

**PAPER # 02 - MATHEMATICS**

1. (d) The mid-point of line segment joining (0, 0) and (-4, -2) is  $\left(\frac{0-4}{2}, \frac{0-2}{2}\right)$  i.e. (-2, -1).
2. (c)  $\tan 45^\circ \cos 60^\circ + \sin 60^\circ \cot 60^\circ$   
 $1 \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} = \frac{1}{2} + \frac{1}{2} = 1$
3. (a) Given equation,  $2x^2 - \sqrt{5}x + 1 = 0$   
 On comparing it with  $ax^2 + bx + c = 0$ , we get  
 $a = 2, b = \sqrt{5}$  and  $c = 1$   
 $\therefore D = (\sqrt{5})^2 - 4(2)(1)$  [ $\because D = b^2 - 4ac$ ]  
 $= 5 - 8 = -3$
4. (b) We have,  $\sqrt{3} \sin \theta = \cos \theta$   
 $\Rightarrow \frac{\sin \theta}{\cos \theta} = \frac{1}{\sqrt{3}} \Rightarrow \tan \theta = \frac{1}{\sqrt{3}}$   
 $\Rightarrow \tan \theta = \tan 30^\circ \quad \theta = 30^\circ$
5. (d) Given,  $AB \parallel EW$   
 $\therefore \frac{DA}{AE} = \frac{DB}{BW}$  [by Thales theorem]  
 $\Rightarrow \frac{DA}{DE - DA} = \frac{DB}{DW - DB}$   
 $\Rightarrow \frac{4}{12 - 4} = \frac{DB}{24 - DB}$   
 $\Rightarrow \frac{4}{8} = \frac{DB}{24 - DB}$   
 $\Rightarrow 24 - DB = 2DB$   
 $\Rightarrow 24 = 3DB$   
 $\Rightarrow DB = \frac{24}{3} = 8 \text{ cm}$
6. (b) Let 4 be the event 'getting an even number.'  
 Clearly, event A occurs, if we obtain any one of 2, 4, 6 as an outcome.  
 $\therefore$  Number of outcomes favourable to A = 3  
 Hence,  $P(A) = \frac{3}{6} = \frac{1}{2}$
7. (a) Length of the arc  $= \frac{\theta}{360^\circ} \times 2\pi r$   
 $\Rightarrow 4.4 = \frac{30^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times r$   
 $\Rightarrow 4.4 = \frac{1}{12} \times \frac{44}{7} \times r$   
 $\Rightarrow r = \frac{4.4 \times 12 \times 7}{44} = 8.4 \text{ cm}$
8. (c) We know that  
 Product of zeroes  $= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$   
 $\therefore \alpha\beta = \frac{7}{4}$
9. (c) Total number of cards = 52  
 Kings which are red in colour = 2  
 $P(\text{king of red colour}) = \frac{2}{52} = \frac{1}{26}$
10. (a) If point P lies inside the circle then no tangent can be drawn.
11. (b) Let  $\alpha$  and  $\beta$  be the zeros of the polynomial  $f(x) = ax^2 + bx + c$ . Then,  
 $\alpha + \beta = -\frac{b}{a}$  and  $\alpha\beta = \frac{c}{a}$   
 Let S and P denote respectively the sum and product of the zeros of a polynomial whose zeros are  $\frac{1}{\alpha}$  and  $\frac{1}{\beta}$ . Then,  
 $S = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{-\frac{b}{a}}{\frac{c}{a}} = -\frac{b}{c}$  and  
 $P = \frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{\frac{c}{a}} = \frac{a}{c}$   
 Hence, the required polynomial  $g(x)$  is given by  
 $g(x) = k(x^2 - Sx + P) = k\left(x^2 + \frac{bx}{c} + \frac{a}{c}\right)$ ,  
 where k is any non-zero constant.

12. (a) Class mark, frequency of the class

$$x = \frac{\sum fx}{\sum f} = \frac{\sum(A \times B)}{\sum A}$$

where B is the class mark.

Class mark =  $\frac{1}{2}$  (upper limit + lower limit)

and A is the frequency of the class.

13. (a) Let d be the common difference of the AR According to the question,

$$a_{17} - a_{10} = 7$$

$$\Rightarrow (a + 16d) - (a + 9d) = 7 \Rightarrow 7d = 7 \Rightarrow d = 1$$

14. (b) The mode is the most frequent observation. Here, the mode is 14 with a frequency of 15.

15. (d)  $(x + 2)(3x - 5) = 0$

$$\Rightarrow x + 2 = 0 \text{ or } 3x - 5 = 0$$

$$\Rightarrow x = -2 \text{ or } x = \frac{5}{3}$$

Hence, the roots of the given equation are -2 and  $\frac{5}{3}$ .

16. (d) Given, equation  $2x^2 - 6x + 7 = 0$   
On comparing it with  $ax^2 + bx + c = 0$ , we get

$$a = 2, b = -6 \text{ and } c = 7$$

$$\therefore D = b^2 - 4ac = (-6)^2 - 4(2)(7)$$

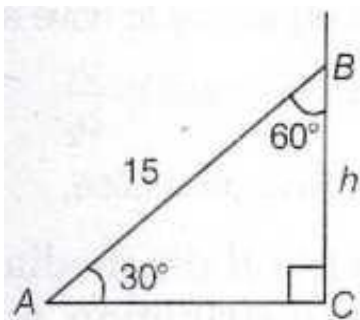
$$= 36 - 56 = -20 < 0$$

So, the roots are imaginary.

17. (c) Given,  $\angle ABC = 60^\circ$

In  $\triangle ABC$ ,  $\angle BAC + \angle ABC + \angle ACB = 180^\circ$

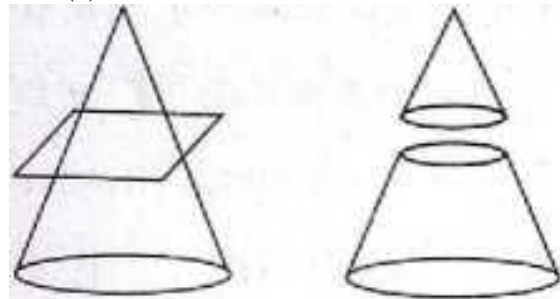
$$\Rightarrow \angle BAC = 180^\circ - 90^\circ - 60^\circ = 30^\circ$$



$$\text{So, } \sin 30^\circ = \frac{BC}{AB} = \frac{h}{15}$$

$$\Rightarrow \frac{1}{2} = \frac{h}{15} \Rightarrow h = \frac{15}{2} \text{ m}$$

18. (c) Circle



19. (c). Assertion Given  $x + y - 8 = 0$  and  $x - y - 2 = 0$

$$\text{Here, } a_1 = 1, b_1 = 1, c_1 = 8$$

$$\text{and } a_2 = 1, b_2 = -1, c_2 = -2$$

$$\text{So, } \frac{a_1}{a_2} = \frac{1}{1}, \frac{b_1}{b_2} = \frac{1}{-1} \text{ and } \frac{c_1}{c_2} = \frac{-8}{-2} = 4$$

$$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

So, the system of equations has a unique solution and the Assertion is true.

**Reason** For equations to have a unique solution,

$$\frac{a_1}{a_2} \text{ should not be equal to } \frac{b_1}{b_2}.$$

$\therefore$  The given Reason is false.

20. (a) Reason is clearly true.

Using the relation given in reason, we have

$$2 \text{ Mean} = 3 \text{ Median} - \text{Mode}$$

$$= 3 \times 150 - 154$$

$$= 296$$

$$\therefore \text{Mean} = \frac{296}{2} = 148, \text{ which is true.}$$

Thus, both Assertion and Reason are true and Reason is the correct explanation of Assertion.

21. Given,  $x = a \cos \theta$  and  $y = b \sin \theta$

$$\therefore b^2 x^2 + a^2 y^2 = b^2 (a \cos \theta)^2 + a^2 (b \sin \theta)^2 \quad (1)$$

$$= a^2 b^2 \cos^2 \theta + a^2 b^2 \sin^2 \theta$$

$$= a^2 b^2 (\cos^2 \theta + \sin^2 \theta)$$

$$= a^2 b^2 (1) \quad [\because \cos^2 A + \sin^2 A = 1]$$

$$= a^2 b^2 (1)$$

22. Let us assume that  $\frac{2}{5\sqrt{3}}$  is a rational number.

$\therefore \frac{2}{5\sqrt{3}} = \frac{p}{q}$ , where  $p, q$  ( $q \neq 0$ ) are integers and  $p, q$  are coprimes. (1)

$$\Rightarrow \frac{2q}{5p} = \sqrt{3}$$

Since, 2, 5,  $p$  and  $q$  are integers.

$\therefore \frac{2q}{5p}$  is rational, so  $\sqrt{3}$  is rational.

But this contradicts the fact that  $\sqrt{3}$  is irrational.

Hence,  $\frac{2}{\sqrt{3}}$  is an irrational number.

**Hence proved. (1)**  
**OR**

Let us assume that  $6 - 2\sqrt{3}$  is rational number.

Then, it will be of the form  $\frac{a}{b}$ , where  $a, b$  are coprime integers and  $b \neq 0$ .

$$\text{Now, } 6 - 2\sqrt{3} = \frac{a}{b}$$

On rearranging, we get

$$6 - \frac{a}{b} = 2\sqrt{3} \quad (1)$$

Since, 6 and  $\frac{a}{b}$  are rational. So, their difference will be rational.

$\therefore 2\sqrt{3}$  is rational.

But we know that,  $\sqrt{3}$  is irrational.

So, this contradicts the fact that  $\sqrt{3}$  is irrational.

Therefore, our assumption is wrong.

Hence,  $6 - 2\sqrt{3}$  is irrational.

**Hence proved. (1)**

23. We have,  $p(x) = 5x^2 - 7x + 1$ , whose zeroes are  $\alpha$  and  $\beta$ .

$$\therefore \text{Sum of zeroes, } \alpha + \beta = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$= -\frac{(-7)}{5} = \frac{7}{5} \dots(i) \quad (1)$$

and product of zeroes,  $\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$$= \frac{1}{5} \dots(ii)$$

$$\text{Now, } \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{7/5}{1/5}$$

[from Eqs. (i) and (ii)]

$$= 7 \quad (1)$$

24. We have,  $\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}$

$$\Rightarrow \frac{(x+2) + 2(x+1)}{(x+1)(x+2)} = \frac{4}{x+4} \quad (1)$$

$$\Rightarrow (x+4)(3x+4) = 4(x^2 + 3x + 2)$$

$$\Rightarrow x^2 - 4x - 8 = 0$$

On comparing it with  $ax^2 + bx + c = 0$ , we get

$$a = 1, b = -4 \text{ and } c = -8$$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-(-4) \pm \sqrt{16 - 4(1)(-8)}}{2} = x = \frac{4 \pm \sqrt{48}}{2}$$

$$\Rightarrow x = 2 \pm 2\sqrt{3} \quad (1)$$

25.  $\therefore \triangle AGF \sim \triangle DBG \dots(i)$

[by AA similarity criterion]

Now, in  $\triangle AGF$  and  $\triangle EFC$ , we get

$$\angle FAG = \angle CEF \text{ [each } 90^\circ]$$

and  $\angle AFG = \angle ECF$  [corresponding angles because  $GF \parallel BC$  and  $AC$  is the transversal]

$$\therefore \triangle AGF \sim \triangle EFC \dots(ii) \quad (1)$$

From Eqs. (i) and (ii), we get

$$\triangle DBG \sim \triangle EFC$$

$$\Rightarrow \frac{BD}{EF} = \frac{DG}{EC}$$

$$\Rightarrow \frac{BD}{DE} = \frac{DE}{EC} \text{ [}\because \text{ DEFG is a square]}$$

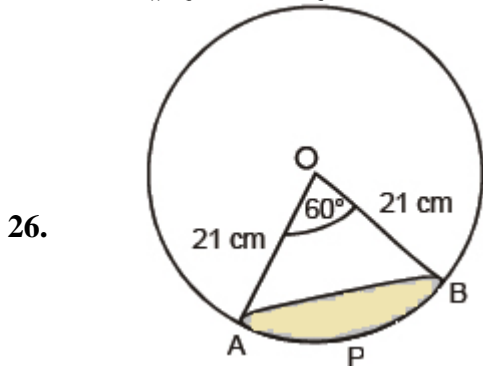
$$\therefore DE^2 = BD \times EC \text{ Hence proved. (1)}$$

**OR**

Given,

In  $\triangle PQO$ ,  $DE \parallel OQ$

So by using Basic Proportionality Theorem,  
 $PD/DO = PE/EQ$  ..... ..(i)  
 Again given, in  $\Delta POR$ ,  $DF \parallel OR$ ,  
 So by using Basic Proportionality Theorem,  
 $PD/DO = PF/FR$  ..... (ii)  
 From equation (i) and (ii), we get,  
 $PE/EQ = PF/FR$   
 Therefore, by converse of Basic Proportionality Theorem,  
 $EF \parallel QR$ , in  $\Delta PQR$ .



26.

Given,  
 Radius = 21 cm  
 $\theta = 60^\circ$

- (i) Length of an arc =  $\theta/360^\circ \times \text{Circumference}$  ( $2\pi r$ )  
 $\therefore$  Length of an arc AB =  $(60^\circ/360^\circ) \times 2 \times (22/7) \times 21$   
 $= (1/6) \times 2 \times (22/7) \times 21$   
 Or Arc AB Length = 22cm
- (ii) It is given that the angle subtended by the arc =  $60^\circ$   
 So, the area of the sector making an angle of  $60^\circ$   
 $= (60^\circ/360^\circ) \times \pi r^2 \text{ cm}^2$   
 $= 441/6 \times 22/7 \text{ cm}^2$   
 Or, the area of the sector formed by the arc APB is  $231 \text{ cm}^2$
- (iii) Area of segment APB = Area of sector OAPB – Area of  $\Delta OAB$   
 Since the two arms of the triangle are the radii of the circle and thus are equal, and one angle is  $60^\circ$ ,  $\Delta OAB$  is an equilateral triangle. So, its area will be  $\sqrt{3}/4 \times a^2$  sq. Units.  
 The area of segment APB =  $231 - (\sqrt{3}/4) \times (OA)^2$

- $= 231 - (\sqrt{3}/4) \times 21^2$   
 Or, the area of segment APB =  $[231 - (441 \times \sqrt{3})/4] \text{ cm}^2$
27. Given A circle inscribed in a  $\Delta PQR$  such that  
 $PQ = PR$   
**To prove**  $QT = TR$   
**Proof** We know that the tangents from an external points to a circle are equal in length.  
 $PS = PU$  [tangents from P] ... (i)  
 $QS = QT$  [tangents from O] ... (ii)  
 $RT = RU$  [tangents from R] ... (iii)  
 Now,  $PQ = PR$  [given] ( $1 \frac{1}{2}$ )  
 $\Rightarrow PQ - PS = PR - PS$   
 [subtracting PS from both sides]  
 $\Rightarrow PQ - PS = PR - PU$  [from Eq. (i)]  
 $\Rightarrow QS = RU$   
 $\Rightarrow QT = RU$  [from Eq. (ii)]  
 $\Rightarrow QT = RT$  [from Eq. (iii)]  
 Hence proved. ( $1 \frac{1}{2}$ )
28. There are 6 possible outcomes (1, 2, 3, 4, 5 and 6) in a single throw of a die.
- (i) We know that even prime number is only 2.  
 So, number of favourable outcomes = 1  
 $\therefore P(\text{getting an even prime number}) = \frac{1}{6}$  ( $1 \frac{1}{2}$ )
- (ii) The numbers divisible by 2 are 2, 4 and 6.  
 So, number of favourable outcomes = 3  
 $\therefore P(\text{getting a number divisible by 2}) = \frac{3}{6} = \frac{1}{2}$
- OR**
- Number of red cards = 26  
 Number of queens = 4  
 But, out of these 4 queens, 2 are red.  
 $\therefore$  Number of queens which are not red = 2  
 Now, number of cards which are red or queen  
 $= 26 + 2 = 28$  (1)  
 $\therefore P(\text{getting either red card or queen}) = \frac{\text{Number of card which are red or queen}}{\text{Total number of cards}}$   
 $= \frac{28}{52} = \frac{7}{13}$  (1)

Now, P (not getting either red card or queen)  
 = 1 - P (getting either red card or queen)  
 $= 1 - \frac{7}{13} = \frac{13-7}{13} = \frac{6}{13}$

29. Here, class intervals are not in inclusive form.

So, we first convert them in inclusive form by subtracting  $h/2$  from the lower limit and adding  $h/2$  to the upper limit of each class, where  $h$  is the difference between the lower limit of a class and the upper limit of the preceding class.

The given frequency distribution in inclusive form is as follows.

Age (in yr)	Number of cases
4.5-14.5	6
14.5-24.5	11
24.5-34.5	21
34.5-44.5	23
44.5-54.5	14
54.5-64.5	5

(1)

We observe that the class 34.5-44.5 has the maximum frequency.

So, it is the modal class such that  $I = 34.5$ ,  $h = 10$ ,  $f_1 = 23$ ,  $f_0 = 21$  and  $f_2 = 14$

$$\therefore \text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h,$$

$$\Rightarrow \text{Mode} = 34.5 + \frac{23 - 21}{46 - 21 - 14} \times 10$$

$$= 34.5 + \frac{2}{11} \times 10 = 36.31 \quad (1)$$

30. The given equations are  $10x + 3y = 75 \dots(i)$   
 $6x - 5y = 11 \dots(ii)$   
 Multiplying Eq. (i) by 5 and Eq. (ii) by 3, we get

$$50x + 15y = 375 \dots(iii)$$

$$18x - 15y = 33 \dots(iv) \quad (1)$$

Adding Eqs. (iii) and (iv), we get

$$68x = 408$$

$$\Rightarrow x = \frac{408}{68} \Rightarrow x = 6 \quad (1)$$

Putting  $x = 6$  in Eq.(i), we get

$$(10 \times 6) + 3y = 75$$

$$\Rightarrow 60 + 3y = 75$$

$$\Rightarrow 3y = 75 - 60$$

$$\Rightarrow 3y = 15$$

$$\Rightarrow y = 5$$

$$\therefore x = 6 \text{ and } y = 5 \quad (1)$$

OR

The given equations are

$$11x + 15y + 23 = 0 \dots (i)$$

$$7x - 2y - 20 = 0 \dots(ii)$$

Multiplying Eq. (i) by 2 and Eq. (ii) by 15 and adding the results, we get

$$22x + 105y = -46 + 300$$

$$\Rightarrow 127x = 254$$

$$\Rightarrow x = \frac{254}{127} = 2 \quad (1)$$

Putting  $x = 2$  in Eq. (i), we get

$$22 + 15y = -23$$

$$\Rightarrow 15y = -23 - 22$$

$$\Rightarrow 15y = -45$$

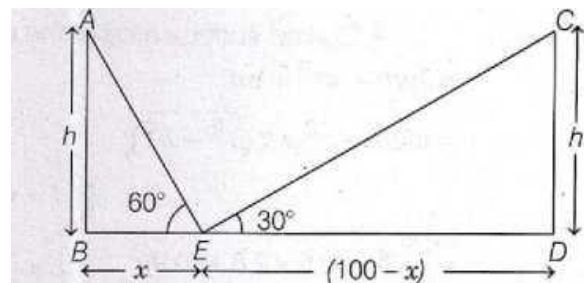
$$\Rightarrow y = \frac{-45}{15} \Rightarrow y = -3$$

Hence,  $x = 2$  and  $y = -3 \quad (2)$

31. Let AB and CD be two pillars of equal height  $h$  and distance between them be  $BD = 100$  m.

Let E be a point on the road such that  $BE = x$ ,

$DE = (100 - x)$ ,  $\angle AEB = 60^\circ$  and  $\angle CED = 30^\circ$ .



In right angled  $\triangle ABE$ ,

$$\frac{AB}{BE} = \tan 60^\circ$$

$$\Rightarrow \frac{h}{x} = \sqrt{3} \quad [\because \tan 60^\circ = \sqrt{3}]$$

$$\Rightarrow h = \sqrt{3} x \dots (i) \quad (1)$$

In right angled  $\triangle CDE$ ,

$$\frac{CD}{DE} = \tan 30^\circ$$

$$\Rightarrow \frac{h}{100-x} = \frac{1}{\sqrt{3}} \dots(ii) (1)$$

From Eqs. (i) and (ii); we get

$$\sqrt{3} = \frac{100-x}{\sqrt{3}}$$

$$\Rightarrow 3x = 100 - x \quad \Rightarrow 4x = 100$$

$$\therefore x = 25$$

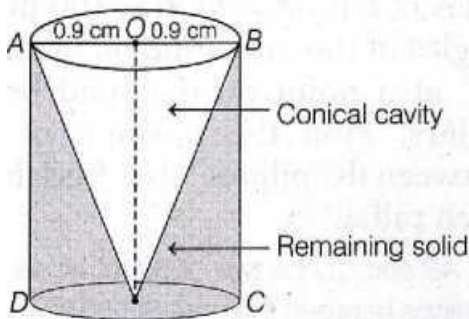
On putting  $x = 25$  in Eq. (i), we get

$$h = \sqrt{3} \times 25$$

$$= 25 \times 1.732 = 43.3 \text{ m}$$

Hence, height of each pillar is 43.3 m and position of the point from pillar making an angle of  $60^\circ$  is 25 m. (1)

32. Lets be the total surface area of the remaining solid.



Then,  $S$  = Curved surface area of the cylinder + Area of the base of the cylinder + Curved surface area of the cone

$$= 2\pi rh + \pi r^2 + \pi r l \quad (1)$$

$$= \pi [2rh + r^2 + r\sqrt{r^2 + h^2}]$$

$$[\because l = \sqrt{r^2 + h^2}]$$

$$= \frac{22}{7} [5.04 + 0.81 + 0.9\sqrt{0.81 + 7.84}]$$

$$= \frac{22}{7} [5.85 + 0.9\sqrt{8.65}]$$

$$= \frac{22}{7} [5.85 + 0.9 \times 2.94]$$

$$= \frac{22}{7} \times [5.85 + 2.64] = \frac{186.78}{7}$$

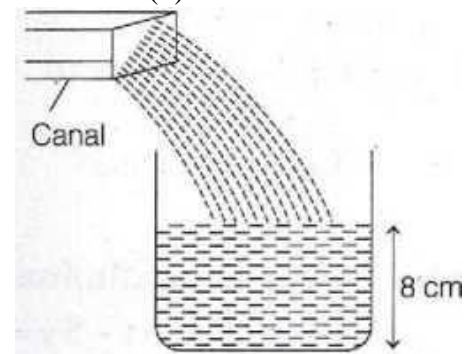
$$= 26.68 \text{ cm}^2 \quad (2)$$

OR

Given, speed of flow of water = 10 km/h  
 =  $10 \times 1000 \text{ m/h}$  [ $\because 1 \text{ km} = 1000 \text{ m}$ ]  
 $\Rightarrow$  Length of water flow in 1 h =  $10 \times 1000 \text{ m}$   
 $\Rightarrow$  Length of water flow in 30 min (i.e. in  $\frac{1}{2}$  h)

$$= \frac{1}{2} \times 10 \times 1000$$

$$= 5000 \text{ m (1)}$$



(1)

Now, volume of water flowing in 30 min  
 = Volume of cuboid of length 5000 m, width 6 m and depth 1.5 m  
 $= 5000 \times 6 \times 1.5 \text{ m}^3 = 45000 \text{ m}^3$  (1)

Hence, the required area covered for irrigation with 8 cm or m of standing water

$$= \frac{4500}{8} \times 100 = 562500 \text{ m}^2$$

$$= \frac{562500}{1000} \text{ hec} [\because 1 \text{ hec} = 10000 \text{ m}^2]$$

$$= 56.25 \text{ hec (2)}$$

33. Given, equations are  $5x - y = 5$  ... (i)

and  $3x - y = 3$  ... (ii)

Table for  $5x - y = 5$  or  $y = 5x - 5$  is

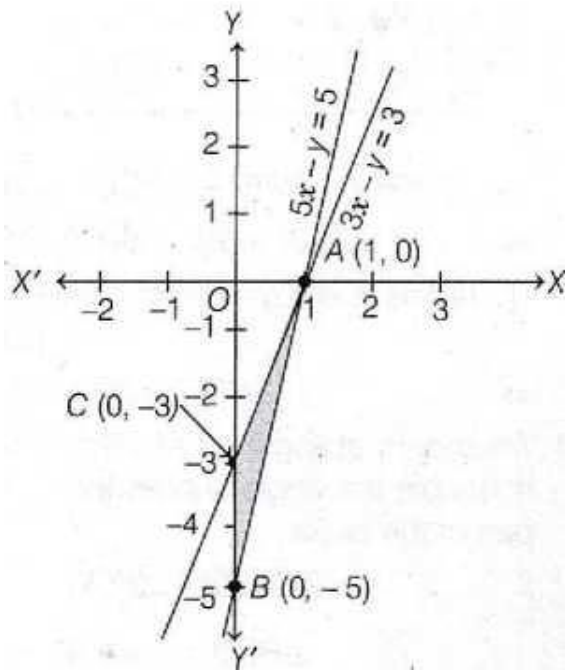
X	1	0
Y	0	-5
Points	A (1,0)	B (0, -5)

Plot the points A(1, 0) and B(0, -5) on a graph paper and join these points to form line AB. (1)

Table for  $3x - y = 3$  or  $y = 3x - 3$  is

X	1	0
Y	0	-3
Points	A (1, 0)	C (0, -3)

Plot the points A (1, 0) and C (0, -3) on the same graph paper and join these points to form line AC. (1)



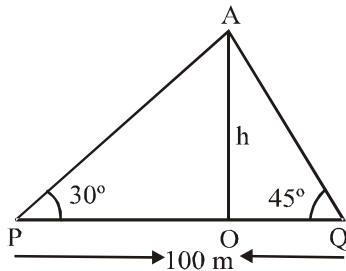
(2)

Hence, the triangle formed by given lines is  $\triangle ABC$  whose vertices are  $A(1, 0)$ ,  $B(0, -5)$  and  $C(0, -3)$ .

(1)

34. Let  $OA$  be the tree of height  $h$  metre. In triangles  $POA$  and  $QOA$ , we have

$$\tan 30^\circ = \frac{OA}{OP} \text{ and } \tan 45^\circ = \frac{OA}{OQ}$$



$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{OP} \text{ and } 1 = \frac{h}{OQ}$$

$$\Rightarrow OP = \sqrt{3}h \text{ and } OQ = h$$

$$\Rightarrow OP + OQ = \sqrt{3}h + h$$

$$\Rightarrow PQ = (\sqrt{3} + 1)h$$

$$\Rightarrow 100 = (\sqrt{3} + 1)h \text{ [}\because PQ = 100 \text{ m]}$$

$$\Rightarrow h = \frac{100}{\sqrt{3} + 1} \text{ m}$$

$$\Rightarrow h = \frac{100(\sqrt{3} - 1)}{2} \text{ m}$$

$$\Rightarrow h = 50(1.732 - 1) \text{ m} = 36.6 \text{ m}$$

Hence, the height of the tree is 36.6 m

OR

Let  $P$  and  $Q$  be the positions of two aeroplanes when  $Q$  is vertically below  $P$  and  $OP = 4000$  m. Let the angles of elevation of  $P$  and  $Q$  at a point  $A$  on the ground be  $60^\circ$  and  $45^\circ$  respectively.

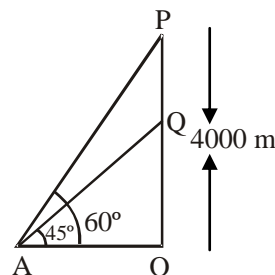
In triangles  $AOP$  and  $AOQ$ , we have

$$\tan 60^\circ = \frac{OP}{OA} \text{ and } \tan 45^\circ = \frac{OQ}{OA}$$

$$\Rightarrow \sqrt{3} = \frac{4000}{OA} \text{ and } 1 = \frac{OQ}{OA}$$

$$\Rightarrow OA = \frac{4000}{\sqrt{3}} \text{ and } OQ = OA$$

$$\Rightarrow OQ = \frac{4000}{\sqrt{3}} \text{ m}$$

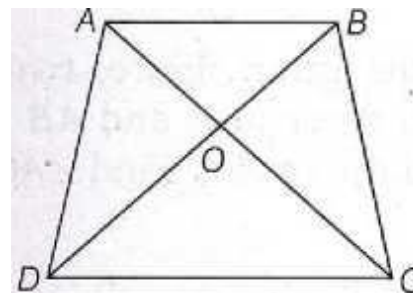


$\therefore$  Vertical distance between the aeroplanes =  $PQ = OP - OQ$

$$= \left( 4000 - \frac{4000}{\sqrt{3}} \right) \text{ m} = 4000 \frac{(\sqrt{3} - 1)}{\sqrt{3}} \text{ m}$$

$$= 1690.53 \text{ m}$$

35. Given  $ABCD$  is a trapezium in which  $AB \parallel DC$ .



To prove  $\frac{OA}{OC} = \frac{OB}{OD}$  (2)

**Proof** In  $\triangle OAS$  and  $\triangle ODC$ , we have

$AB \parallel DC$

Then,  $\angle OAB = \angle OCD$  [alternate interior angles]

$\angle AOB = \angle DOC$  [vertically opposite angles]

and  $\angle ABO = \angle CDO$  [alternate interior angles]

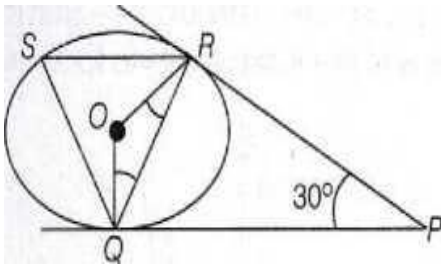
$\therefore \Delta OAB \sim \Delta OCD$  [by AAA similarity criterion]

Hence,  $\frac{OA}{OC} = \frac{OB}{OD}$

[if two triangles are similar, then their corresponding sides are proportional]

**Hence proved. (1)**

36. (i) In quadrilateral POOR, we have



$\angle QPR + \angle PRO + \angle PQO + \angle ROQ = 360^\circ$   
 $\Rightarrow 30^\circ + 90^\circ + 90^\circ + \angle ROQ = 360^\circ$

[ $\because$  radius is always perpendicular to the tangent at point of contact]

$\Rightarrow \angle ROQ = 360^\circ - 210^\circ = 150^\circ$

(ii) We know that angle subtended by an arc at centre is double the angle subtended by it at any other part of the circle.

$2 \angle RSQ = \angle ROQ$

$\angle RSQ = \frac{1}{2} \times 150^\circ = 75^\circ$

(iii) In  $\Delta QOR$ ,  $OQ = OR$  [radii]

$\angle ORQ = \angle OQR$

Now,  $\angle ROQ + \angle ORQ + \angle OQR = 180^\circ$

$\Rightarrow 2\angle OQR = 180^\circ - 150^\circ$

$\Rightarrow 2 \angle OQR = 30^\circ$

$\Rightarrow \angle OQR = 15^\circ$

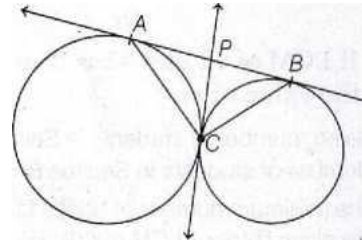
Again,  $\angle OQP = 90^\circ$  [ $\because OQ \perp QP$ ]

$\Rightarrow \angle OQR + \angle RQP = 90^\circ$

$\Rightarrow \angle RQP = 90^\circ - 15^\circ = 75^\circ$

**OR**

Draw a tangent to the circles at point C. Let it meets AB at P



Then,  $PA = PC$  and  $PS = PC$

[the tangents from an external points to a circle are equal in length]

$PA = PC \Rightarrow \angle PAC = \angle PCA$

$PB = PC \Rightarrow \angle PBC = \angle PCB$

$\therefore \angle PAC + \angle PBC = \angle PCA + \angle PCB = \angle ACB$

$\Rightarrow \angle PAC + \angle PBC + \angle ACB = 2\angle ACB$

$\Rightarrow 180^\circ = 2\angle ACB$

$\Rightarrow \angle ACB = 90^\circ$

37. (i) For first metre, the charge is Rs. 100  
 i.e. first term,  $a = 100$

As, there is increasing of Rs. 25 for each subsequent metres, therefore common difference,  $d = 25$

So, the AP thus formed is

100, 125, 150, ...

(ii) Labour charge to dig the well is the 15th term of AP.

We know,  $a_n = a + (n - 1)d$

$\therefore a_{15} = 100 + (15 - 1)25$

$= 100 + 14 \times 25 = 450$

$\therefore$  Labour charge = Rs. 450

(iii) Money saved by Ram = Rs. 450 - Rs. 400 = Rs. 50

**OR**

We know that  $S_n = \frac{n}{2} [2a + (n-1)d]$



$$\begin{aligned} \text{Sum of 15 terms, } S_{15} &= \frac{15}{2} [2 \times 100 + 14 \times 25] \\ &= \frac{15}{2} [200 + 350] = \frac{15}{2} \times 550 = 4125 \end{aligned}$$

38. (i) Given, number of students in Section A = 32

Number of students in Section B = 36

The minimum number of books to be acquired for the class library = LCM of (32, 36)

$$\begin{aligned} &= 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \\ &= 2^5 \times 3^2 \\ &= 32 \times 9 = 288 \end{aligned}$$

- (ii) The prime factors of 36 are  
 $36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$
- (iii) HCF (867, 255) = 51

**OR**

Given, LCM (12, 42) = 10m + 4

Factors of 12 =  $2 \times 2 \times 3$

and factors of 42 =  $2 \times 3 \times 7$

Now, LCM (12, 42) =  $2 \times 2 \times 3 \times 7 = 84$

$$\therefore 84 = 10m + 4$$

$$\Rightarrow 84 - 4 = 10m$$

$$\therefore m = \frac{80}{10} = 8$$