0.75

$y_c = 69.9 + 0.75x$

Thus, trend values are

$$y_{2010} = 69.9 + 0.75(-4) = 66.90$$

$$y_{2012} = 69.9 + 0.75(-2) = 68.40$$

$$y_{2013} = 69.9 + 0.75(-1) = 69.15$$

$$y_{2014} = 69.9 + 0.75(0) = 69.90$$

$$y_{2015} = 69.9 + 0.75(1) = 70.65$$

$$y_{2016} = 69.9 + 0.75(2) = 71.40$$

$$y_{2019} = 69.9 + 0.75\,(5) = 73.65$$

35. CAGR is the mean annual growth rate of an investment over a specified period of time longer than one year.

$$\text{CAGR} = [\left[\frac{\text{Ending investment amount}}{\text{Start amount}}\right]^{\frac{1}{\text{no. of years}}} - 1$$

P.V. = ₹ 10,000

F.V. = ₹ 14,000

n = 6 years

$$\text{So CAGR} = \left(\frac{14000}{10000}\right)^{\frac{1}{6}} - 1 = (1.4)^{\frac{1}{6}} - 1$$

$$= 1.058 - 1 = 0.058$$

Hence, CAGR = 5.8%

36. n = 100 p = $\frac{6}{100}$, m = np

Here m = 100 x $\frac{6}{100}$ = 6.

$$P(r) = e^{-m}\frac{m^r}{r!}$$

(i) $P(0) = e^{-m}\frac{m^0}{0!} = e^{-6} = 0.0024$

(ii) $P(2) = e^{-m}\dfrac{m^2}{2!} = e^{-6}\times\dfrac{36}{2} = 0.0432$

(iii)(a) $P(0) + P(1) = e^{-6} + e^{-6}\dfrac{m^1}{1!} = e^{-6} + 6e^{-6} = 7e^{-6} = 0.0168$

OR

(iii)(b) Mean = Variance = m = np = 6

37. (i) $Z = 10x + 20y$

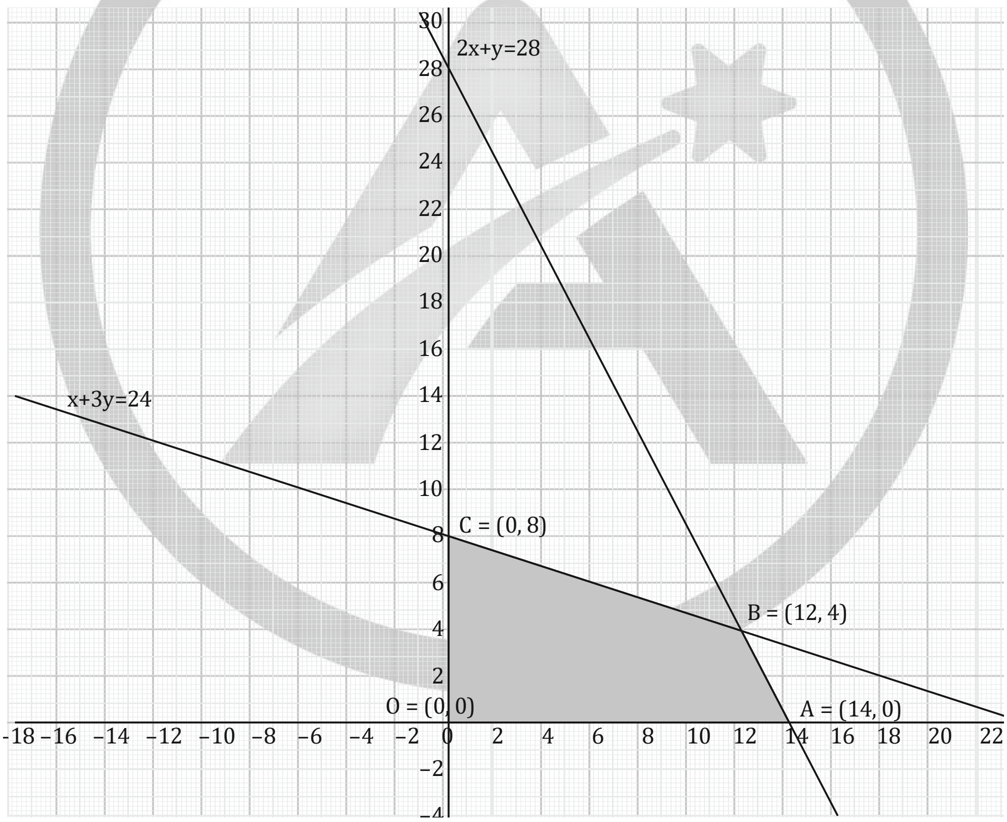
(ii) $x + 3y \le 24$

(iii) (a) other constraints are

$2x + y \le 28$

$x \ge 0$

$y \ge 0$

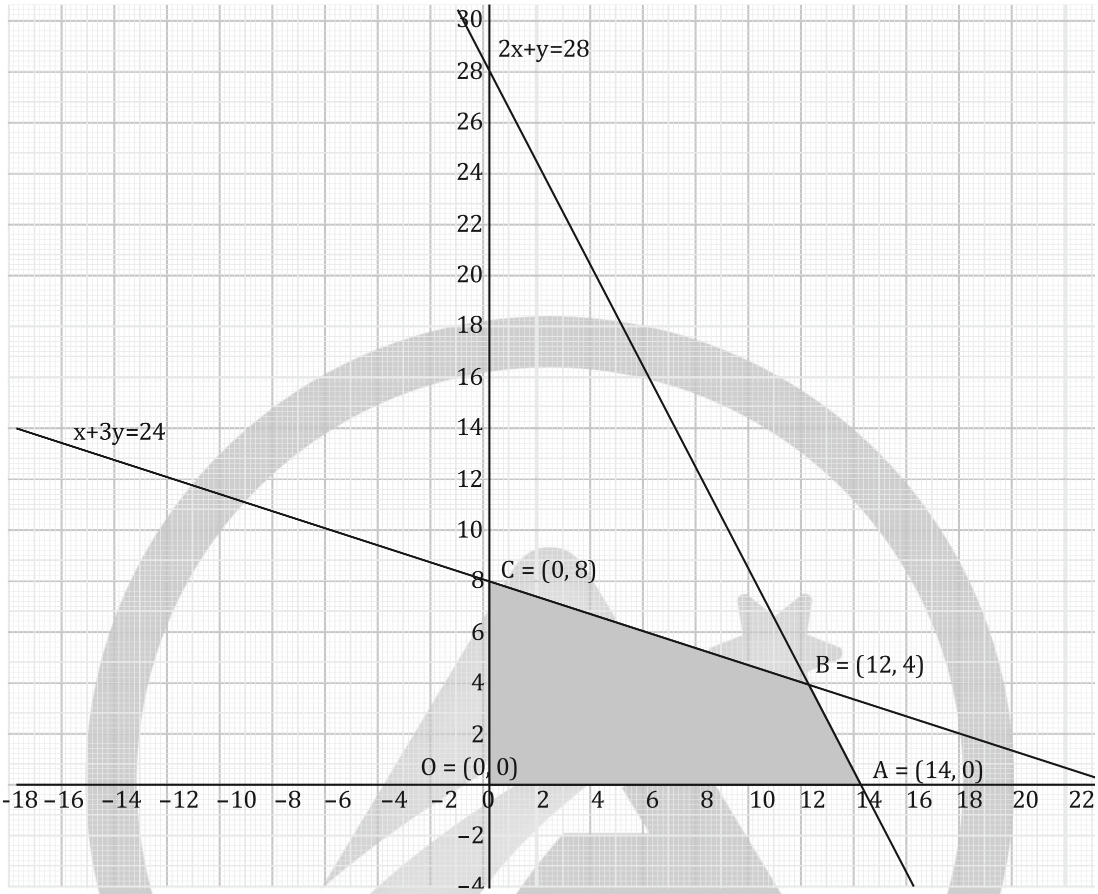


Corner Points	Value of Z
O (0, 0)	0
A (14, 0)	140
B (12, 4)	200 → Max Value
C (0, 8)	160

$\therefore$ P is maximum at B(12, 4); which is ₹ 200

OR

(iii) (b)



Corner Points	Value of Z
O (0, 0)	0
A (14, 0)	140
B (12, 4)	200 $\rightarrow$ Max Value
C (0, 8)	160

12 bats and 4 rackets

38. Given P = ₹ 30,00,000, i = $\dfrac{7.5}{1200}$ = 0.00625

and n = 12 × 20 = 240 months

(i) $\text{EMI} = \dfrac{pi}{1-(1+i)^{-n}}$

$$= \frac{30,00,000 \times 0.00625}{1-(1.00625)^{-240}-1}$$

$$= \frac{30,00,000 \times 0.00625 \times 4.4608}{3.4608}$$

= ₹ 24167.82

(ii) Interest paid on 150th instalment

$$= \frac{\text{EMI} \times \left[(1+i)^{240-150+1} - 1\right]}{(1+i)^{240-150+1}}$$

$$= \frac{24167 \times [1.7629 - 1]}{1.7629}$$

= ₹ 10458.70

⇒ Principal paid in 150th instalment = EMI − interest

= ₹ (24167.82 – 10458.70)

= ₹ 13709.12

(iii) (a) Total Interest paid = n × EMI × P

= ₹ (240 × 24167.82 – 30,00,000)

= ₹ 28,00,276.80

OR

(iii) (b) Total amount paid = n × EMI

= 240 × 2416.81

= ₹ 5800276.8