SET-2

CLASS : X

SUBJECT – MATHEMATICS

SOLUTIONS

1. (a) 52
2. (d) (3, 0)
3. (b) $\frac{4}{5}$
4. (c) BD × CD = AD^2
5. (b) $\frac{\sin A}{1+\cos A}$
6. (a) 3
7. (d) 86cm
8. (d) $\frac{9}{36}$
9. (a) $\frac{5}{12}$
10. (a) 98 kg
11. (d) 400 cu.cm
12. (a) 6
13. (c) $(3)^{\frac{2}{3}}$
14. (a) 15cm
15. (a) $\frac{1}{6}$
16. (b) 12
17. (b) 25
18. (d) 100^o
19. (b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
20. (d) Assertion (A) is false but reason (R) is true.

21 (a) a + 14d = 47 $\quad\therefore$ d = 3

a + 14 (3) = 47 $\quad$ a = 5

Let a_n = 95 $\quad$ a + (n – 1) d = 95

5 + (n – 1) 3 = 95 $\quad$ n = 31

Total 31 terms are there.

(b) a = 119, d = –3

For first negative term $a_n < 0$

a + (n – 1)d < 0

119 + (n – 1) (–3) < 0

119 – 3n + 3 < 0 $\quad$ 122 < 3n

$\frac{122}{3} < n$ $\quad$ 40.67 < n

41st term is first negative term.

22. (a) $\sin\theta = \sqrt{2}\cos\theta - \cos\theta$

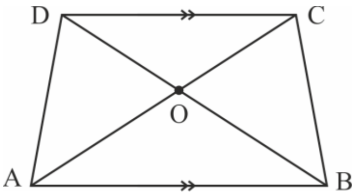
$\sin\theta = \left(\sqrt{2}-1\right)\cos\theta$

$\frac{\sin\theta}{\cos\theta} = \sqrt{2}-1$ $\quad$ $\tan\theta = \sqrt{2}-1$

(b) $2\times(2)^2 + x.\left(\frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4}\left(\frac{1}{\sqrt{3}}\right)^2 = 10$

$8+\frac{3x}{4}-\frac{1}{4}=10$ $\quad$ $\frac{3x-1}{4} = 2$

3x = 8 + 1 $\quad$ x = 3

23. 

In ΔOAB and ΔOCD,

∠OAB = ∠OCD $\quad$ ∠OBA = ∠ODC

{Alternate interior angles}

ΔOAB ~ ΔOCD

(By AA similarity)

$\frac{OB}{OD} = \frac{AB}{CD} \Rightarrow \frac{2}{1} = \frac{AB}{CD}$ $\quad$ AB = 2CD

Hence proved.

24. r = 7cm $\quad$ θ = 60º

Length of arc,

$\ell = \frac{\pi r\theta}{180^o} = \frac{22}{7}\times\frac{7\times 60^o}{180^o} = \frac{22}{3}$ cm

Area of sector

$\Rightarrow \frac{1}{2}\times\ell\times r = \frac{1}{2}\times\frac{22}{3}\times 7 = \frac{154}{6}\text{cm}^2$

$= \frac{77}{3}\text{cm}^2$

25. a = 1, b = –(k + 6), c = 2(2k – 1)

Sum of zeroes = $\frac{1}{2}\times$ Product of zeroes

$\frac{-b}{a}=\frac{1}{2}\times\frac{c}{a}$ $\quad$ $-2b = c$

$-2\{-(k+6)\} = 2(2k-1)$

$2(k+6) = 2(2k-1)$ $\quad$ $k+6 = 2k-1$

$k = 7$

26. (a) Let $4-3\sqrt{2}$ be a rational number.

$4-3\sqrt{2}=\frac{a}{b}$

(where a and b are co–prime integers)

$4-\frac{a}{b}=3\sqrt{2}$ $\quad$ $\frac{4b-a}{3b}=\sqrt{2}$

$\therefore$ a and b are integers $\frac{4b-a}{3b}$ is rational

$\sqrt{2}$ is rational

But given that $\sqrt{2}$ is irrational, It contradict our assumption.

Hence $3-\sqrt{2}$ is an irrational number.

(b) HCF (990, 945) = 45

45 fruits in each basket.

27. Let fixed charge be x Rs.

Per kilometer charge y Rs.

$x + 10y = 105$ $\quad$ $x + 15y = 155$

On solving

$x = 5$, $y = 10$

Fixed charge is 5Rs.

Charges per km is 10 Rs.

Charges per 25km = $x + 25y$ = 255Rs.

28. For two dice,

Total outcome = 36

(i) For even sum,

Favorable outcome = 18

P (even sum) $=\frac{18}{36}=\frac{1}{2}$

(ii) For even product

Favorable outcome = 27

P (even product) $=\frac{27}{36}=\frac{3}{4}$

29. (a) L.H.S. $=\frac{\sin A-\cos A+1}{\sin A+\cos A-1}$

On dividing by cosA

$\Rightarrow \frac{\tan A-1+\sec A}{\tan A+1-\sec A} \Rightarrow \frac{\tan A-1+\sec A}{\tan A-\sec A+1}$

$\Rightarrow \frac{\tan A-1+\sec A}{\tan A-\sec A+(\sec A+\tan A)(\sec A-\tan A)}$

[$\therefore$ 1 = (secA + tanA) (secA – tanA)]

$\Rightarrow \frac{\tan A-1+\sec A}{(\sec A-\tan A)[-1+\sec A+\tan A]}$

$\Rightarrow \frac{1}{\sec A-\tan A}$

= R.H.S. (Hence proved)

(b) $(\sin^2\theta)^3 + (\cos^2\theta)^3$

$\therefore a^3 + b^3 = (a+b)^3 - 3ab(a+b)$

$\Rightarrow (\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta$

$(\sin^2\theta + \cos^2\theta)$

$\Rightarrow 1 - 3\sin^2\theta\cos^2\theta$ ($\therefore \sin^2\theta + \cos^2\theta = 1$)

$\Rightarrow$ R.H.S. Hence proved

30. $\alpha+\beta=\frac{-(4)}{1}=4$ $\quad$ $\alpha\beta=\frac{c}{a}=-5$

(i) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

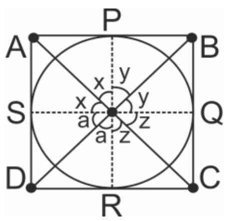
$= (4)^2 - 2(-5)$ $\quad$ $= 16 + 10$

$= 26$

(ii) $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\beta+\alpha}{\alpha\beta}$ $\quad$ $=\frac{4}{(-5)}$

$=\frac{-4}{5}$

31.



To prove:-

$\angle AOB+\angle COD=\angle AOD+\angle BOC=180^o$

Construction:- Join OP, OQ, OR and OS where P, Q, R and S are points of contact of tangents AB, BC, CD and AD respectively.

Proof:- Tangents drawn from an external point to the circle subtends equal angles at the center of the circle.

$\angle AOP=\angle AOS=x$

$\angle BOP=\angle BOQ=y$

$\angle COQ=\angle COR=z$

$\angle DOR = \angle DOS = a$

$2(x + y + a + z) = 360^o$ (complete angle)

$x + y + a + z = 180^o$

$\angle AOB + \angle COD = 180^o$

or $\angle AOD + \angle BOC = 180^o$

32. (a) Let speed of fast train be x km/hr

Speed of slow train be (x – 10) km/hr

$$\frac{600}{x-10} - \frac{600}{x} = 3$$

$$\frac{600[x - x + 10]}{x(x-10)} = 3$$

6000 = 3 (x) (x – 10)

2000 = x (x – 10)

40 × 50 = x (x – 10)

50 × (50 – 10) = x(x – 10)

On comparing x = 50

Speed of fast train be 50 km/hr

Speed of slow train be 40 km/hr

(b) $$\frac{2x(2x+3)+1(x-3)+3x+9}{(x-3)(2x+3)} = 0$$

$4x^2 + 6x + x - 3 + 3x + 9 = 0$

$4x^2 + 10x + 6 = 0$

$2(2x^2 + 5x + 3) = 0$

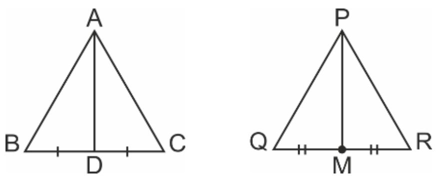
$2x^2 + 3x + 2x + 3 = \frac{0}{2} = 0$

x (2x + 3) + 1(2x + 3) = 0

(2x + 3) (x + 1) = 0

$x = \frac{-3}{2}, -1 \qquad \therefore x \neq \frac{-3}{2} \qquad x = -1.$

33. (a)



$\therefore \Delta ABC \sim \Delta PQR \qquad \angle B = \angle Q$

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{2BD}{2QM} = \frac{BD}{QM}$$

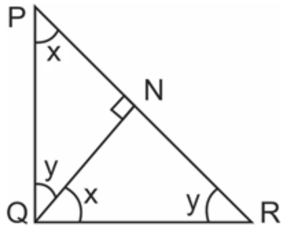
In ΔABD and ΔPQM,

$\angle B = \angle Q \qquad \frac{AB}{PQ} = \frac{BD}{QM}$

$\Delta ABD \sim \Delta PQM$ (By SAS similarity)

$$\frac{AB}{PQ} = \frac{AD}{PM}$$

(b)



Given that $PN \times NR = QN^2$

$\frac{PN}{QN} = \frac{QN}{NR} \qquad \angle PNQ = \angle QNR = 90$

$\Delta PNQ \sim \Delta QNR$ (By SAS Similarity)

$\angle QPN = \angle RQN = x^o$

$\angle PQN = \angle QRN = y^o$

$\angle P + \angle Q + \angle R = 180^o$

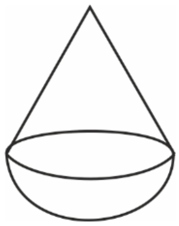
In ΔPQR, $\angle P + \angle Q + \angle R = 180^o$

$x + x + y + y = 180^o$

$2(x + y) = 180^o \qquad x + y = 90^o$

$\angle PQR = 90^o \qquad$ Hence proved.

34.



For cone, r = 7cm h = 3.5cm

Hemisphere r = 7cm

Volume of solid = Volume of cone + volume of hemisphere

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 \qquad = \frac{1}{3}\pi r^2 (h + 2r)$

$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 (3.5 + 2 \times 7)$

$= \frac{154}{3} \times 17.5 \qquad = 898.34 cm^3$

35.

Height (in cm)	No. of girls	CF
0 – 140	4	4
140 – 145	11	15
145 – 150	29	44
150 – 155	40	84
155 – 160	**46**	**130**
160 – 165	51	181

N = 181

Median class is 155 – 160.

$\ell = 155,\ \frac{N}{2} = 90.5,$ CF = 84,

f = 46, h = 5

$$\text{Median} = \ell + \left(\frac{\frac{N}{2} - CF}{f}\right) \times h$$

$$= 155 + \left(\frac{90.5 - 84}{46}\right) \times 5$$

$$= 155 + \frac{6.5 \times 5}{46} \qquad = 155 + \frac{32.5}{46}$$

= 155.7 (Approx)

36. (i) Distance between A (1, 9) and B (5, 13)

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(5-1)^2 + (13-9)^2}$$

$$= \sqrt{16+16} \qquad = \sqrt{32} \quad = 4\sqrt{2} \text{ units}$$

(ii) Mid point of M(5, 11) and Q(10, 3)

$$\Rightarrow \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \qquad = \left(\frac{15}{2}, 7\right)$$

(iii) (a) Point denoted by (5, 2) is Q.

Point denoted by (9, 4) is E.

(b) GFQR is a rectangle. (l = 5 units, b = 2 units)

Area = 10 sq. unit

Perimeter = 14 unit

37. (i) a_{10} = a + 9d = 30 + 9 × 10

= 120

120 seats will be in 10^{th} row.

(ii) For 17 rows. Middle row is 9^{th} row.

a_9 = a + 8d = 30 + 80 = 110

110 seats in middle row.

(iii) (a) a = 30 d = 10

S_n = 1500

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$1500 = \frac{n}{2}\left[60 + (n-1)10\right]$$

3000 = n(60 + 10n – 10)

3000 = 50n + $10n^2$

$10n^2$ + 50n – 3000 = 0

$10(n^2 + 5n - 300) = 0$

$n^2 + 5n - 300 = 0$

(n + 20) (n – 15) = 0

n = 15, –20 (rejected)

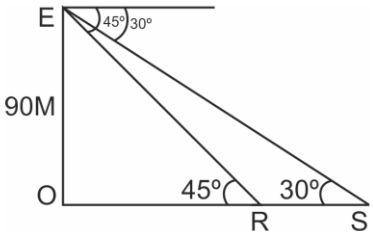
15 rows are there.

(b) $S_{10} = \frac{10}{2}\left[60 + (10-1)10\right]$

= 5 × 150 = 750

Seats left = 1500 – 750 = 750.

38.



(i) $\frac{OE}{OR} = \tan 45^\circ = 1$

OE = OR = 90m

(ii) In ΔEOS

$$\frac{OE}{OS} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\frac{90}{OS} = \frac{1}{\sqrt{3}} \qquad OS = 90\sqrt{3}$$

RS = OS – OR

$$= 90\sqrt{3} - 90 \qquad = 90\left(\sqrt{3} - 1\right)m$$

(iii) (a) Distance travelled eagle to catch rat = SE

$$\text{Sin}30^\circ = \frac{90}{SE} \qquad \frac{1}{2} = \frac{90}{SE}$$

Distance covered by eagle = 180m

OR

(b) Speed of eagle $= \frac{\text{Distance}}{\text{time}}$

$$= \frac{180}{10} \qquad = 18 \text{ m/sec.}$$