PRACTICE PAPER-1 (SOLUTION)

CLASS – XII

SUBJECT : MATHEMATICS

SECTION – A

1. (c)

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$; $a_{ij} \in \{0, 1\}$

Here $\sum a_{ij} = 2, 3, 5, 7$

Hence, total matrices = ${}^9C_2 + {}^9C_3 + {}^9C_5 + {}^9C_7 = 282$

2. (c)

$\because \quad A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$

$\therefore \quad |A| = x(yz - x^2) - y(y^2 - zx) + z(xy - z^2)$

$\Rightarrow \quad |A| = 3xyz - x^3 - y^3 - z^3$

given $A^2 = I_3$

$\Rightarrow \quad |A^2| = 1 \Rightarrow |A|^2 = 1$

$\Rightarrow \quad (3xyz - x^3 - y^3 - z^3)^2 = 1$

$\Rightarrow \quad x^3 + y^3 + z^3 - 3xyz = 1 \qquad [\because x + y + z > 0]$

$\Rightarrow \quad x^3 + y^3 + z^3 = 1 + 3xyz$

$\Rightarrow \quad x^3 + y^3 + z^3 = 1 + 3 \times 2 = 7 \qquad [\because xyz = 2]$

3. (c)

Given $|\vec{a}| = 2, |\vec{b}| = 3$

and $|2\vec{a} - \vec{b}| = 5 \Rightarrow |2\vec{a} - \vec{b}|^2 = 25$

$\Rightarrow \quad (2\vec{a} - \vec{b}).(2\vec{a} - \vec{b}) = 25$

$\Rightarrow \quad 4|\vec{a}|^2 + |\vec{b}|^2 - 4\vec{a}.\vec{b} = 25 \qquad \because |\vec{a}|^2 = \vec{a} \cdot \vec{a}$

$\Rightarrow \quad 4(2)^2 + (3)^2 - 4\vec{a}.\vec{b} = 25 \qquad (\because |\vec{a}| = 2, |\vec{b}| = 3)$

$\Rightarrow \quad \vec{a}\cdot\vec{b} = 0$

$\therefore \quad |2\vec{a}+\vec{b}|^2 = 4|\vec{a}|^2 + |\vec{b}|^2 + 4\vec{a}.\vec{b}$

$= 4(2)^2 + (3)^2 + 4(0) = 25$

$\therefore \quad |2\vec{a}+\vec{b}| = 5$

4. **(b)**

Given $f(x) = \begin{cases} \dfrac{\log(1+5x) - \log(1+\alpha x)}{x} & ; \quad \text{if } x \neq 0 \\ 10 & ; \quad \text{if } x = 0 \end{cases}$

since f(x) is continuous at x = 0, then

$\lim_{x\to 0} f(x) = f(0)$

$\Rightarrow \quad \lim_{x\to 0} 5.\dfrac{\log(1+5x)}{5x} - \lim_{x\to 0} \alpha\cdot\dfrac{\log(1+\alpha x)}{\alpha x} = 10$

$\Rightarrow \quad 5 \times 1 - \alpha \times 1 = 10 \qquad \left(\because \lim_{x\to 0} \log\dfrac{(1+x)}{x} = 1\right)$

$\Rightarrow \quad \alpha = -5$

5. **(a)**

Let $I = \int \dfrac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx = \int \dfrac{(\cos x - \sin x)}{\sqrt{9 - (\sin x + \cos x)^2}} dx$

put sinx + cosx = t $\Rightarrow$ (cosx – sinx)dx = dt

$\therefore \quad I = \int \dfrac{1}{\sqrt{9 - t^2}} dt$

$= \sin^{-1}\left(\dfrac{t}{3}\right) + C = \sin^{-1}\left(\dfrac{\sin x + \cos x}{3}\right) + C$

Now $\quad \sin^{-1}\left(\dfrac{\sin x + \cos x}{3}\right) + C = a\sin^{-1}\left(\dfrac{\sin x + \cos x}{b}\right) + C \qquad$ (given)

So, $\quad a = 1, b = 3$

6. **(a)**

Given differential equation $\left(1 + 3\dfrac{d^2y}{dx^2}\right)^{5/2} = 2\dfrac{d^3y}{dx^3}$

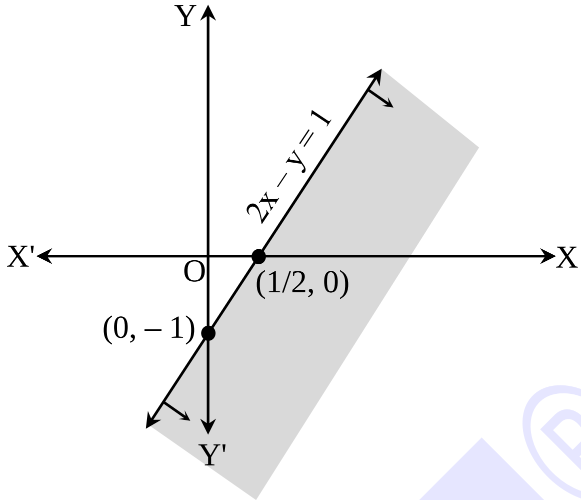
$\Rightarrow \quad \left(1 + 3\dfrac{d^2y}{dx^2}\right)^5 = 4\left(\dfrac{d^3y}{dx^3}\right)^2$

So, order = m = 3, degree = n = 2

$\therefore \quad m - n = 3 - 2 = 1$

7. **(d)**

The solution region of the inequality $2x - y \geq 1$ is an open half plane containing the line $2x - y = 1$.



8. **(d)**

Given $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

the projection of $\vec{a}$ on $\vec{b} = \vec{a} \cdot \hat{b}$

$$= (2\hat{i} - \hat{j} + \hat{k}) \cdot \frac{(\hat{i} - 2\hat{j} + \hat{k})}{\sqrt{6}} = \frac{2+2+1}{\sqrt{6}} = \frac{5}{\sqrt{6}}$$

9. **(c)**

Let $I = \int_0^1 \frac{dx}{(1 + x + x^2)}$

$$= \int_0^1 \frac{dx}{\left\{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}\right\}} = \left[\frac{2}{\sqrt{3}} \tan^{-1} \frac{\left(x + \frac{1}{2}\right)}{\frac{\sqrt{3}}{2}}\right]_0^1$$

$$= \frac{2}{\sqrt{3}}\left[\tan^{-1}\sqrt{3} - \tan^{-1}\frac{1}{\sqrt{3}}\right] = \frac{2}{\sqrt{3}}\left[\frac{\pi}{3} - \frac{\pi}{6}\right] = \frac{\pi}{3\sqrt{3}}$$

10. **(c)**

Given $A^{-1} = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$

$|A^{-1}| = 18 - 20 = -2$

$\text{adj}(A^{-1}) = \begin{bmatrix} 6 & -4 \\ -5 & 3 \end{bmatrix}$

$\therefore \quad (A^{-1})^{-1} = \frac{1}{|A^{-1}|}(\text{adj}A^{-1}) = -\frac{1}{2}\begin{bmatrix} 6 & -4 \\ -5 & 3 \end{bmatrix}$

$\therefore \quad A = (A^{-1})^{-1} = \begin{bmatrix} -3 & 2 \\ 5/2 & -3/2 \end{bmatrix}$

11. **(d)**

Corner points	Z = 5x + 10y
A(60, 0)	$Z_A = 300 + 0 = 300$
B(40, 20)	$Z_B = 200 + 200 = 400$
C(60, 30)	$Z_C = 300 + 300 = 600$ (Max.)
D(120, 0)	$Z_D = 600 + 0 = 600$ (Max.)

Since, feasible region is bounded, so maximum value of Z occurs at all points of line segment joining the points (60, 30) and (120, 0).

12. **(b)**

Given $A \neq O$, $B \neq O$ such that $AB = O$

$\Rightarrow \quad |A| = 0$ and $|B| = 0$

13. **(b)**

Given $A + B = \begin{bmatrix} 4 & -3 \\ 1 & 6 \end{bmatrix}$ and $A - B = \begin{bmatrix} -2 & -1 \\ 5 & 2 \end{bmatrix}$

$\Rightarrow \quad A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$

$\Rightarrow \quad |A| = 4 + 6 = 10, |B| = 6 - 2 = 4$

$|AB| = |A||B| = 10 \times 4 = 40$

14. **(c)**

Given $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P(A' \cup B') = \frac{1}{4}$

$\Rightarrow \quad P(A \cap B)' = \frac{1}{4} \Rightarrow P(A \cap B) = 1 - \frac{1}{4} = \frac{3}{4}$

$\because \quad P(A \cap B) \neq 0$, So A and B are not mutually exclusive

given $P(A) \times P(B) = \frac{1}{2} \times \frac{7}{12} = \frac{7}{24} \neq P(A \cap B)$

So, A and B are not independent

15. (b)

Given $\frac{dy}{dx} = 2^{x+y} = 2^x.2^y$

$\Rightarrow \quad 2^{-y}dy = 2^x dx$

$\Rightarrow \quad \int 2^{-y}\,dx = \int 2^x\,dx$

$\Rightarrow \quad -\frac{2^{-y}}{\log 2} = \frac{2^x}{\log 2} + \log C$

$(2^x + 2^{-y}) = -\log 2 \cdot \log C$

$2^x + 2^{-y} = C$, where $-\log 2 \cdot \log C = C$

16. (d)

$\because\ y = \cos^{-1}x$

$\Rightarrow \quad x = \cos y$

$\Rightarrow \quad 1 = -\sin y\frac{dy}{dx}$

$\Rightarrow \quad \frac{dy}{dx} = -\operatorname{cosec} y \quad ...(i)$

$\Rightarrow \quad \frac{d^2y}{dx^2} = \operatorname{cosec} y \cot y \times \frac{dy}{dx}$

$\therefore \quad \frac{d^2y}{dx^2} = -\cot y \operatorname{cosec}^2 y$ [using (i)]

17. (d)

Given $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$ and $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 0$

$\because \quad |\vec{a}+\vec{b}+\vec{c}|^2 = (\vec{a}+\vec{b}+\vec{c})\cdot(\vec{a}+\vec{b}+\vec{c})$

$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$

$= 3 + 2(0) = 3$

$\therefore \quad |\vec{a}+\vec{b}+\vec{c}| = \sqrt{3}$

18. (c) Eq. of a line through points A(3, –1, 2) and B(0, 5, –2) is

$\frac{x-3}{0-3} = \frac{y+1}{5+1} = \frac{z-2}{-2-2}$ i.e. $\frac{x-3}{-3} = \frac{y+1}{6} = \frac{z-2}{-4}$

coordinates of any point P on the line is $(-3\lambda + 3, 6\lambda - 1, -4\lambda + 2)$, where λ is constant

given $6\lambda - 1 = 5 \Rightarrow \lambda = 1$

So, z-coordinate of the point P is

$-4 \times 1 + 2 = -2$

19. **(b)**

Assertion: $\text{cosec}^{-1} x$ is defined if $x \le -1$ or $x \ge 1$

i.e. $\text{cosec}^{-1} 2x$ will be defined if $x \le -\frac{1}{2}$ or $x \ge \frac{1}{2}$

So, domain of $\text{cosec}^{-1} 2x$ is $\left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$

Hence, A is true.

Reason: The range of $\text{cosec}^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

let $\text{cosec}^{-1}(-2) = \theta$

$\Rightarrow \text{cosec}\theta = -2 = \text{cosec}\left(-\frac{\pi}{6}\right) \Rightarrow \theta = -\frac{\pi}{6} \Rightarrow \text{cosec}^{-1}(-2) = -\frac{\pi}{6}$

Hence, Both A and R is true but R is not the correct explanation of A.

20. **(a)**

Assertion: given lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ i.e. $\frac{x-1}{-3} = \frac{y-2}{(2p/7)} = \frac{z-3}{2}$ ...(i)

and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ i.e. $\frac{x-1}{(-3p/7)} = \frac{y-5}{1} = \frac{z-6}{(-5)}$ ...(ii)

Since, lines (i) and (ii) are at right angle, then

$(-3) \times \left(\frac{-3p}{7}\right) + \frac{2p}{7} \times 1 + 2 \times (-5) = 0$

$\Rightarrow \quad \frac{9p}{7} + \frac{2p}{7} - 10 = 0 \Rightarrow \frac{11p}{7} = 10 \Rightarrow p = \frac{70}{11}$

Reason: Statement is true.

Hence A and R both are true and R is correct explanation of A

SECTION – B

21. Let $y = \sin\left[2\cos^{-1}\left(\frac{-3}{5}\right)\right]$

put $\cos^{-1}\left(-\frac{3}{5}\right) = \theta \Rightarrow \cos\theta = -\frac{3}{5}$, where $\theta \in [0, \pi]$

Since $\theta \in [0, \pi]$, So $\sin\theta > 0$

$\therefore \quad \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{9}{25}} = \frac{4}{5}$

$\therefore \quad y = \sin[2\theta] = 2\sin\theta\cos\theta$

$\Rightarrow \quad y = 2\left(\frac{4}{5}\right)\left(\frac{-3}{5}\right) = -\frac{24}{25}$

OR

Given function $f : R \to R$, $f(x) = 3 - 4x$

Let $x_1, x_2 \in R$ (domain) such that if

$f(x_1) = f(x_2) \Rightarrow 3 - 4x_1 = 3 - 4x_2$

$\Rightarrow$ $x_1 = x_2$

$\therefore$ f is injective.

Now let $y \in R$ (codomain) and $y = f(x)$

$\Rightarrow$ $y = 3 - 4x$

$\Rightarrow$ $x = \frac{3-y}{4} \in R$ (domain), $\forall\ y \in R$ (codomain)

Such that $f(x) = f\left(\frac{3-y}{4}\right) = \left\{3 - \frac{4(3-y)}{4}\right\} = y$

Thus, every element of codomain has pre-image in domain

$\therefore$ f is onto

Hence, f is one-one and onto i.e. f is bijective.

22. Let AB be the lamp post, the lamp being at B

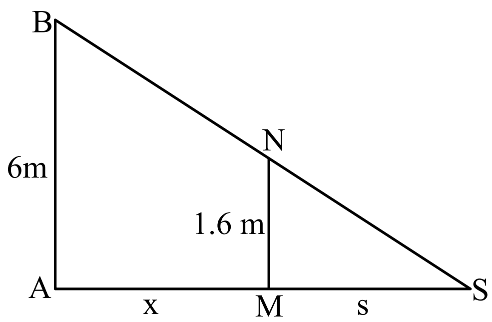
Then AB = 6 m

at any time t, let MN be the position of the man and MS his shadow.

Then MN = 1.6 m

let AM = x m and MS = s m

given $\frac{dx}{dt} = 1.1$ m/s



Here $\Delta SAB \sim \Delta SMN$

$\therefore$ $\frac{AS}{MS} = \frac{AB}{MN}$

$\Rightarrow$ $\frac{x+s}{s} = \frac{6}{1.6} = \frac{15}{4}$

$\Rightarrow$ $x = \frac{11}{4}s$

$\Rightarrow \quad \frac{dx}{dt} = \frac{11}{4}\frac{ds}{dt}$

$\Rightarrow \quad 1.1 = \frac{11}{4}\cdot\frac{ds}{dt}$

$\Rightarrow \quad \frac{ds}{dt} = \frac{1.1 \times 4}{11} = 0.4 \text{ m/s}$

Hence, the length of the shadow is increasing at the rate of 0.4 m/s.

23. Let θ be the angle between $\vec{a}$ and $\vec{b}$.

given $\quad \vec{a} + \vec{b} + \vec{c} = \vec{0}$

$\Rightarrow \quad \vec{a} + \vec{b} = -\vec{c}$

$\Rightarrow \quad (\vec{a} + \vec{b})\cdot(\vec{a} + \vec{b}) = (-\vec{c})\cdot(-\vec{c})$

$\Rightarrow \quad |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = |\vec{c}|^2 \qquad (\because \vec{a}\cdot\vec{a} = |\vec{a}|^2, \vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a})$

$\Rightarrow \quad |\vec{a}|^2 + |b|^2 + 2|\vec{a}||\vec{b}|\cos\theta = |\vec{c}|^2$

$\Rightarrow \quad 9 + 25 + 2 \times 3 \times 5 \times \cos\theta = 49 \qquad (\because |\vec{a}| = 3, |\vec{b}| = 5, |\vec{c}| = 7)$

$\Rightarrow \quad 30\cos\theta = 15$

$\Rightarrow \quad \cos\theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$

Hence, the angle between $\vec{a}$ and $\vec{b}$ is 60°

OR

Eq. of given line is

$6x - 2 = 3y + 1 = 2z - 2$

$\Rightarrow \quad \frac{x - \frac{1}{3}}{\left(\frac{1}{6}\right)} = \frac{y + \frac{1}{3}}{\left(\frac{1}{3}\right)} = \frac{z-1}{\left(\frac{1}{2}\right)}$

$\Rightarrow \quad \frac{x - \frac{1}{3}}{1} = \frac{y + \frac{1}{3}}{2} = \frac{z-1}{3}$

Here, The direction ratios of the line are 1, 2, 3

Now, The direction ratios of a lines parallel to given line are also 1, 2, 3.

The Cartesian equation of the line passes through (2, –1, –1) and parallel to given line, is

$\frac{x-2}{1} = \frac{y+1}{2} = \frac{z+1}{3}$

24. Given $\tan^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right)=a \Rightarrow \frac{x^2-y^2}{x^2+y^2}=\tan a$

$\Rightarrow \quad x^2-y^2=(x^2+y^2)\tan a$

diff. both sides w.r.to x

$$2x-2y\frac{dy}{dx}=\left(2x+2y\frac{dy}{dx}\right)\tan a$$

$$\Rightarrow \quad 2y(1+\tan a)\frac{dy}{dx}=2x(1-\tan a)$$

$$\Rightarrow \quad \frac{dy}{dx}=\frac{x(1-\tan a)}{y(1+\tan a)}$$

25. Given $\vec{a}=2\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=-\hat{i}+2\hat{j}+\hat{k}$, $\vec{c}=3\hat{i}+\hat{j}$

$\therefore \quad \vec{a}+\lambda\vec{b}=(2-\lambda)\hat{i}+(2+2\lambda)\hat{j}+(3+\lambda)\hat{k}$

Now, $(\vec{a}+\lambda\vec{b})\perp\vec{c} \Rightarrow (\vec{a}+\lambda\vec{b})\cdot\vec{c}=0$

$\Rightarrow \{(2-\lambda)\hat{i}+(2+2\lambda)\hat{j}+(3+\lambda)\hat{k}\}\cdot(3\hat{i}+\hat{j})=0$

$\Rightarrow 3(2-\lambda)+(2+2\lambda)=0$

$\Rightarrow \lambda=8$

SECTION – C

26. Let $I=\int\frac{2^x}{\sqrt{1-4^x}}dx$

put $2^x=t \Rightarrow (2^x\log 2)dx=dt$

$$\therefore \quad I=\frac{1}{(\log 2)}\int\frac{1}{\sqrt{1-t^2}}dt$$

$$=\frac{1}{(\log 2)}\cdot\sin^{-1}t+C$$

$$=\frac{1}{(\log 2)}\sin^{-1}(2^x)+C$$

27. Let E_1 : event that A is selected.

E_2 : event that B is selected.

then $P(E_1)=\frac{1}{3}$, $P(E_2)=\frac{2}{5}$

$\Rightarrow \quad P(\bar{E}_1)=\frac{2}{3}, P(\bar{E}_2)=\frac{3}{5}$

Req. prob. = P(only one of them is selected)

= P((E_1 and not E_2) or (E_2 and not E_1))

$$= P((E_1 \cap \bar{E}_2) \cup (\bar{E}_1 \cap E_2))$$

$$= P(E_1 \cap \bar{E}_2) + P(\bar{E}_1 \cap E_2)$$

$$= P(E_1) \cdot P(\bar{E}_2) + P(\bar{E}_1) \cdot P(E_2)$$

$$= \left(\frac{1}{3} \times \frac{3}{5}\right) + \left(\frac{2}{3} \times \frac{2}{5}\right)$$

$$= \frac{1}{5} + \frac{4}{15} = \frac{7}{15}$$

OR

Since, X denotes the number of defective bulbs, then the possible values of X are 0, 1, 2 or 3

∴ P(X = 0) = P(none of the bulbs is defective)

$$= \frac{^7C_3}{^{10}C_3} = \frac{7 \times 6 \times 5}{10 \times 9 \times 8} = \frac{7}{24}$$

P(X = 1)= P(1 defective and 2 non-defective bulbs)

$$= \frac{^3C_1 \times {}^7C_2}{^{10}C_3} = \left(3 \times \frac{7 \times 6}{2 \times 1} \times \frac{3 \times 2 \times 1}{10 \times 9 \times 8}\right) = \frac{21}{40}$$

P(X = 2) = P(2 defective and 1 non-defective bulbs)

$$= \frac{^3C_2 \times {}^7C_1}{^{10}C_3} = \left(\frac{3 \times 2}{2 \times 1} \times 7 \times \frac{3 \times 2 \times 1}{10 \times 9 \times 8}\right) = \frac{7}{40}$$

P(X = 3) = P(3 defective bulbs)

$$= \frac{^3C_3}{^{10}C_3} = \left(1 \times \frac{3 \times 2 \times 1}{10 \times 9 \times 8}\right) = \frac{1}{120}$$

Thus the probability distribution of X is

X	0	1	2	3
P(X)	$\frac{7}{24}$	$\frac{21}{40}$	$\frac{7}{40}$	$\frac{1}{120}$

∴ Mean (μ) = $\sum x_i \times P(x)$

$$= 0 \times \frac{7}{24} + 1 \times \frac{21}{40} + 2 \times \frac{7}{40} + 3 \times \frac{1}{120}$$

$$= \frac{63 + 42 + 3}{120} = \frac{108}{120} = \frac{9}{10}$$

28. Let $I=\int_0^1 \frac{\log(1+x)}{(1+x^2)}dx$

put x = tanθ ⇒ dx = sec²θdθ

when x = 0, θ = 0 & x = 1, θ = $\frac{\pi}{4}$

$$\therefore \quad I=\int_0^{\pi/4}\frac{\log(1+\tan\theta)}{(1+\tan^2\theta)}\cdot\sec^2\theta d\theta$$

$$I=\int_0^{\pi/4}\log(1+\tan\theta)d\theta \qquad ...(i)$$

$$I=\int_0^{\pi/4}\log\left[1+\tan\left(\frac{\pi}{4}-\theta\right)\right]d\theta \qquad \left(\because \int_0^a f(x)dx=\int_0^a f(a-x)dx\right)$$

$$I=\int_0^{\pi/4}\log\left[1+\frac{(1-\tan\theta)}{(1+\tan\theta)}\right]d\theta=\int_0^{\pi/4}\log\left[\frac{2}{1+\tan\theta}\right]d\theta$$

$$I=\int_0^{\pi/4}\log 2d\theta-\int_0^{\pi/4}\log(1+\tan\theta)d\theta$$

$$I=\log 2[\theta]_0^{\pi/4}-I \qquad \text{[using (i)]}$$

$$2I=\frac{\pi}{4}\log 2$$

$$\therefore \quad I=\frac{\pi}{8}\log 2$$

OR

Let $I=\int_{-\pi/4}^{\pi/4}|\sin x|dx$

$$I=\int_{-\pi/4}^{0}(-\sin x)dx+\int_0^{\pi/4}\sin xdx$$

$$I=[\cos x]_{-\pi/4}^{0}+[-\cos x]_0^{\pi/4}$$

$$I=\left\{\cos 0-\cos\left(-\frac{\pi}{4}\right)\right\}+\left\{-\cos\frac{\pi}{4}+\cos 0\right\}$$

$$I=\left(1-\frac{1}{\sqrt{2}}\right)+\left(-\frac{1}{\sqrt{2}}+1\right)$$

$$I=2-\frac{2}{\sqrt{2}}=(2-\sqrt{2})$$

29. Given diff. equation is $(x+2y^3)\frac{dy}{dx}=y$

$$\Rightarrow \quad \frac{dx}{dy}=\frac{x+2y^3}{y}$$

$$\Rightarrow \quad \frac{dx}{dy}-\frac{1}{y}\cdot x=2y^2$$

this is linear diff. equation of the form $\frac{dx}{dy}+Px=Q$

Here, $P=-\frac{1}{y}$, $Q=2y^2$

I.F. = $e^{\int P dy} = e^{\int -\frac{1}{y} dy} = e^{-\log y} = \frac{1}{y}$

So, the solution of given diff. equation is

x × IF = $\int (Q \times IF) dy + C$

$$\Rightarrow \quad x \times \frac{1}{y} = \int \left(2y^2 \times \frac{1}{y}\right) dy + C$$

$$\Rightarrow \quad \frac{x}{y} = \int 2y dy + C$$

$$\Rightarrow \quad x = y^3 + Cy$$

OR

Given diff. equation is $(x\sqrt{x^2+y^2} - y^2)dx + xydy = 0$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y^2 - x\sqrt{x^2+y^2}}{xy}$$

$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x} - \sqrt{\frac{x^2}{y^2} + 1}$, which is homogeneous diff. equation

Put $\quad \frac{y}{x} = v$ i.e. $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = v - \sqrt{\frac{1}{v^2} + 1}$$

$$\Rightarrow \quad \frac{xdv}{dx} = -\frac{\sqrt{1+v^2}}{v}$$

$$\Rightarrow \quad \frac{v}{\sqrt{1+v^2}} dv = -\frac{dx}{x}$$

on integrating both sides

$$\int \frac{v}{\sqrt{1+v^2}} dv = -\int \frac{dx}{x}$$

$$\sqrt{1+v^2} = -\log|x| + C$$

$$\Rightarrow \quad \sqrt{x^2+y^2} = -x\log|x| + Cx$$

$$\Rightarrow \quad \sqrt{x^2+y^2} + x\log|x| = Cx$$

which is required solution.

30. given problem is

Maximum $Z = 400x + 300y$

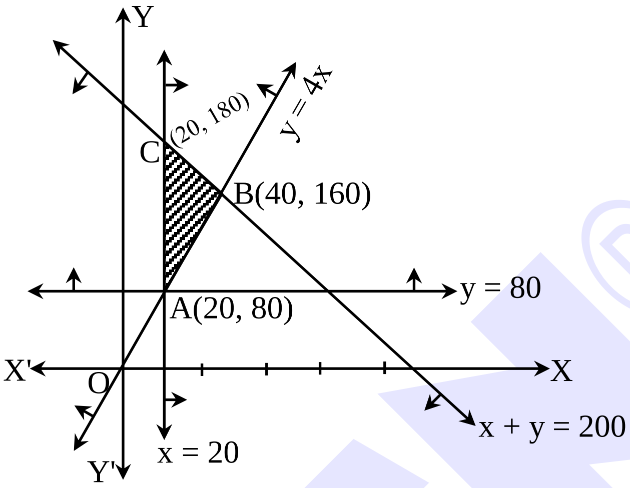
Subject to $x + y \leq 200$ ...(i)

$x \geq 20$ ...(ii)

$y \geq 4x$ ...(iii)

$y \geq 80$ ...(iv)

First we find the feasible region by using given constraints (i) to (iv).



Here, obtained feasible region is bounded with its corner points are A(20, 80), B(40, 160) and C(20, 180)

Corner points	**Z = 400x + 300y**
A(20, 80)	$Z_A = 8000 + 24000 = 32000$
B(40, 160)	$Z_B = 16000 + 48000 = 64000$ (Max.)
C(20, 180)	$Z_C = 8000 + 54000 = 62000$

Since, feasible region is bounded.

So, maximum value of Z is 64000 at point B(40, 160).

31. Let $I = \int \frac{x^2+1}{(x+1)^2} dx$

$$\frac{x^2+1}{(x+1)^2} = 1 - \frac{2x}{(x+1)^2} = 1 - \frac{A}{(x+1)} - \frac{B}{(x+1)^2} \quad \text{(Let)}$$

$\Rightarrow \quad x^2 + 1 = (x + 1)^2 - A(x + 1) - B$ ...(i)

put $x = -1 \Rightarrow B = -2$

on equating constant terms, we get

$-A - B + 1 = 1 \Rightarrow A + B = 0 \Rightarrow A = -B = 2$

$$\therefore \quad \frac{x^2+1}{(x+1)^2} = 1 - \frac{2}{(x+1)} + \frac{2}{(x+1)^2}$$

$$\therefore \quad I = \int 1\,dx - 2\int \frac{1}{(x+1)} dx + 2\int \frac{1}{(x+1)^2} dx$$

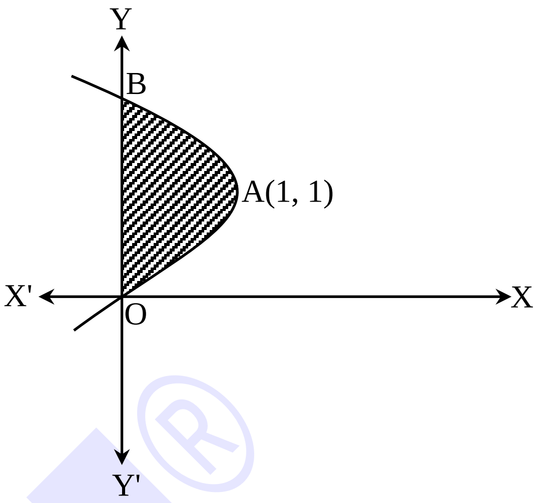
$$\Rightarrow \quad I = x - 2\log|x + 1| - \frac{2}{(x+1)} + C$$

SECTION – D

32. Given curve $y^2 = 2y - x$

$\Rightarrow \quad y^2 - 2y + 1 = -x + 1$

$\Rightarrow \quad (y-1)^2 = -(x-1)$ ...(i)



curve (i) is represents left handed parabola with vertex (1, 1)

put x = 0 in equation (i)

$y^2 - 2y = 0$

$\Rightarrow \quad y(y-2) = 0$

$\Rightarrow \quad y = 0, y = 2$

Thus, the curve (i) meets y-axis at (0, 0) and (0, 2)

Req. Area = area of the region ABOA

$$= \int_0^2 x\,dy = \int_0^2 (2y - y^2)dy$$

$$= \left[2\cdot\frac{y^2}{2} - \frac{y^3}{3}\right]_0^2$$

$$= \left[\left(4 - \frac{8}{3}\right) - 0\right] = \frac{4}{3} \text{ Sq. units.}$$

33. Given relation R on set A = {1, 2, 3, 4, 5, 6} such that

R = {(a, b) : |a – b| is even}

Reflexive: Let $a \in A$ such that

if $(a, a) \in R \Rightarrow |a - a| = 0$ is even

which is true, $\forall a \in A$

So, R is reflexive

Symmetric: Let $a, b \in A$ such that

if $(a, b) \in R \quad \Rightarrow |a - b|$ is even

$\Rightarrow |b - a|$ is also even

$\Rightarrow (b, a) \in R$

So, R is symmetric

Transitive: Let $a, b, c \in R$ such that

if $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow |a - b|$ and $|b - c|$ both are even

let $|a - b| = 2k_1$ and $|b - c| = 2k_2$, where $k_1, k_2 \in N$

$\Rightarrow a - b = \pm 2k_1$, ...(i) and $b - c = \pm 2k_2$ ...(ii)

on adding equation (i) and (ii)

$a - c = \pm 2(k_1 + k_2)$

$\Rightarrow \quad |a - c| = 2(k_1 + k_2)$

$\Rightarrow \quad |a - c|$ is even $\quad (\because k_1 + k_2 \in N)$

$\Rightarrow \quad (a, c) \in R$

So, R is transitive

Since R is reflexive, symmetric and transitive on A

Hence R is an equivalence relation on A

Further, Let $x \in A$ such that $(x, 2) \in R$

$\Rightarrow$ $|x - 2|$ is even

$\Rightarrow$ $x - 2 = 0, 2, 4, 6.....$

$\Rightarrow$ $x = 2, 4, 6$ $(\because x \in A)$

$\therefore$ Req. set = {2, 4, 6}

OR

Given a relation R on set N such that

$R = \{(x, y): x \in N, 2x + y = 20\}$

$\therefore$ $R = \{(1, 18), (2, 16), (3, 14), (4, 12), (5, 10), (6, 8), (7, 6), (8, 4), (9, 2)\}$

So, Domain of R = {1, 2, 3,.....9}

Range of R = {2, 4, 6,.....18}

Further $\because (1, 1) \notin R$, where $1 \in N$

So, R is not reflexive

$\because (1, 18) \in R$ but $(18, 1) \notin R$, where $1, 18 \in N$

So, R is not symmetric

again, $(6, 8) \in R$ and $(8, 4) \in R$ but $(6, 4) \notin R$

So, R is not transitive

Hence, R is neither reflexive nor symmetric nor transitive on set N.

34. The equation of given line are

$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ ...(i)

and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ ...(ii)

any points on the lines (i) and (ii) are $P(3\lambda + 3, -\lambda + 8, \lambda + 3)$ and $Q(-3\mu - 3, 2\mu - 7, 4\mu + 6)$ respectively

The dr's of line PQ are $-3\mu - 3\lambda - 6, 2\mu + \lambda - 15, 4\mu - \lambda + 3$, if PQ is the line of shortest distance, then PQ is perpendicular to both lines (i) and (ii)

$3(-3\mu - 3\lambda - 6) + (-1)(2\mu + \lambda - 15) + 1(4\mu - \lambda + 3) = 0$

and $-3(-3\mu - 3\lambda - 6) + 2(2\mu + \lambda - 15) + 4(4\mu - \lambda + 3) = 0$

$\Rightarrow$ $-11\lambda - 7\mu = 0$ ...(iii)

and $7\lambda + 29\mu = 0$ ...(iv)

from equation (iii) and (iv)

$\lambda = 0$ and $\mu = 0$

put values of λ and μ in P and Q

So, P(3, 8, 3) and Q(–3, –7, 6)

$\therefore$ S.D. = $PQ = \sqrt{(3+3)^2 + (8+7)^2 + (3-6)^2}$

$$= \sqrt{36+225+9} = \sqrt{270} = 3\sqrt{30} \text{ units}$$

The equation of line of shortest distance PQ is

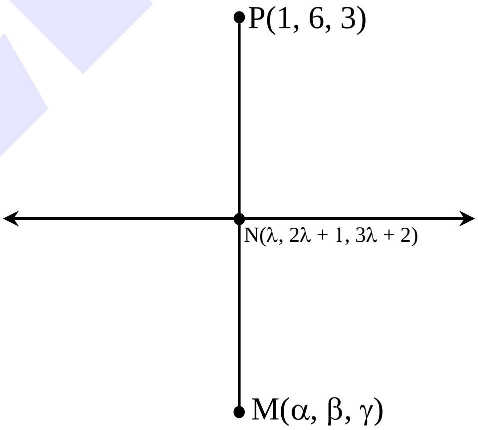
$$\frac{x-3}{-3-3} = \frac{y-8}{-7-8} = \frac{z-3}{6-3}$$

$$\Rightarrow \quad \frac{x-3}{2} = \frac{y-8}{5} = \frac{z-3}{-1}$$

OR

The equation of given line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ ...(i)

Let N be the foot of perpendicular drawn from point P(1, 6, 4) on the line, then



$N(\lambda, 2\lambda + 1, 3\lambda + 2)$

The dr's of line PN are $\lambda - 1, 2\lambda - 5, 3\lambda - 1$

Since PN is perpendicular to line (i), then

$1(\lambda - 1) + 2(2\lambda - 5) + 3(3\lambda - 1) = 0$

$\Rightarrow \lambda = 1$

So the coordinates of point N(1, 3, 5)

Let $M(\alpha, \beta, \gamma)$ be the image of point P(1, 6, 3) in the line (i), then N is mid-point of PM.

$$\therefore \quad \frac{\alpha+1}{2} = 1, \frac{\beta+6}{2} = 3, \frac{\gamma+3}{2} = 5$$

$\Rightarrow \quad \alpha = 1, \beta = 0, \gamma = 7$

Hence, the image of point P(1, 6, 3) in the line (i) is M(1, 0, 7).

35. Given $A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6 \end{bmatrix}$

$$\Rightarrow \quad |A| = \begin{vmatrix} 3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6 \end{vmatrix} = 3(12 - 6) - 4(0 + 3) + 2(0 - 2) = 2$$

$\because$ $|A| \neq 0$ i.e. A is non-singular matrix

So, A^{-1} be exists

Here, the cofactors of element of |A| are

$A_{11} = 6, A_{12} = -3, A_{13} = -2$

$A_{21} = -28, A_{22} = 16, A_{23} = 10$

$A_{31} = -16, A_{32} = 9, A_{33} = 6$

$$\therefore \quad \text{adj } A = \begin{bmatrix} 6 & -3 & -2 \\ -28 & 16 & 10 \\ -16 & 9 & 6 \end{bmatrix}^T = \begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & 9 \\ -2 & 10 & 6 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|}\cdot(\text{adj}A)$$

$$\Rightarrow \quad A^{-1} = \frac{1}{2}\begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & 9 \\ -2 & 10 & 6 \end{bmatrix} \quad \text{...(i)}$$

Now, given system of linear equations is

$3x + 4y + 2z = 8$

$2y - 3z = 3$

$x - 2y + 6z = -2$

This system can be written in matrix equation form as

$$\begin{bmatrix} 3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix}$$

$\Rightarrow \quad AX = B$

$\Rightarrow \quad X = A^{-1}B$

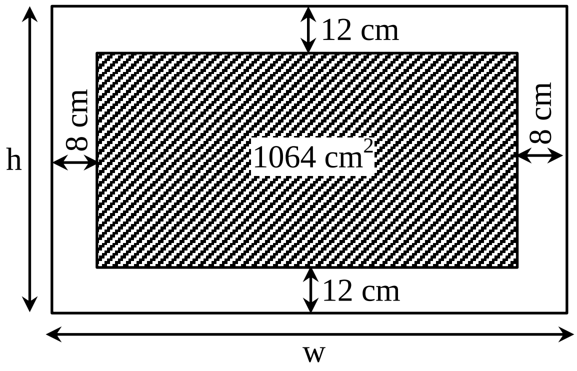
$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & 9 \\ -2 & 10 & 6 \end{bmatrix}\begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 48-84+32 \\ -24+48-18 \\ -16+30-12 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -4 \\ 6 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$$

$\therefore \quad x = -2, y = 3, z = 1$

SECTION – E

36. (i) Let area of the poster is A, then



$A = 1064 + 2(w \times 12) + 2(h \times 8) - 4(8 \times 12)$

$A = 1064 + 24w + 16h - 384$

$A = 680 + 24w + 16h$ ...(i)

(ii) $\because$ $A = w \times h$ ...(ii)

$\Rightarrow$ $680 + 24w + 16h = wh$

[using equation (i)]

$\Rightarrow$ $(h - 24)w = 16h + 680$

$\Rightarrow$ $w = \dfrac{(16h+680)}{(h-24)}$ ...(iii)

put value of w in equation (ii)

$A = \dfrac{(16h+680)}{(h-24)} \times h$

$A = \dfrac{(16h^2+680h)}{(h-24)}$

(iii) $\because$ $A = \dfrac{(16h^2+680h)}{(h-24)}$

$\Rightarrow$ $\dfrac{dA}{dh} = \dfrac{(h-24)(32h+680)-(16h^2+680h)}{(h-24)^2} = \dfrac{16h^2 - 24\times 32h - 24\times 680}{(h-24)^2}$

$\Rightarrow$ $\dfrac{dA}{dh} = \dfrac{16(h^2-48h-1020)}{(h-24)^2}$

for minimum value of A

$\dfrac{dA}{dh} = \dfrac{16(h^2-48-1020)}{(h-24)^2} = 0$

$\Rightarrow$ $h^2 - 48h - 1020 = 0$

$\Rightarrow$ $h = -\dfrac{(-48)\pm\sqrt{(-48)^2-4(1)(-1020)}}{2} = \dfrac{48\pm\sqrt{2304+4080}}{2}$

$\Rightarrow$ $h = \dfrac{48\pm\sqrt{6384}}{2} = 24\pm\sqrt{1581} = 24\pm 39.94$

$\Rightarrow$ $h = 63.94$ or -15.94

$\Rightarrow$ $h = 63.94$ ($\because$ h cannot be negative)

Further $\frac{d^2A}{dh^2} = \frac{(h-24)^2 \times 16(2h-48) - 16(h^2-48h-1020)\times 2(h-24)}{(h-24)^4}$

$$= \frac{32(h-24)^3 - 32(h-24)(h^2-48h-1020)}{(h-24)^4}$$

$$= \frac{32[(h-24)^2 - (h^2-48h-1020)]}{(h-24)^3}$$

$$= \frac{32(1596)}{(h-24)^3}$$

at h = 63.94

$\frac{d^2A}{dh^2} > 0$

So, A is min. at h = 63.94

when h = 63.94 then,

$w = \frac{(16 \times 63.94 + 680)}{(63.94 - 24)}$ [using equation (iii)]

$w = \frac{1703.04}{39.94} = 42.63$

Thus A is minimum when h = 63.94 cm and w = 42.63 cm

OR

(iii) $\because$ $A = \frac{(16h^2 + 680h)}{(h-24)}$

$\Rightarrow$ $\frac{dA}{dh} = \frac{(h-24)(32h+680) - (16h^2+680h)}{(h-24)^2} = \frac{16h^2 - 24 \times 32h - 24 \times 680}{(h-24)^2}$

$\Rightarrow$ $\frac{dA}{dh} = \frac{16(h^2-48h-1020)}{(h-24)^2}$

for minimum value of A

$\frac{dA}{dh} = \frac{16(h^2-48-1020)}{(h-24)^2} = 0$

$\Rightarrow$ $h^2 - 48h - 1020 = 0$

$\Rightarrow$ $h = -\frac{(-48) \pm \sqrt{(-48)^2 - 4(1)(-1020)}}{2} = \frac{48 \pm \sqrt{2304 + 4080}}{2}$

$\Rightarrow$ $h = \frac{48 \pm \sqrt{6384}}{2} = 24 \pm \sqrt{1581} = 24 \pm 39.94$

$\Rightarrow$ h = 63.94 or – 15.94

$\Rightarrow$ h = 63.94 ($\because$ h cannot be negative)

Further $\frac{d^2A}{dh^2} = \frac{(h-24)^2 \times 16(2h-48) - 16(h^2-48h-1020)\times 2(h-24)}{(h-24)^4}$

$= \frac{32(h-24)^3 - 32(h-24)(h^2-48h-1020)}{(h-24)^4}$

$= \frac{32[(h-24)^2 - (h^2-48h-1020)]}{(h-24)^3}$

$= \frac{32(1596)}{(h-24)^3}$

at h = 63.94

$\frac{d^2h}{dh^2} > 0$

So, A is min. at h = 63.94

when h = 63.94 then,

$w = \frac{(16\times 63.94 + 680)}{(63.94-24)}$

$w = \frac{1703.04}{39.94} = 42.63$

minimum area of the poster, is

$A_{min.} = 42.63 \times 63.94$ [using equation (ii)]

$= 2775.76\ cm^2$

37. (i) Given AC + BC = 10 ...(i)

In right angle ΔABC

$AB^2 + BC^2 = AC^2$

$\Rightarrow$ $AB^2 + BC^2 = (10 - BC)^2$ (using equation (i)

$\Rightarrow$ $AB^2 + BC^2 = 100 - 20BC + BC^2$

$\Rightarrow$ $AB^2 = 100 - 20\ BC$

$\Rightarrow$ $BC = \frac{100 - AB^2}{20}$

$\Rightarrow$ $BC = \frac{100 - x^2}{20}$ ...(ii) ($\because$ AB = x)

(ii) Area of triangle ABC = $\frac{1}{2} \times AB \times BC$

$\Rightarrow$ $S = \frac{1}{2} \times x \times \frac{(100-x^2)}{20} = \frac{(100x - x^3)}{40}$ (using equation (ii))

$$\Rightarrow \quad \frac{dS}{dx} = \frac{(100-3x^2)}{40}$$

for critical point of S

$$\frac{dS}{dx} = 0$$

$$\Rightarrow \quad \frac{100-3x^2}{40} = 0$$

$$\Rightarrow \quad x^2 = \frac{100}{3}$$

$$\Rightarrow \quad x = \frac{10}{\sqrt{3}} \text{ or } \frac{10\sqrt{3}}{3} \qquad (\because x \text{ not be negative})$$

(iii) $\because \quad \frac{dS}{dx} = \frac{100-3x^2}{40}$

$$\Rightarrow \quad \frac{d^2S}{dx^2} = -\frac{6x}{40} = -\frac{3}{20}x$$

$$\text{at } x = \frac{10}{\sqrt{3}}, \frac{d^2S}{dx^2} = -\frac{3}{20} \times \frac{10}{\sqrt{3}} = -\frac{3}{2\sqrt{3}} < 0$$

So, S is maximum at $x = \frac{10}{\sqrt{3}}$

Hence, maximum area of the triangle is

$$S_{max} = \frac{1}{40}\left(100 \times \frac{10}{\sqrt{3}} - \frac{1000}{3\sqrt{3}}\right) = \frac{1}{40} \times \frac{2000}{3\sqrt{3}}$$

$$S_{max} = \frac{50}{3\sqrt{3}} \text{ Sq. units}$$

OR

(iii) $\because \quad \frac{dS}{dx} = \frac{100-3x^2}{40}$

$$\Rightarrow \quad \frac{d^2S}{dx^2} = -\frac{6x}{40} = -\frac{3}{20}x$$

$$\text{at } x = \frac{10}{\sqrt{3}}, \frac{d^2S}{dx^2} = -\frac{3}{20} \times \frac{10}{\sqrt{3}} = -\frac{3}{2\sqrt{3}} < 0$$

So, S is maximum at $x = \frac{10}{\sqrt{3}}$

$\because$ area of triangle is maximum at $x = \frac{10}{\sqrt{3}}$

$\therefore$ $BC = \frac{100 - \left(\frac{10}{\sqrt{3}}\right)^2}{20}$ [using equation. (ii)]

$\Rightarrow$ $BC = \frac{200}{60} = \frac{10}{3}$

$\because$ $AC + BC = 10$ [using (i)]

$\Rightarrow$ $AC = 10 - BC$

$\Rightarrow$ $AC = 10 - \frac{10}{3} = \frac{20}{3}$

Hence, length of hypotenuse, when area of triangle be maximum, is $AC = \frac{20}{3}$ units

38. Consider the following events

E : A is fails, F : B is fails

then, $P(E) = 0.2$, $P(E \cap F) = 0.15$ and $P(\bar{E} \cap F) = 0.15$

(i) $\because$ $P(\bar{E} \cap F) = 0.15$

$\Rightarrow$ $P(F) - P(E \cap F) = 0.15$

$\Rightarrow$ $P(F) = 0.15 + P(E \cap F)$

$\Rightarrow$ $P(F) = 0.15 + 0.15 = 0.30$

Hence P(B fails) = P(F) = 0.30

Further

$\because$ $P(E \cap \bar{F}) = P(E) - P(E \cap F)$

$= 0.2 - 0.15$

$= 0.05$

Hence P(A fails alone) = $P(E \cap \bar{F}) = 0.05$

(ii) P(whole system fail) = $P(E \cup F)$

$= P(E) + P(F) - P(E \cap F)$

$= 0.2 + 0.3 - 0.15$

$= 0.5 - 0.15 = 0.35$

Hence, P(whole system fails) = 0.35

again P(B fails/A has failed) = P(F/E)

$$= \frac{P(E \cap F)}{P(E)} = \frac{0.15}{0.2} = 0.75$$

Hence P(B fails/A as failed) = 0.75