PRACTICE PAPER-1

CLASS – XII

SUBJECT: MATHEMATICS

Time : 3 Hrs. **Max. Marks : 80**

General Instructions :

1. This question paper contains **FIVE SECTIONS – A, B, C, D and E.** Each section is compulsory. However, there are internal choices in some question.
2. **SECTION A** has **18 MCQ's** and **02 Assertion-Reason** based questions of **1 mark** each.
3. **SECTION B** has **5 Very Short Answer (VSA)-type** questions of **2 marks** each.
4. **SECTION C** has **6 Short Answer (SA)-type** questions of **3 marks** each.
5. **SECTION D** has **4 Long Answer (LA)-type** questions of **5 marks** each.
6. **SECTION E** has **3 source based/case based/passage based/integrated units of assessment** (**4 marks** each) with sub parts.

SECTION – A

The following questions are multiple-choice questions with one correct answer. Each question carries 1 mark.

1. The number of matrices of order 3 × 3, whose entries are either 0 or 1 and the sum of all entries is a prime number, is

(a) 512 (b)283 (c) 282 (d) 230

2. Let $A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$, where x, y and z are real numbers such that x + y + z > 0 and xyz = 2. If $A^2 = I_3$, then the value of $x^3 + y^3 + z^3$, is

(a) 5 (b) 6 (c) 7 (d) 8

3. If $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $|2\vec{a} - \vec{b}| = 5$, then $|2\vec{a} + \vec{b}|$ equals:

(a) 17 (b) 7 (c) 5 (d) 1

4. Let the function $f(x)=\begin{cases}\dfrac{\log(1+5x)-\log(1+\alpha x)}{x} & ; \text{ if } x\neq 0\\ 10 & ; \text{ if } x=0\end{cases}$, is continuous at x = 0, then the value of α, is

(a) 5 (b) – 5 (c) 10 (d) – 10

5. If $\int \frac{\cos x-\sin x}{\sqrt{8-\sin 2x}}dx = a\sin^{-1}\left(\frac{\sin x+\cos x}{b}\right)+C$, where C is integration constant, then the value of a and b are respectively

(a) a = 1, b = 3 (b) a = 3, b = 1 (c) a = 2, b = 2 (d) $a=\frac{1}{3}, b=3$

6. If m and n are order and degree respectively, of the differential equation $\left(1+3\frac{d^2y}{dx^2}\right)^{5/2}=2\frac{d^3y}{dx^3}$, then m – n is equals

(a) 1 (b)2 (c) 3 (d) – 1

7. The solution set of the inequality 2x – y ≥ 1, is

(a) an open half plane containing origin.

(b) an closed half plane not containing origin.

(c) an open half plane not containing the line 2x – y = 1

(d) an open half plane containing the line 2x – y = 1

8. The projection of the vector $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$ on the vector $\vec{b}=\hat{i}-2\hat{j}+\hat{k}$, is

(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{3}{\sqrt{5}}$ (c) $\frac{4}{\sqrt{5}}$ (d) $\frac{5}{\sqrt{6}}$

9. The value of $\int_0^1 \frac{dx}{(1+x+x^2)}$, is

(a) $\frac{\pi}{\sqrt{3}}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{3\sqrt{3}}$ (d) $\frac{2\pi}{3\sqrt{3}}$

10. If A is an invertible matrix and $A^{-1}=\begin{bmatrix}3 & 4\\ 5 & 6\end{bmatrix}$, then A = ?

(a) $\begin{bmatrix}6 & -4\\ -5 & 3\end{bmatrix}$ (b) $\begin{bmatrix}1/3 & 1/4\\ 1/5 & 1/6\end{bmatrix}$ (c) $\begin{bmatrix}-3 & 2\\ 5/2 & -3/2\end{bmatrix}$ (d) $\begin{bmatrix}3/2 & -5/2\\ -2 & 3\end{bmatrix}$

11. The corner points of the bounded feasible region of an LPP are A(60, 0), B(40, 20), C(60, 30) and D(120, 0). The maximum value of objective function Z = 5x + 10y occurs at

(a) (60, 0) only
(b) (40, 20) only
(c) (60, 30) and (120, 0) only
(d) all points of line segment joining the points (60, 30) and (120, 0).

12. If A and B are two non-zero square matrices of same order such that AB = O, then

(a) $|A| = 0$ or $|B| = 0$
(b) $|A| = 0$ and $|B| = 0$
(c) $|A| \neq 0$ and $|B| \neq 0$
(d) None of these

13. If A and B are two square matrices such that

$A+B=\begin{bmatrix} 4 & -3 \\ 1 & 6 \end{bmatrix}$ and $A-B=\begin{bmatrix} -2 & -1 \\ 5 & 2 \end{bmatrix}$, then $|AB| = ?$

(a) 27
(b) 40
(c) – 27
(d) – 40

14. If A and B are two events such that $P(A)=\frac{1}{2}, P(B)=\frac{7}{12}$ and $P(A'\cup B')=\frac{1}{4}$ then events A and B are

(a) independent
(b) mutually exclusive
(c) not independent
(d) independent and mutually exclusive

15. The general solution of the differential equation $\frac{dy}{dx}=2^{x+y}$, is

(a) $2^x + 2^y = C$
(b) $2^x + 2^{-y} = C$
(c) $2^x - 2^{-y} = C$
(d) $2^{-x} + 2^y - C$

16. If $y = \cos^{-1}x$, then $\frac{d^2y}{dx^2}$ in terms of y is

(a) $\cot y \operatorname{cosec}^2 y$
(b) $-\cot^2 y \operatorname{cosec} y$
(c) $\cot^2 y \operatorname{cosec} y$
(d) $-\cot y \operatorname{cosec}^2 y$

17. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular units vectors, then $|\vec{a}+\vec{b}+\vec{c}| = ?$

(a) 1
(b) $\sqrt{2}$
(c) 3
(d) $\sqrt{3}$

18. Let P is any point on the line joining the points A(3, –1, 2) and B(0, 5, –2). If y-coordinate of P is 5, then its z-coordinate is

(a) 0 (b) 5 (c) – 2 (d) 2

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

19. **Assertion (A) :** The domain of the function $\operatorname{cosec}^{-1}2x$ is $\left(-\infty,-\frac{1}{2}\right]\cup\left[\frac{1}{2},\infty\right)$

Reason (R) : $\operatorname{cosec}^{-1}(-2)=-\frac{\pi}{6}$

20. **Assertion (A) :** If lines $\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}$ and $\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angle then the value of p is $\frac{70}{11}$.

Reason (R) : The lines $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ are perpendicular, if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

SECTION – B

This section comprises of very short answer type-questions (VSA) of 2 marks each

21. Evaluate: $\sin\left[2\cos^{-1}\left(\frac{-3}{5}\right)\right]$

OR

Show that the function f : R → R, f(x) = 3 – 4x is injective. Is the function bijective ? Justify your answer.

22. A man 160 cm tall, walks away from a source of light situated at the top of a pole 6 m high, at the rate of 1.1 m/s. How fast is the length of his shadow increasing when he is 1m away from the pole ?

23. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $|\vec{a}| = 3, |\vec{b}| = 5, |\vec{c}| = 7$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, find the angle between $\vec{a}$ and $\vec{b}$.

OR

Find the direction ratios of the line 6x – 2 = 3y + 1 = 2z – 2 also find the carteisan equation of a line parallel to this line and passing through the point (2, –1, –1)

24. If $\tan^{-1}\left(\frac{x^2 - y^2}{x^2 + y^2}\right) = a$, then show that $\frac{dy}{dx} = \frac{x(1 - \tan a)}{y(1 + \tan a)}$

25. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ such that $(\vec{a} + \lambda\vec{b}) \perp \vec{c}$, then find the value of λ.

SECTION – C

This section comprises of short answer type questions (SA) of 3 marks each

26. Evaluate: $\int \frac{2^x}{\sqrt{1 - 4^x}} dx$

27. A and B appear for an interview for two posts. The probability of A's selection is $\frac{1}{3}$ and that of B's selection is $\frac{2}{5}$. Find the probability that only one of them will be selected.

OR

Three defective bulbs are mixed with 7 good ones. Let X denotes the number of defective bulbs when 3 bulbs are drawn at random. Find the mean of X.

28. Evaluate: $\int_0^1 \frac{\log(1 + x)}{(1 + x^2)} dx$

OR

Evaluate: $\int_{-\pi/4}^{\pi/4} |\sin x| dx$

29. Solve the differential equation: $(x+2y^3)\frac{dy}{dx}=y$

OR

Solve the differential equation: $(x\sqrt{x^2+y^2}-y^2)dx+xydy=0$

30. Solve the following linear programming problem graphically:

Maximize Z = 400x + 300y

Subject to x + y ≤ 200, x ≥ 20, y ≥ 4x, y ≥ 80

31. Evaluate: $\int\frac{x^2+1}{(x+1)^2}dx$

SECTION – D

This section comprises of long answer-type questions (LA) of 5 marks each

32. Find the area of the region bounded by the curve $y^2 = 2y - x$ and the y-axis.

33. Show that the relation R defined on the set A = {1, 2, 3, 4, 5, 6}, given by R = {(a, b): |a – b| is even} is an equivalence relation. Also find the set of all elements related to 2.

OR

Let R be a relation on the set of natural numbers N defined by R = {(x, y) : x ∈ N, 2x + y = 20}. Find the domain and range of the relation R. also verify whether R is reflexive, symmetric and transitive.

34. Find the length and the equation of the line of shortest distance between the lines

$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$.

OR

Find the image of the point (1, 6, 3) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$.

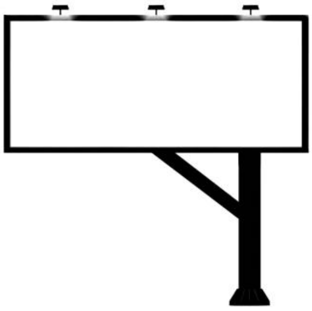
35. If $A=\begin{bmatrix}3 & 4 & 2\\ 0 & 2 & -3\\ 1 & -2 & 6\end{bmatrix}$, find A^{-1} and hence solve the following system of linear equation:

3x + 4y + 2z = 8, 2y – 3z = 3, x – 2y + 6z = – 2.

SECTION – E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)

36. A company wants to form a poster for advertisement Purpose. The top and bottom margins of poster should be 12 cm and the side margins should be 8 cm. Also, The area for printing the advertisement should be 1064 cm^2.



Based on the above information, answer the following question:

(i) If w cm be the width and h cm be the height of poster, then the area of poster, expressed in terms of w and h.

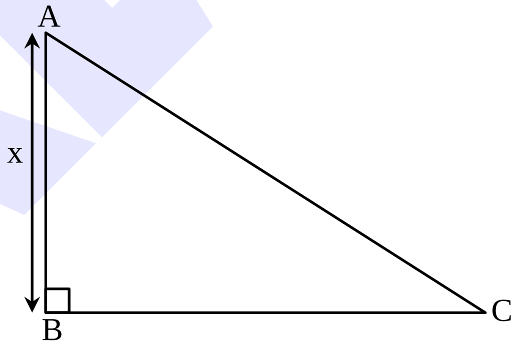
(ii) Find the area of the poster in terms of h.

(iii) Find the values of h and w, so that area of the poster is minimum.

OR

(iii) Find the minimum area of the poster.

37. The sum of the length of hypotenuse and a side of a right-angle triangle ABC such that AC + BC = 10



Based on the above information answer the following questions:

(i) Find the length of base BC in terms of x.

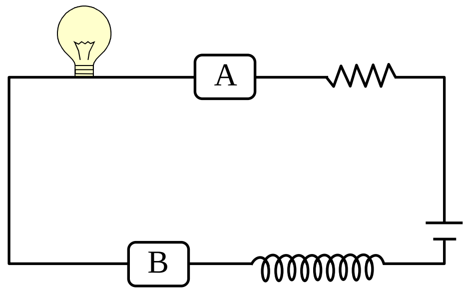
(ii) If S be the area of the triangle, then find the critical point of S.

(iii) Find the maximum area of the triangle ABC

OR

(iii) Find the length of hypotenuse, when area of triangle is maximum.

38. An electronic assembly consists of two sub-systems say A and B as shown below.



From, previous testing procedures, the following probabilities are assumed to be know

P(A fails) = 0.2, P(B fails alone) = 0.15, P(A and B fail) = 0.15.

On the basis of above information answer the following questions:

(i) Find the probability that B fails, also find the probabilities that, A fail alone.

(ii) Find the probability that whole system fails, also find the probability that B fails while given that A fails.