MATHEMATICS
(STANDARD)
ANSWER AND SOLUTIONS

SECTION-A

1. Option (4)
720 mins

2. Option (3)
$4\sqrt{3}-3$

3. Option (3)
6 sq. units

4. Option (1)
$k \le 8$

5. Option (2)
90

6. Option (4)
IV quadrant

7. Option (2)
5 : 1

8. Option (1)
$50°$

9. Option (1)
60 cm^2

10. Option (1)
$25°$

11. Option (1)
1

12. Option (4)
$\frac{\sqrt{7}}{\sqrt{8}}$

13. Option (4)
$30\sqrt{3}\,m$

14. Option (4)
9π cm^2

15. Option (3)
132 cm

16. Option (4)
$\frac{17}{16}$

17. Option (2)
$\frac{22}{46}$

18. Option (2)
8

19. Option (3)
Assertion (A) is true but Reason (R) is false.

20. Option (4)
Assertion (A) is false but Reason (R) is true.

SECTION-B

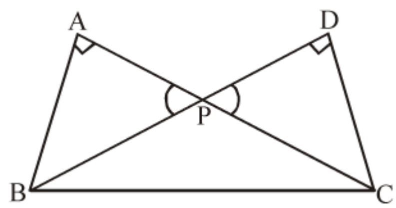
21. Here, Length = 825 cm
Breadth = 675 cm
and Height = 450 cm
Also, $825 = 5 \times 5 \times 3 \times 11$
$675 = 5 \times 5 \times 3 \times 3 \times 3$
and $450 = 2 \times 3 \times 3 \times 5 \times 5$
HCF $= 5 \times 5 \times 3 = 75$

Therefore, the length of the longest rod which can measure the three dimensions of the room exactly is 75 cm.

22. In ΔAPB and ΔDPC
$\angle A = \angle D$ [Each 90°]
$\angle APB = \angle DPC$ [Vertically opposite angles]
$\Delta APB \sim \Delta DPC$ [AA similarity]

$$\frac{AP}{DP} = \frac{BP}{PC}$$

$\Rightarrow AP \times PC = BP \times DP$



23. In ΔPOA and ΔPOB

$PA = PB$ (Tangents drawn from an external point to a circle are equal)

$OP = OP$ (Common)

$OA = OB$ (Radius)

$\Delta POA \cong \Delta POB$ (by SSS cong.)

$\angle APO = \angle BPO$ (by cpct)

$\angle APB = 80°$

$$\angle APO = \frac{80°}{2} = 40°$$

In ΔPAO

$\angle POA = 180° - (90° + 40°) = 50°$

24. Since $\sin(A - C) = \frac{1}{2}$

$A - C = 30°$

But, $A + C = 90°$ (as, $A + B + C = 180°$)

So, $C = 30°$ and $A = 60°$

OR

Given, $\sin\theta = \frac{2mn}{m^2 + n^2}$, we have

$$\cos\theta = \frac{m^2 - n^2}{m^2 + n^2} \text{ and } \cot\theta = \frac{m^2 - n^2}{2mn}$$

So, $$\frac{\sin\theta\cot\theta}{\cos\theta} = \frac{\frac{2mn}{m^2 + n^2} \times \frac{m^2 - n^2}{2mn}}{\frac{m^2 - n^2}{m^2 + n^2}}$$

$= 1$

25. Radii of two concentric circle = 7 cm and 14 cm

and $\angle AOC = 40°$

Reflex $\angle AOC = 360° - 40° = 320°$

Area of shaded region

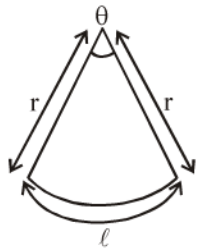
$$= \frac{320}{360} \times \frac{22}{7}(14^2 - 7^2) = \frac{320}{360} \times \frac{22}{7} \times 7 \times 21$$

$= 410.67 \text{ cm}^2$

OR

Let θ be the central angle of the sector and r be the radius of the circle.

Then, length of arc of sector $(\ell) = \frac{\theta}{360°} \times 2\pi r$



So, perimeter of the sector $= \ell + r + r$

$$= \frac{\theta}{360°} \times 2\pi r + 2r$$

$$\Rightarrow 16.4 = \frac{\theta}{360°} \times 2\pi \times (5.2) + 2 \times 5.2$$

[$\because$ r = 5.2 cm]

$$\Rightarrow \pi\theta = \frac{1080}{5.2} \quad \text{....(i)}$$

Now, area of the sector $= \frac{\theta}{360°} \times \pi(5.2)^2$

$$= \frac{1}{360°} \times \frac{1080}{5.2} \times (5.2)^2 \quad \text{[Using (i)]}$$

$= 15.6 \text{ cm}^2$

Hence, area of the sector is 15.6 cm^2

SECTION-C

26. Let us assume, to the contrary, that $\sqrt{5}$ is rational.

So, we can find coprime integers a and b ($\neq 0$) such that

$$\sqrt{5} = \frac{a}{b}, \; b \neq 0, \; a, b \in I$$

$$\Rightarrow \sqrt{5}\,b = a$$

Squaring on both sides, we get

$$5b^2 = a^2$$

Therefore, 5 divides a^2.

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Therefore, 5 divides a

So, we can write a = 5c for some integer c.

Substituting for a, we get

$5b^2 = 25c^2$

$\Rightarrow b^2 = 5c^2$

This means that 5 divides b^2, and so 5 divides b.

Therefore, a and b have at least 5 as a common factor.

But this contradicts the fact that a and b have no common factor other than 1.

This contradict our assumption that $\sqrt{5}$ is rational.

So, we conclude that $\sqrt{5}$ is irrational.

27. $f(x) = 6x^2 + x - 2.$

Here a = 6, b = 1, c = –2

Sum of zeroes, $(\alpha + \beta) = \frac{-b}{a} = \frac{-1}{6}$

Product of zeroes, $(\alpha\beta) = \frac{c}{a} = \frac{-2}{6} = \frac{-1}{3}$

Now $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\beta\alpha} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\beta\alpha}$

$$= \frac{\left(\frac{-1}{6}\right)^2 - 2\times\left(\frac{-1}{3}\right)}{\frac{-1}{3}} = \frac{\frac{1}{36} + \frac{2}{3}}{\frac{-1}{3}} = \frac{1+24}{36} \times -3$$

$$= \frac{-25}{12}$$

28. The given equation are 71x + 37y = 253 ...(i)

and 37x + 71y = 287 ....(ii)

It is clear from above equations that coefficients of x and y are interchanged in both equations.

On adding equations (i) and (ii), we get

108x + 108y = 540

$\Rightarrow$ 108(x + y) = 540

$\Rightarrow x + y = \frac{540}{108} = 5$ ....(iii)

On subtracting equation (ii) from equation (i), we get

34x – 34y = –34 $\Rightarrow$ 34(x – y) = –34

$\Rightarrow x - y = \frac{-34}{34}$

$\Rightarrow$ x – y = –1 ....(iv)

On adding equations (iii) and (iv), we get

$x = \frac{4}{2} = 2$

On substituting the value of x in equation (iii), we get

2 + y = 5 $\Rightarrow$ y = 5 – 2 = 3

Hence, x = 2 and y = 3

OR

Let the fixed charge be Rs.x and the charge per kilometer be Rs.y.

According to first condition, x + 6y = 58 ...(i)

According to second condition,

x + 10y = 90 ....(ii)

On subtracting equation (ii) from equation (i), we get

x + 6y = 58

x + 10y = 90

– – –

–4y = –32

$\Rightarrow y = \frac{-32}{4} = 8$

On substituting the value of y in equation (i), we get

x + 6 × 8 = 58

$\Rightarrow$ x = 58 – 48 $\Rightarrow$ x = 10

Hence, charge per km is Rs.8 and fixed charge is Rs.10

29. Assume the radius of the circle as x cm and since NVUW is a square, WU = UV = x cm.

Uses the pythagoras theorem in ΔSUT and find the length of ST as $\sqrt{(400+100)} = 10\sqrt{5}$ cm

Find the length of VT as (10 – x) cm and use properties of tangents to write YT = VT

Use properties of tangents to equate SY and SW to write the equation as : $20 - x = 10\sqrt{5} - (10 - x)$

$\Rightarrow 20 - x = 10\sqrt{5} - 10 + x$

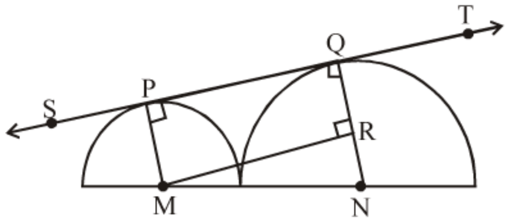
$\Rightarrow 2x = 30 - 10\sqrt{5}$

$\Rightarrow x = (15 - 5\sqrt{5})$ cm

Hence radius of circle is $(15 - 5\sqrt{5})$ cm

OR

Joints MP, NQ and uses the perpendicularity of radius to tangents to draw MR parallel line to PQ. The rough figure may look as follows :



Finds RN as $16-9=7$ cm and MN as $9+16=25$ cm

Uses the pythagoras theorem in ΔMRN to find the length of MR as $\sqrt{(25^2-7^2)}$ = 24 cm.

Writes the since MRQP is a rectangle, PQ = MR = 24 cm.

30. LHS = $\dfrac{\sin A-2\sin^3 A}{2\cos^3 A-\cos A}$

$=\dfrac{\sin A(1-2\sin^2 A)}{\cos A(2\cos^2 A-1)}$

$=\dfrac{\sin A}{\cos A}\left(\dfrac{1-2(1-\cos^2 A)}{2\cos^2 A-1}\right)$

$=\tan A\left(\dfrac{1-2+2\cos^2 A}{2\cos^2 A-1}\right)$

$=\tan A\left(\dfrac{2\cos^2 A-1}{2\cos^2 A-1}\right)$

= tanA

= RHS

31.

Marks	Number of students (f_i)	Cumulative frequency
0-10	10	10
10-20	x	10 + x
20-30	25	35 + x
30-40	30	65 + x
40-50	y	65 + x + y
50-60	10	75 + x + y
Total	100	

It is given that,

N = 100

$\Rightarrow$ 75 + x + y = 100

$\Rightarrow$ x + y = 25 ....(i)

Also, it is given that median is 32.

So, the median class is 30-40

For this class,

$\ell = 30$, f = 30, cf = 35 + x, $\dfrac{N}{2}$ = 50 and h = 10

Now, Median = $\ell+\left(\dfrac{\frac{N}{2}-cf}{f}\right)\times h$

$\Rightarrow$ 32 = 30 + $\left(\dfrac{50-(35+x)}{30}\right)$ × 10

$\Rightarrow$ 2 × 3 = 15 – x

$\Rightarrow$ x = 15 – 6 = 9

Putting the value of 'x' in equation (i), we get

y = 25 – 9 = 16

Hence, the values of 'x' and 'y' are 9 and 16 respectively.

SECTION-D

32. Let the original speed of the train be x km/h.

Then, time taken to cover the journey of

480 km = $\dfrac{480}{x}$ hours

Time taken to cover the journey of 480 km

with speed of (x – 8) km/h = $\dfrac{480}{x-8}$ hours

Now according to questions,

$\dfrac{480}{x-8}-\dfrac{480}{x}=3$

$\Rightarrow 480\left[\dfrac{x-x+8}{x(x-8)}\right]=3$

$\Rightarrow 3x(x - 8) = 3840$

$\Rightarrow x(x - 8) = 1280$

$\Rightarrow x^2 - 8x - 1280 = 0$

$\Rightarrow x^2 - 40x + 32x - 1280 = 0$

$\Rightarrow x(x - 40) + 32(x - 40) = 0$

$\Rightarrow (x + 32)(x - 40) = 0$

$\Rightarrow x + 32 = 0$ or $x - 40 = 0$

$\because x = -32$ (not possible)

$\therefore x = 40$

Thus, the original speed of the train is 40 km/h.

OR

Let the time taken by larger pipe alone to fill the tank = x hours

Therefore, the time taken by the smaller pipe

= x + 10 hours

water filled by larger pipe running for 4 hours

$= \frac{4}{x}$ litres

Water filled by smaller pipe running for 9 hours

$= \frac{9}{x+10}$ litres

We know that, $\frac{4}{x} + \frac{9}{x+10} = \frac{1}{2}$

Which on simplification gives :

$x^2 - 16x - 80 = 0$

$x^2 - 20x + 4x - 80 = 0$

$x(x - 20) + 4(x - 20) = 0$

$(x + 4)(x - 20) = 0$

$x = -4, 20$

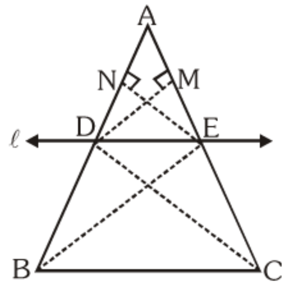
x cannot be negative.

Thus, x = 20

x + 10 = 30

Larger pipe would alone fill the tank in 20 hours and smaller pipe would fill the tank alone in 30 hours.

33.



We are given a triangle ABC in which a line parallel to side BC intersects other two sides AB and AC at D and E respectively.

We need to prove that $\frac{AD}{DB} = \frac{AE}{EC}$.

Let us join BE and CD and then draw

DM ⊥ AC and EN ⊥ AB.

Now, area of ΔADE $= (\frac{1}{2} \times \text{base} \times \text{height})$

$= \frac{1}{2} \times AD \times EN$

area of ΔADE is denoted as ar(ADE).

So, ar(ADE) $= \frac{1}{2} \times AD \times EN$

Similarly, ar(BDE) $= \frac{1}{2} \times DB \times EN$

ar(ADE) $= \frac{1}{2} \times AE \times DM$

and ar(DEC) $= \frac{1}{2} \times EC \times DM$

Therefore, $\frac{ar(ADE)}{ar(BDE)} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB}$.(i)

$\frac{ar(ADE)}{ar(DEC)} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC}$ ...(ii)

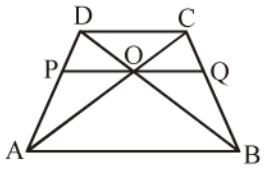
Note that ΔBDE and ΔDEC are on the same base and between the same parallels BC and DE.

So, ar(BDE) = ar(DEC) ....(iii)

Thereofore, form (i), (ii) and (iii), we have :

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence, proved



In ΔDAB

$PO \parallel AB$

$$\frac{DP}{PA} = \frac{DO}{OB} \quad ....(1)$$

In ΔDBC

$OQ \parallel CD$

$$\frac{CQ}{QB} = \frac{DO}{OB} \quad ....(2)$$

from (1) and (2)

$$\frac{DP}{PA} = \frac{CQ}{BQ}$$

34. Radius of the base of cylinder (r) = 2.8 m

= Radius of the base of the cone (r)

Height of the cylinder (h) = 3.5 m

Height of the cone (H) = 2.1 m

Slant height of conical part (ℓ) = $\sqrt{r^2 + H^2}$

$= \sqrt{(2.8)^2 + (2.1)^2}$

$= \sqrt{7.84 + 4.41}$

$= \sqrt{12.25}$

= 3.5 m

Area of canvas used to make tent

= CSA of cylinder + CSA of cone

$= 2 \times \pi \times 2.8 \times 3.5 + \pi \times 2.8 \times 3.5$

= 61.6 + 30.8

= 92.4 m^2

Cost of 1500 tents at Rs.120 per sq.m

$= 1500 \times 120 \times 92.4$

= 16,632,000

Share of each school to set up the tents

$= \frac{16632000}{50}$

= Rs.332,640

OR

(i) Total surface area of the block

= TSA of cube + CSA of hemisphere

= Base area of hemisphere

$= 6a^2 + 2\pi r^2 - \pi r^2$

$= 6a^2 + \pi r^2$

$= \left(6 \times 6^2 + \frac{22}{7} \times 2.1 \times 2.1\right)$ = 229.86 sq. cm

(ii) Volume of the block

$= 6^3 + \frac{2}{3} \times \frac{22}{7} \times (2.1)^3$ cu. cm

= (216 + 19.40) cu.cm

= 235.40 cu.cm

35. The frequency distribution for calculating the mean, for the given data is

Classes	Frequency (f_i)	Clas mark (x_i)	$d_i = x_i - A$ where A = 50	$f_i d_i$
0-20	5	10	–40	–200
20-40	f_1	30	–20	$-20f_1$
40-60	10	50 = A	0	0
60-80	f_2	70	20	$20f_2$
80-100	7	90	40	280
100-120	8	110	60	480
	$\Sigma f_i = 30 + f_1 + f_2$			$\Sigma f_i d_i = 560 + 20f_2 - 20f_1$

We know that,

$30 + f_1 + f_2 = 50$

$\Rightarrow f_1 + f_2 = 20$

$\Rightarrow f_2 = 20 - f_1$ ...(i)

Now, mean $= A + \frac{\Sigma f_i d_i}{\Sigma f_i}$

$\Rightarrow 62.8 = 50 + \dfrac{560 + 20f_2 - 20f_1}{50}$

$\Rightarrow 12.8 = \dfrac{560 + 20(20 - f_1) - 20f_1}{50}$ [Using (i)]

$\Rightarrow 640 = 960 - 40\, f_1$

$\Rightarrow 40f_1 = 320$

$\Rightarrow f_1 = 8$

$\therefore\ f_2 = 20 - 8 = 12$

$\therefore\ f_1 = 8,\ f_2 = 12$

SECTION-E

36. (i) $a_6 = 16,000$

$a + (n - 1)d = 16,000$

$a + (6 - 1)d = 16,000$

$a + 5d = 16,000$ ....(i)

$a_9 = 22,600$

$a + (n - 1)d = 22,600$

$a + (9 - 1)d = 22,600$

$a + 8d = 22,600$ ....(ii)

Solving equation (i) and (ii)

$a + 5d = 16,000$

$a + 8d = 22,600$

$-\quad -\quad -$

$-3d = -6,600$

$d = 2,200$

Now, putting d = 2,200 in equation (i)

$a + 5d = 16,000$

$a + 5 \times 2,200 = 16,000$

$a + 11,000 = 16,000$

$a = 5,000$

(ii) Production during 8^{th} year is (a + 7d)

$= 5000 + 7(2200) = 20400$

(iii) Production during first 3 year

$= 5000 + 7200 + 9400$

$= 21600$

OR

$a_n = 29,200$

$a + (n - 1)d = 29,200$

$(n - 1)2,200 = 29,200 - 5,000$

$2200n - 2,200 = 24,200$

$2200n = 26,400$

$n = \dfrac{26,400}{2,200}$

$n = 12$

In 12^{th} year, the production is Rs.29,200

37. (i) $LB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\Rightarrow LB = \sqrt{(0-5)^2 + (7-10)^2}$

$LB = \sqrt{(5)^2 + (3)^2} \Rightarrow LB = \sqrt{25+9}$

$LB = \sqrt{34}$

Hence the distance is $\sqrt{34}$ km

(ii) Coordinate of Kota (K) is

$\left(\dfrac{3\times0+2\times5}{3+2}, \dfrac{3\times7+2\times10}{3+2}\right)$

$= \left(\dfrac{10+0}{5}, \dfrac{21+20}{5}\right) = \left(2, \dfrac{41}{5}\right)$

(iii) L(5, 10), N(2, 6), P(8, 6)

$LN = \sqrt{(2-5)^2 + (6-10)^2} = \sqrt{(3)^2 + (4)^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$

$NP = \sqrt{(8-2)^2 + (6-6)^2} = \sqrt{(6)^2 + (0)^2} = 6$

$PL = \sqrt{(8-5)^2 + (6-10)^2} = \sqrt{(3)^2 + (4)^2} = 5$

as LN = PL ≠ NP, so ΔLNP is an isosceles triangle.

OR

Let A(0, b) be a point on the y-axis then AL = AP

$\Rightarrow \sqrt{(5-0)^2+(10-b)^2} = \sqrt{(8-0)^2+(6-b)^2}$

$\Rightarrow (5)^2 + (10 - b)^2 = (8)^2 + (6 - b)^2$

$\Rightarrow 25 + 100 - 20b + b^2 = 64 + 36 - 12b + b^2$

$\Rightarrow 8b = 25 \Rightarrow b = \frac{25}{8}$

So, the coordinates on y-axis is $\left(0, \frac{25}{8}\right)$

38. (i) Since, $\cos A = \frac{1}{2}$

$\Rightarrow \cos A = \cos 60°$

$\Rightarrow \cos A = \cos 60°$

$\Rightarrow A = 60°$

Then $12\cot^2 A - 2 = 12(\cot 60°)^2 - 2$

$= 12\left(\frac{1}{\sqrt{3}}\right)^2 - 2$

$= 12 \times \frac{1}{3} - 2$

$= 4 - 2 = 2$

(ii) Since, AC ⊥ BC, then ∠C = 90°

$\sin C \times \cos A = \sin 90° \times \frac{AC}{AB}$

$= 1 \times \frac{4}{8} = \frac{1}{2}$

(iii) In ΔABC

$\tan 30° = \frac{AC}{BC}$

$\frac{1}{\sqrt{3}} = \frac{4}{BC}$

$BC = 4\sqrt{3}$ m

$\sin 30° = \frac{AC}{AB}$

$\frac{1}{2} = \frac{4}{AB}$

AB = 8 m

OR

In ΔABC

$\tan 60° = \frac{AC}{BC}$

$\sqrt{3} = \frac{4}{BC}$

$BC = \frac{4\sqrt{3}}{3}$ m

$\sin 60° = \frac{AC}{AB}$

$\frac{\sqrt{3}}{2} = \frac{4}{AB}$

$AB = \frac{8}{\sqrt{3}} = \frac{8\sqrt{3}}{3}$ m