MATHEMATICS

(BASIC)

ANSWER AND SOLUTIONS

SECTION-A

1. Option (4)
both (1) and (2)

2. Option (4)
2,520

3. Option (3)
no real roots

4. Option (2)
infinite solution

5. Option (3)
k = 4

6. Option (2)
5 : 1

7. Option (1)
AAA similarity criterion

8. Option (3)
$\angle B = \angle D$

9. Option (1)
25°

10. Option (3)
$\sqrt{3}$

11. Option (1)
30°

12. Option (3)
$\frac{\sqrt{3}}{2}$

13. Option (3)
9π

14. Option (2)
33 cm

15. Option (3)
$\frac{1}{365}$

16. Option (3)
30 – 40

17. Option (2)
17.5

18. Option (4)
$147\ \pi\ cm^2$

19. Option (4)
Assertion (A) is false but Reason (R) is true.

20. Option (1)
Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

SECTION-B

21. Given system of equations is

$$cx + 3y + (3 - c) = 0$$

and $12x + cy - c = 0$

Condition for equations to have infinitely many solutions is :

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Here, $a_1 = c,\ b_1 = 3,\ c_1 = 3 - c$
$a_2 = 12,\ b_2 = c,\ c_2 = -c$

$$\therefore \quad \frac{c}{12} = \frac{3}{c} = \frac{3-c}{-c}$$

$\Rightarrow \quad c^2 = 36$

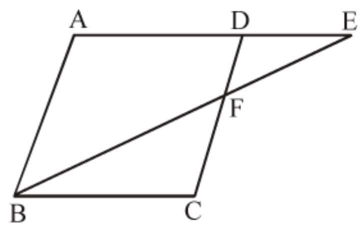
$\Rightarrow \quad c = 6 \text{ or } c = -6 \quad ...(1)$

Also, $\quad -3c = 3c - c^2$

$\Rightarrow \quad c = 6 \text{ or } c = 0 \quad ...(2)$

From (1) and (2), we get, c = 6

22.



Given : ABCD is a parallelogram with side AD extending at E. BE is joined intersecting CD at F.

In ΔABE and ΔCFB

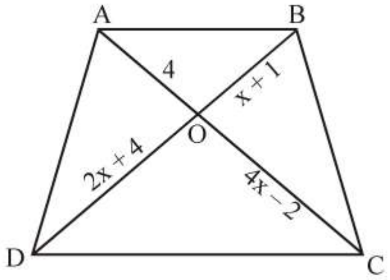
$\angle A = \angle C$ [opposite $\angle$s of parallelgram]

$\angle CFB = \angle ABE$ [Alternate $\angle$s]

$\Delta ABE \sim \Delta CFB$ [By AA similarity]

Hence proved

OR



In ΔABO and ΔCDO

$\angle OAB = \angle OCD$ [Alternate $\angle$s]

$\angle AOB = \angle COD$ [Vertically opposite $\angle$s]

$\Delta AOB \cong \Delta COD$ [by AA similarly]

$$\frac{OA}{OC} = \frac{OB}{OD}$$

$$\frac{4}{4x-2} = \frac{x+1}{2x+4}$$

$$\frac{4}{2(2x-1)} = \frac{(x+1)}{2(x+2)}$$

$4(x + 2) = (2x - 1)(x + 1)$

$4x + 8 = 2x^2 + 2x - x - 1$

$2x^2 - 3x - 9 = 0$

$2x^2 - 6x + 3x - 9 = 0$

$2x(x - 3) + 3(x - 3) = 0$

$(x - 3)(2x + 3) = 0$

$x = 3$ or $x = \frac{-3}{2}$ (not possible)

So, $x = 3$

23. $\angle PQO = 90°$ [$\because$ Tangent $\perp$ radius]

Also, $\angle QOP = 180° - 120° = 60°$ [Linear pair]

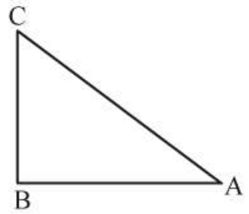
In ΔPQO,

$\angle OPQ = 180° - \angle OQP - \angle QOP$

$= 180° - 90° - 60°$

$= 30°$

24.



Given that, $\tan A = \frac{3}{4}$

Let $BC = 3k$, $AB = 4k$

$\Rightarrow AC = 5k$

$$\sin A = \frac{3k}{5k} = \frac{3}{5}$$

$$\cos A = \frac{4k}{5k} = \frac{4}{5}$$

$$\frac{1}{\sin A} + \frac{1}{\cos A} = \frac{5}{3} + \frac{5}{4}$$

$$= \frac{20+15}{12}$$

$$= \frac{35}{12}$$

25. It is given that circumference of the circle is 176 m

$\Rightarrow 2\pi r = 176$

$$\Rightarrow 2 \times \frac{22}{7} \times r = 176$$

$$\Rightarrow r = \frac{176 \times 7}{2 \times 22} = 28 \text{ cm}$$

Also, in a quadrant $\theta = 90°$

$$\text{Area of quadrant} = \frac{\theta}{360°} \times \pi r^2$$

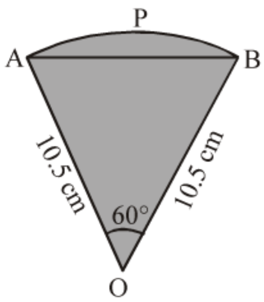
$$= \frac{90°}{360°} \times \frac{22}{7} \times 28 \times 28$$

$= 616 \text{ m}^2$

OR

We have, radius (r) = 10.5 cm

and angle (θ) = 60°



$= \frac{\theta}{360°} \times 2\pi r$

$= \frac{60°}{360°} \times 2 \times \frac{22}{7} \times 10.5$

= 11 cm

Now, the perimeter of the sector OAPBO

= OA + length of an arc APB + BO

= (10.5 + 11 + 10.5) cm

= 32 cm

SECTION-C

26. Let us assume, to the contrary, that $(5+2\sqrt{3})$ is rational.

So, we can find coprime integers a and b (≠ 0) such that

$(5+2\sqrt{3}) = \frac{a}{b}, b \neq 0, a, b \in I$

$\Rightarrow \sqrt{3} = \frac{a-5b}{2b}$

This contradict the given fact that $\sqrt{3}$ is irrational.

So, we conclude that $(5+2\sqrt{3})$ is irrational.

27. Let α and β be the zeroes of $5x^2 + 2x - 3$.

Then, $\alpha + \beta = \left(-\frac{b}{a}\right) = -\frac{2}{5}$ and $\alpha\beta = \frac{c}{a} = \frac{-3}{5}$

According to the question, $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are the zeroes of the required polynomial.

Now $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-\frac{2}{5}}{-\frac{3}{5}} = \frac{2}{3}$

and $\frac{1}{\alpha}.\frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{-\frac{3}{5}} = \frac{-5}{3}$

Thus, a quadratic polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is

$k\left\{x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \frac{1}{\alpha}.\frac{1}{\beta}\right\}$

$\Rightarrow k\{x^2 - \frac{2}{3}x - \frac{5}{3}\}$

i.e., $k_1(3x^2 - 2x - 5)$ [Let $\frac{k}{3} = k_1$]

28. Let number of right answers be x

Let number of wrong answers be y

As per question

$4x - y = 70$ ....(i)

$5x - 2y = 80$ ....(ii)

2 × eq.(i) – eq.(ii),

$8x - 2y = 140$

$5x - 2y = 80$

– + –

$3x = 60$

$\Rightarrow x = 20$

Substituting the value of x in equation(i) to get vlaue of y,

$4(20) - y = 70$

$\Rightarrow 80 - y = 70$

$\therefore y = 10$

Hence, total number of questions are

= 20 + 10 = 30

OR

Let father's age be x-years

And son's age be y-years.

According to the questions,

$x + 2y = 70$ ....(1)

And $2x + y = 95$ ....(2)

Multiplying equation (2) by 2 and subtracting from (1)

$x + 2y = 70$

$4x + 2y = 190$

$- \quad - \quad -$

$-3x = -120$

$x = 40$

$40 + 2y = 70$

$2y = 30$

$y = 15$

Thus, father's age = 40 years and son's age = 15 years

29. In ΔOPQ and ΔOPR

$PQ = PR$ [Tangent drawn from an external point]

$OP = OP$ [Common]

$OQ = OR$ [Radius of same circle]

$\Delta OPR \cong \Delta OPQ$ [by SSS cong.]

$\angle OPR = \angle OPQ = 45°$ [by cpct]

$\angle OQP = 90°$ [Tangent $\perp$ radius]

$\angle ORP = 90°$ [Tangent $\perp$ radius]

$\angle RPQ = \angle OPR + \angle OPQ = 90°$

$PQ = PR$ [Tangent from external point are equal]

$OR = OQ$ [Radius of same circle]

ORPQ is square

Hence proved

30. Given, $\sin(A + 2B) = \frac{\sqrt{3}}{2} \Rightarrow A + 2B = 60°$

$\cos(A + 4B) = 0 \Rightarrow A + 4B = 90°$

Solving, we get $A = 30°$ and $B = 15°$

OR

$4(\sin^4 30° + \cos^4 60°) - 3(\cos^2 45° - \sin^2 90°)$

$$= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right] - 3\left[\left(\frac{1}{\sqrt{2}}\right)^2 - (1)^2\right]$$

$$= 4\left[\frac{1}{16} + \frac{1}{16}\right] - 3\left[\frac{1}{2} - 1\right]$$

$$= 4 \times \frac{2}{16} - 3 \times \left(-\frac{1}{2}\right)$$

$$= \frac{1}{2} + \frac{3}{2}$$

$$= \frac{4}{2} = 2$$

31. Total number of balls $= x + 2x + 3x = 6x$

(i) P(not a red ball) = P(a white or a blue ball)

$$= \frac{2x + 3x}{6x} = \frac{5}{6}$$

(ii) P(a white ball) $= \frac{2x}{6x} = \frac{1}{3}$

(iii) P(a blue or a white) = P(not a red ball)

$= \frac{5}{6}$ [by (i) above]

SECTION-D

32. Let the speed of car at A be x km/h

And the speed of car at B y km/h

Case 1 : $8x - 8y = 80$

or, $x - y = 10$

Case 2 : $\frac{4}{3}x + \frac{4}{3}y = 80$

or, $x + y = 60$

On solving, $x = 35$ and $y = 25$

Hence, speed of cars at A and B are 35 km/h and 25 km/h respectively.

OR

Let cost of 1 pencil = x and cost of 1 pen = y

According to question

$5x + 7y = 250$ .....(1)

$7x + 5y = 302$ .....(2)

Adding equation (1) and (2)

$12x + 12y = 552$

$x + y = 46$ .....(3)

Subtracting equation (1) and (2)

$-2x + 2y = -52$

$x - y = 26$ .....(4)

Adding equation (3) and (4)

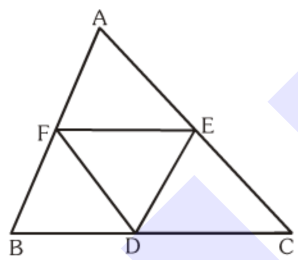
$2x = 72$

$x = 36$

From (3)

$y = 46 - 36 = 10$

33. Given A ΔABC in which D, E, F are the midpoints of BC, CA and AB respectively.



To prove $\Delta AFE \sim \Delta ABC$,

$\Delta FBD \sim \Delta ABC$,

$\Delta EDC \sim \Delta ABC$,

and $\Delta DEF \sim \Delta ABC$

Proof We shall first show that $\Delta AFE \sim \Delta ABC$

Since F and E are the midpoints of AB and AC respectively, so by the converse of Thales theorem,

we have : FE ∥ BC.

$\therefore$ $\angle AFE = \angle B$ [corresponding $\angle$s]

Now, in ΔAFE and ΔABC, we have :

$\angle AFE = \angle B$ [corresponding $\angle$s]

and $\angle A = \angle A$ (common)

$\therefore$ $\Delta AFE \sim \Delta ABC$ [by AA-similarity].

Similarly, $\Delta FBD \sim \Delta ABC$ and $\Delta EDC \sim \Delta ABC$.

Now, we shall show that $\Delta DEF \sim \Delta ABC$.

In the same manner as above, we can prove that ED ∥ AF and DF ∥ EA.

$\therefore$ AFDE is a ∥gm.

$\therefore$ $\angle EDF = \angle A$ [opposite angles of a ∥gm]

Similarly, BDEF is a ∥gm.

$\therefore$ $\angle DEF = \angle B$ [opposite angles of a ∥gm]

Thus, in ΔDEF and ΔABC, we have :

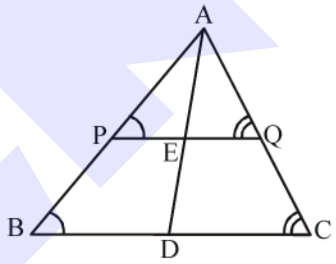
$\therefore$ $\angle EDF = \angle A$ and $\angle DEF = \angle B$

$\therefore$ $\Delta DEF \sim \Delta ABC$ [by AA-similarity]

Hence, the result follows.

OR

Given A ΔABC in which P and Q are the points on AB and AC respectively such that PQ ∥ BC and AD is the median, cutting BC at E.



To prove PE = EQ

Proof

In ΔAPE and ΔABD, we have :

$\angle PAE = \angle BAD$ (common)

$\angle APE = \angle ABD$ (corresponding $\angle$s]

$\therefore$ $\Delta APE \sim \Delta ABD$ [by AA-similarity].

But, in similar triangles, the corresponding sides are proportional.

$$\therefore \quad \frac{AE}{AD} = \frac{PE}{BD} \quad ...(1)$$

In ΔAEQ and ΔADC, we have :

$\angle QAE = \angle CAD$ (common)

$\angle AQE = \angle ACD$ (corresponding $\angle$s]

$\therefore$ $\Delta AEQ \sim \Delta ADC$ [by AA-similarity].

But, in similar triangles, the corresponding sides are proportional.

$$\therefore \quad \frac{AE}{AD} = \frac{EQ}{DC} \quad ...(2)$$

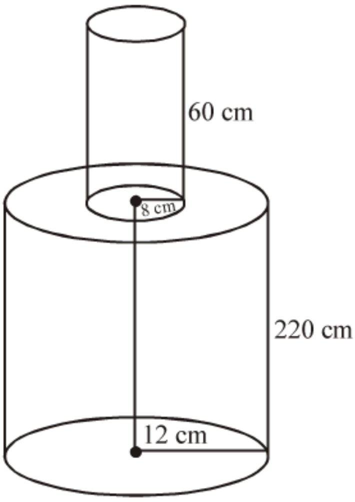
From (1) and (2), we get :

$$\frac{PE}{BD} = \frac{EQ}{DC} \qquad \left[\text{each equal to } \frac{AE}{AD}\right]$$

But, BD = DC [$\because$ Ad is the median]

$\therefore$ PE = EQ.

34.



Volume of Ist cylinder

$= \pi r^2 h$

$= \pi(12)^2 \times 220$

$= 144 \times 220\pi$

$= 31680\pi$

Volume of IInd cylinder

$= \pi R^2 H$

$= \pi 8^2 \times 60$

$= 3840\ \pi$

Total volume $= 31680\pi + 3840\pi$

$= 32520\pi$

$= 32520 \times 3.14$

$= 111532.8\ cm^3$

Mass of 1 cm^3 of iron = 8g

Total weigh = 111532.8 × 8

= 892262.4 gm

= 892.26 kg

OR

Capacity of first glass $= \pi r^2 H - \frac{2}{3}\pi r^3$

$= \pi \times 9(10 - 2)$

$= 72\pi\ cm^3$

Capacity of second glass $= \pi r^2 H - \frac{1}{3}\pi r^2 h$

$= \pi \times 3 \times 3(10 - 0.5)$

$= 85.5\ \pi cm^3$

$\because$ Suresh got 42.39 cm^3 more quantity of juice.

35. Convert the given frequency distribution to cumulative frequency distribution type.

Class interval	Frequency	c.f.
0-10	5	5
10-20	x	5+ x
20-30	20	25 + x
30-40	15	40 + x
40-50	y	40 + x + y
50-60	5	45 + x + y

Given : Median = 28.5

$\therefore$ Median class = 20 – 30

$\because$ Sum of frequencies = 60

$\therefore$ 5 + x + 20 + 15 + y + 5 = 60

$\Rightarrow$ 45 + x + y = 60

$\Rightarrow$ x + y = 15

We know,

$$\text{Median} = \ell + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

Here, median = 28.5 ℓ = 20, $\frac{n}{2}$ = 30, cf = 5 + x

f = 20 and h = 10

$$\therefore 28.5 = 20 + \left(\frac{30 - 5 - x}{20}\right) \times 10$$

$\Rightarrow$ 8.5 × 2 = 25 – x

$\Rightarrow$ x = 25 – 17 = 8

Now, put the value of 'x' in equation (i), we get

$\therefore$ x = 8 and y = 7

SECTION-E

36. (i) From figure, the electrician is required to reach at the point B on the pole AD.

So, BD = AD – AB

= (5 – 1.3) m = 3.7 m

(ii) In ΔBDC,

$$\sin 60^\circ = \frac{BD}{BC}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{3.7}{BC}$$

$$BC = \frac{3.7 \times 2}{\sqrt{3}} = \frac{3.7 \times 2 \times \sqrt{3}}{3}$$

$\Rightarrow$ BC = 4.28 m (approx.)

(iii) In ΔBDC,

$$\because \cot 60^\circ = \frac{DC}{BD}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{DC}{3.7}$$

$$\Rightarrow DC = \frac{3.7}{\sqrt{3}} = \frac{3.7\sqrt{3}}{3}$$

$\Rightarrow$ DC = 2.14 m(approx)

OR

In ΔBDC,

$$\therefore \sin 30^\circ = \frac{BD}{BC}$$

$$\Rightarrow \frac{1}{2} = \frac{3.7}{BC}$$

$\Rightarrow$ BC = 3.7 × 2 = 7.4 m

37. (i) Point A lies in x = 3 and y = 4.

$\therefore$ A(3, 4) is the correct position.

Point D lies at x = 6 and y = 1.

So, the correct position of D is (6, 1)

(ii) Position of A = (3, 4)

Position of B = (6, 7)

$$\therefore \text{Distance of AB} = \left|\sqrt{(3-6)^2 + (4-7)^2}\right|$$

$$= \left|\sqrt{(-3)^2 + (-3)^2}\right|$$

$$= \sqrt{18} = 3\sqrt{2} \text{ units}$$

OR

A — P — B
(3, 4) 3 : 1 (6, 7)

$$P\left(\frac{18+3}{4}, \frac{21+4}{4}\right)$$

$$P\left(\frac{21}{4}, \frac{25}{4}\right)$$

(iii) Position of B = (6, 7)

and position of C = (9, 4)

$$\therefore \text{Mid-point of B and C} = \left(\frac{6+9}{2}, \frac{7+4}{2}\right)$$

$$= \left(\frac{15}{2}, \frac{11}{2}\right)$$

38. (i) Since the top and the bottom rungs are apart by

$$2\frac{1}{2}\text{m} = \frac{5}{2}\text{m}$$

$$= \frac{5}{2} \times 100 \text{ cm} = 250 \text{ cm}$$

(ii) The distance between the two rungs is 25 cm

Hence, the total number of rungs

$$= \frac{250}{25} + 1 = 11$$

(iii) The required length of the wood,

$$S_{11} = \frac{11}{2}[25 + 45]$$

$$= \frac{11}{2} \times 70 = 385 \text{ cm}$$

OR

$a_{11} = 25$

a + 10d = 25

45 + 10d = 25

10d = – 20

d = – 2

a_{10} = a + 9d

= 45 + 9 × (– 2) = 45 – 18 = 27 m