ANSWER AND SOLUTIONS

SECTION-A

1. Option (4)
both (1) and (2)

2. Option (4)
2,520

3. Option (3)
no real roots

4. Option (2)
infinite solution

5. Option (3)
k = 4

6. Option (2)
5 : 1

7. Option (1)
AAA similarity criterion

8. Option (3)
$\angle B = \angle D$

9. Option (1)
$25°$

10. Option (3)
$\sqrt{3}$

11. Option (1)
$30°$

12. Option (3)
$\frac{\sqrt{3}}{2}$

13. Option (3)
9π

14. Option (2)
33 cm

15. Option (3)
$\frac{1}{365}$

16. Option (3)
30 – 40

17. Option (2)
17.5

18. Option (4)
$147\ \pi\ cm^2$

19. Option (4)
Assertion (A) is false but Reason (R) is true.

20. Option (4)
Assertion (A) is false but Reason (R) is true.

SECTION-B

21. Given system of equations is

$$cx + 3y + (3 - c) = 0$$

and $12x + cy - c = 0$

Condition for equations to have infinitely many solutions is :

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Here, $a_1 = c,\ b_1 = 3,\ c_1 = 3 - c$
$a_2 = 12,\ b_2 = c,\ c_2 = -c$

$\therefore \quad \frac{c}{12} = \frac{3}{c} = \frac{3-c}{-c}$

$\Rightarrow \quad c^2 = 36$

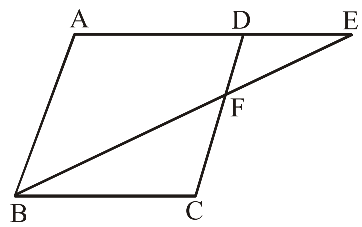
$\Rightarrow \quad c = 6 \text{ or } c = -6 \quad ...(1)$

Also, $\quad -3c = 3c - c^2$

$\Rightarrow \quad c = 6 \text{ or } c = 0 \quad ...(2)$

From (1) and (2), we get, c = 6

22.



Given : ABCD is a parallelogram with side AD extending at E. BE is joined intersecting CD at F.

In ΔABE and ΔCFB

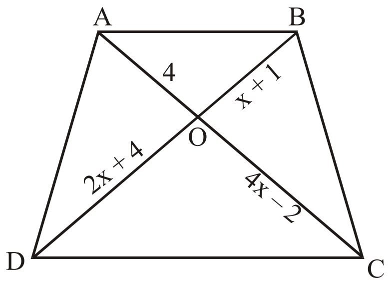
$\angle A = \angle C$ [opposite $\angle$s of parallelgram]

$\angle CFB = \angle ABE$ [Alternate $\angle$s]

$\Delta ABE \sim \Delta CFB$ [By AA similarity]

Hence proved

OR



In ΔABO and ΔCDO

$\angle OAB = \angle OCD$ [Alternate $\angle$s]

$\angle AOB = \angle COD$ [Vertically opposite $\angle$s]

$\Delta AOB \cong \Delta COD$ [by AA similarly]

$$\frac{OA}{OC} = \frac{OB}{OD}$$

$$\frac{4}{4x-2} = \frac{x+1}{2x+4}$$

$$\frac{4}{2(2x-1)} = \frac{(x+1)}{2(x+2)}$$

$4(x + 2) = (2x - 1)(x + 1)$

$4x + 8 = 2x^2 + 2x - x - 1$

$2x^2 - 3x - 9 = 0$

$2x^2 - 6x + 3x - 9 = 0$

$2x(x - 3) + 3(x - 3) = 0$

$(x - 3)(2x + 3) = 0$

$x = 3$ or $x = \frac{-3}{2}$ (not possible)

So, $x = 3$

23. $\angle PQO = 90°$ [$\because$ Tangent $\perp$ radius]

Also, $\angle QOP = 180° - 120° = 60°$ [Linear pair]

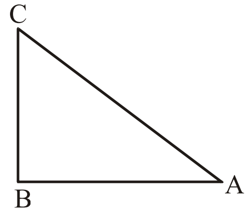
In ΔPQO,

$\angle OPQ = 180° - \angle OQP - \angle QOP$

$= 180° - 90° - 60°$

$= 30°$

24.



Given that, $\tan A = \frac{3}{4}$

Let BC = 3k, AB = 4k

$\Rightarrow$ AC = 5k

$$\sin A = \frac{3k}{5k} = \frac{3}{5}$$

$$\cos A = \frac{4k}{5k} = \frac{4}{5}$$

$$\frac{1}{\sin A} + \frac{1}{\cos A} = \frac{5}{3} + \frac{5}{4}$$

$$= \frac{20+15}{12}$$

$$= \frac{35}{12}$$

25. It is given that circumference of the circle is 176 m

$\Rightarrow 2\pi r = 176$

$\Rightarrow 2 \times \frac{22}{7} \times r = 176$

$\Rightarrow r = \frac{176 \times 7}{2 \times 22} = 28$ cm

Also, in a quadrant $\theta = 90°$

Area of quadrant $= \frac{\theta}{360°} \times \pi r^2$

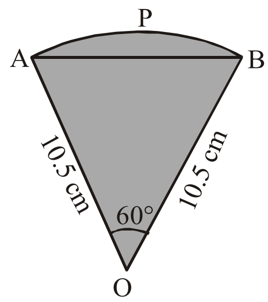
$$= \frac{90°}{360°} \times \frac{22}{7} \times 28 \times 28$$

$= 616\ m^2$

OR

We have, radius (r) = 10.5 cm

and angle (θ) = 60°



$= \frac{\theta}{360°} \times 2\pi r$

$= \frac{60°}{360°} \times 2 \times \frac{22}{7} \times 10.5$

= 11 cm

Now, the perimeter of the sector OAPBO

= OA + length of an arc APB + BO

= (10.5 + 11 + 10.5) cm

= 32 cm

SECTION-C

26. Let us assume, to the contrary, that $(5+2\sqrt{3})$ is rational.

So, we can find coprime integers a and b (≠ 0) such that

$(5+2\sqrt{3}) = \frac{a}{b}, b \neq 0, a, b \in I$

$\Rightarrow \sqrt{3} = \frac{a-5b}{2b}$

This contradict the given fact that $\sqrt{3}$ is irrational.

So, we conclude that $(5+2\sqrt{3})$ is irrational.

27. Let α and β be the zeroes of $5x^2 + 2x - 3$.

Then, $\alpha + \beta = \left(-\frac{b}{a}\right) = -\frac{2}{5}$ and $\alpha\beta = \frac{c}{a} = \frac{-3}{5}$

According to the question, $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the zeroes of the required polynomial.

Now $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-\frac{2}{5}}{-\frac{3}{5}} = \frac{2}{3}$

and $\frac{1}{\alpha}.\frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{-\frac{3}{5}} = \frac{-5}{3}$

Thus, a quadratic polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is

$k\left\{x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \frac{1}{\alpha}.\frac{1}{\beta}\right\}$

$\Rightarrow k\{x^2 - \frac{2}{3}x - \frac{5}{3}\}$

i.e., $k_1(3x^2 - 2x - 5)$ [Let $\frac{k}{3} = k_1$]

28. Let number of right answers be x

Let number of wrong answers be y

As per question

$4x - y = 70$ ....(i)

$5x - 2y = 80$ ....(ii)

2 × eq.(i) – eq.(ii),

$8x - 2y = 140$

$5x - 2y = 80$

– + –

$3x = 60$

$\Rightarrow x = 20$

Substituting the value of x in equation(i) to get vlaue of y,

$4(20) - y = 70$

$\Rightarrow 80 - y = 70$

$\therefore y = 10$

Hence, total number of questions are

= 20 + 10 = 30

OR

Let father's age be x-years

And son's age be y-years.

According to the questions,

$x + 2y = 70$ ....(1)

And $2x + y = 95$ ....(2)

Multiplying equation (2) by 2 and subtracting from (1)

$x + 2y = 70$

$4x + 2y = 190$

$-\quad -\quad -$

$-3x = -120$

$x = 40$

$40 + 2y = 70$

$2y = 30$

$y = 15$

Thus, father's age = 40 years and son's age = 15 years

29. In ΔOPQ and ΔOPR

PQ = PR [Tangent drawn from an external point]

OP = OP [Common]

OQ = OR [Radius of same circle]

$\Delta OPR \cong \Delta OPQ$ [by SSS cong.]

$\angle OPR = \angle OPQ = 45°$ [by cpct]

$\angle OQP = 90°$ [Tangent $\perp$ radius]

$\angle ORP = 90°$ [Tangent $\perp$ radius]

$\angle RPQ = \angle OPR + \angle OPQ = 90°$

PQ = PR [Tangent from external point are equal]

OR = OQ [Radius of same circle]

ORPQ is square

Hence proved

30. Given, $\sin(A + 2B) = \frac{\sqrt{3}}{2} \Rightarrow A + 2B = 60°$

$\cos(A + 4B) = 0 \Rightarrow A + 4B = 90°$

Solving, we get A = 30° and B = 15°

OR

$4(\sin^4 30° + \cos^4 60°) - 3(\cos^2 45° - \sin^2 90°)$

$$= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right] - 3\left[\left(\frac{1}{\sqrt{2}}\right)^2 - (1)^2\right]$$

$$= 4\left[\frac{1}{16} + \frac{1}{16}\right] - 3\left[\frac{1}{2} - 1\right]$$

$$= 4 \times \frac{2}{16} - 3 \times \left(-\frac{1}{2}\right)$$

$$= \frac{1}{2} + \frac{3}{2}$$

$$= \frac{4}{2} = 2$$

31. Total number of balls = x + 2x + 3x = 6x

(i) P(not a red ball) = P(a white or a blue ball)

$$= \frac{2x + 3x}{6x} = \frac{5}{6}$$

(ii) P(a white ball) $= \frac{2x}{6x} = \frac{1}{3}$

(iii) P(a blue or a white) = P(not a red ball)

$= \frac{5}{6}$ [by (i) above]

SECTION-D

32. Let the speed of car at A be x km/h

And the speed of car at B y km/h

Case 1 : $8x - 8y = 80$

or, $x - y = 10$

Case 2 : $\frac{4}{3}x + \frac{4}{3}y = 80$

or, $x + y = 60$

On solving, x = 35 and y = 25

Hence, speed of cars at A and B are 35 km/h and 25 km/h respectively.

OR

Let cost of 1 pencil = x and cost of 1 pen = y

According to question

$5x + 7y = 250$ .....(1)

$7x + 5y = 302$ .....(2)

Adding equation (1) and (2)

$12x + 12y = 552$

$x + y = 46$ .....(3)

Subtracting equation (1) and (2)

$-2x + 2y = -52$

$x - y = 26$ .....(4)

Adding equation (3) and (4)

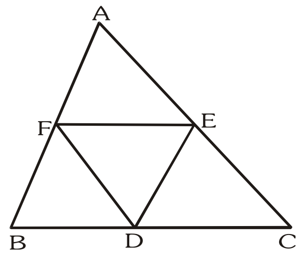
$2x = 72$

$x = 36$

From (3)

$y = 46 - 36 = 10$

33. Given A ΔABC in which D, E, F are the midpoints of BC, CA and AB respectively.



To prove $\Delta AFE \sim \Delta ABC$,
$\Delta FBD \sim \Delta ABC$,
$\Delta EDC \sim \Delta ABC$,
and $\Delta DEF \sim \Delta ABC$

Proof We shall first show that $\Delta AFE \sim \Delta ABC$

Since F and E are the midpoints of AB and AC respectively, so by the converse of Thales theorem,

we have : $FE \parallel BC$.

$\therefore \angle AFE = \angle B$ [corresponding $\angle$s]

Now, in ΔAFE and ΔABC, we have :

$\angle AFE = \angle B$ [corresponding $\angle$s]

and $\angle A = \angle A$ (common)

$\therefore \Delta AFE \sim \Delta ABC$ [by AA-similarity].

Similarly, $\Delta FBD \sim \Delta ABC$ and $\Delta EDC \sim \Delta ABC$.

Now, we shall show that $\Delta DEF \sim \Delta ABC$.

In the same manner as above, we can prove that $ED \parallel AF$ and $DF \parallel EA$.

$\therefore$ AFDE is a ||gm.

$\therefore \angle EDF = \angle A$ [opposite angles of a ||gm]

Similarly, BDEF is a ||gm.

$\therefore \angle DEF = \angle B$ [opposite angles of a ||gm]

Thus, in ΔDEF and ΔABC, we have :

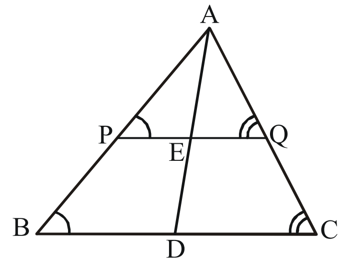
$\therefore \angle EDF = \angle A$ and $\angle DEF = \angle B$

$\therefore \Delta DEF \sim \Delta ABC$ [by AA-similarity]

Hence, the result follows.

OR

Given A ΔABC in which P and Q are the points on AB and AC respectively such that $PQ \parallel BC$ and AD is the median, cutting BC at E.



To prove PE = EQ

Proof

In ΔAPE and ΔABD, we have :

$\angle PAE = \angle BAD$ (common)

$\angle APE = \angle ABD$ (corresponding $\angle$s]

$\therefore \Delta APE \sim \Delta ABD$ [by AA-similarity].

But, in similar triangles, the corresponding sides are proportional.

$$\therefore \frac{AE}{AD} = \frac{PE}{BD} \quad ...(1)$$

In ΔAEQ and ΔADC, we have :

$\angle QAE = \angle CAD$ (common)

$\angle AQE = \angle ACD$ (corresponding $\angle$s]

$\therefore \Delta AEQ \sim \Delta ADC$ [by AA-similarity].

But, in similar triangles, the corresponding sides are proportional.

$$\therefore \frac{AE}{AD} = \frac{EQ}{DC} \quad ...(2)$$

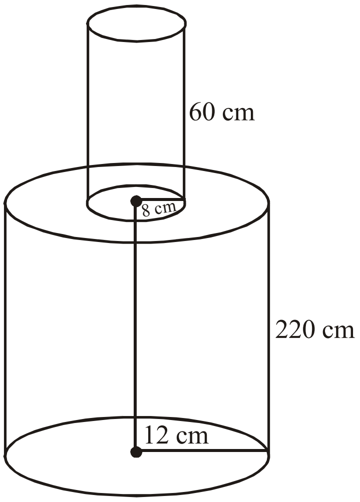
From (1) and (2), we get :

$$\frac{PE}{BD} = \frac{EQ}{DC} \qquad \left[\text{each equal to } \frac{AE}{AD}\right]$$

But, BD = DC [$\because$ Ad is the median]

$\therefore$ PE = EQ.

34.



Volume of I[st] cylinder

$= \pi r^2 h$

$= \pi(12)^2 \times 220$

$= 144 \times 220\pi$

$= 31680\pi$

Volume of II[nd] cylinder

$= \pi R^2 H$

$= \pi 8^2 \times 60$

$= 3840\ \pi$

Total volume $= 31680\pi + 3840\pi$

$= 32520\pi$

$= 32520 \times 3.14$

$= 111532.8\ cm^3$

Mass of 1 cm^3 of iron = 8g

Total weigh = 111532.8×8

= 892262.4 gm

= 892.26 kg

OR

Capacity of first glass $= \pi r^2 H - \frac{2}{3}\pi r^3$

$= \pi \times 9(10 - 2)$

$= 72\pi\ cm^3$

Capacity of second glass $= \pi r^2 H - \frac{1}{3}\pi r^2 h$

$= \pi \times 3 \times 3(10 - 0.5)$

$= 85.5\ \pi cm^3$

$\because$ Suresh got 42.39 cm^3 more quantity of juice.

35. Convert the given frequency distribution to cumulative frequency distribution type.

Class interval	Frequency	c.f.
0-10	5	5
10-20	x	5+ x
20-30	20	25 + x
30-40	15	40 + x
40-50	y	40 + x + y
50-60	5	45 + x + y

Given : Median = 28.5

$\therefore$ Median class = 20 – 30

$\because$ Sum of frequencies = 60

$\therefore 5 + x + 20 + 15 + y + 5 = 60$

$\Rightarrow 45 + x + y = 60$

$\Rightarrow x + y = 15$

We know,

$$\text{Median} = \ell + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

Here, median = 28.5 $\ell = 20$, $\frac{n}{2} = 30$, cf = 5 + x

f = 20 and h = 10

$$\therefore 28.5 = 20 + \left(\frac{30 - 5 - x}{20}\right) \times 10$$

$\Rightarrow 8.5 \times 2 = 25 - x$

$\Rightarrow x = 25 - 17 = 8$

Now, put the value of 'x' in equation (i), we get

$\therefore$ x = 8 and y = 7

SECTION-E

36. (i) From figure, the electrician is required to reach at the point B on the pole AD.

So, BD = AD – AB

= (5 – 1.3) m = 3.7 m

(ii) In ΔBDC,

$\sin 60^\circ = \frac{BD}{BC}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{3.7}{BC}$

$BC = \frac{3.7 \times 2}{\sqrt{3}} = \frac{3.7 \times 2 \times \sqrt{3}}{3}$

$\Rightarrow$ BC = 4.28 m (approx.)

(iii) In ΔBDC,

$\because \cot 60^\circ = \frac{DC}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{DC}{3.7}$

$\Rightarrow DC = \frac{3.7}{\sqrt{3}} = \frac{3.7\sqrt{3}}{3}$

$\Rightarrow$ DC = 2.14 m(approx)

OR

In ΔBDC,

$\therefore \sin 30^\circ = \frac{BD}{BC}$

$\Rightarrow \frac{1}{2} = \frac{3.7}{BC}$

$\Rightarrow$ BC = 3.7 × 2 = 7.4 m

37. (i) Point A lies in x = 3 and y = 4.

$\therefore$ A(3, 4) is the correct position.

Point D lies at x = 6 and y = 1.

So, the correct position of D is (6, 1)

(ii) Position of A = (3, 4)

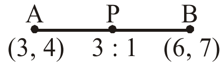
Position of B = (6, 7)

$\therefore$ Distance of AB = $\left|\sqrt{(3-6)^2 + (4-7)^2}\right|$

$= \left|\sqrt{(-3)^2 + (-3)^2}\right|$

$= \sqrt{18} = 3\sqrt{2}$ units

OR



$P\left(\frac{18+3}{4}, \frac{21+4}{4}\right)$

$P\left(\frac{21}{4}, \frac{25}{4}\right)$

(iii) Position of B = (6, 7)

and position of C = (9, 4)

$\therefore$ Mid-point of B and C = $\left(\frac{6+9}{2}, \frac{7+4}{2}\right)$

$= \left(\frac{15}{2}, \frac{11}{2}\right)$

38. (i) Since the top and the bottom rungs are apart by

$2\frac{1}{2} m = \frac{5}{2} m$

$= \frac{5}{2} \times 100$ cm = 250 cm

(ii) The distance between the two rungs is 25 cm

Hence, the total number of rungs

$= \frac{250}{25} + 1 = 11$

(iii) The required length of the wood,

$S_{11} = \frac{11}{2}[25 + 45]$

$= \frac{11}{2} \times 70 = 385$ cm

OR

$a_{11} = 25$

a + 10d = 25

45 + 10d = 25

10d = – 20

d = – 2

$a_{10} = a + 9d$

= 45 + 9 × (– 2) = 45 – 18 = 27 m