MATHEMATICS

(STANDARD)

ANSWER AND SOLUTIONS

SECTION-A

1. Option (1)
1 : 2

2. Option (2)
x = 4, y = 2

3. Option (4)
6

4. Option (4)
0, 8

5. Option (4)
28

6. Option (4)
5 units

7. Option (4)
Infinitely many

8. Option (2)
$\frac{BE}{EC}$

9. Option (3)
50°

10. Option (4)
$3\sqrt{3}$ cm

11. Option (4)
$5\frac{1}{3}$

12. Option (2)
$b^2 - a^2$

13. Option (1)
75 m

14. Option (4)
7 : 22

15. Option (3)
Both I and II

16. Option (2)
10^5

17. Option (1)
$\frac{9}{13}$

18. Option (2)
15.5

19. Option (4)
Assertion (A) is false but Reason (R) is true.

20. Option (2)
Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

SECTION-B

21. Let us assume on the contrary that $\sqrt{3}$ is a rational number. Then, there exist positive integers a and b such that

$\sqrt{3} = \frac{a}{b}$, where a and b are co-prime i.e. their HCF is 1.

Now, $\sqrt{3} = \frac{a}{b}$

$\Rightarrow 3 = \frac{a^2}{b^2}$

$\Rightarrow 3b^2 = a^2$

$\Rightarrow$ 3 divides a^2 [$\because$ 3 divides $3b^2$]

$\Rightarrow$ 3 divides a [By theorem]

$\Rightarrow$ a = 3c for some integer c

$\Rightarrow a^2 = 9c^2$

$\Rightarrow 3b^2 = 9c^2$ [$\because a^2 = 3b^2$]

$\Rightarrow b^2 = 3c^2$

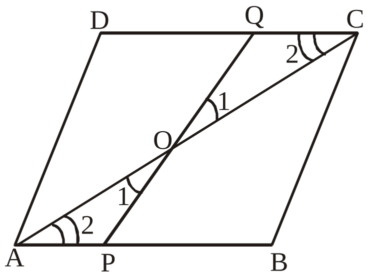
$\Rightarrow$ 3 divides b^2 [$\because$ 3 divides $3c^2$]

$\Rightarrow$ 3 divides b [By theorem]

From (i) and (ii), we observe that a and b have atleast 3 as common factor. But this contradicts the fact that a and b are co-prime. This means that our assumption is not correct.

Hence, $\sqrt{3}$ is an irrational number.

22. Given : $\frac{AP}{PB} = \frac{2}{3}$ and $\frac{DQ}{QC} = \frac{3}{4}$



In ΔAOP and ΔQOC

$\angle AOP = \angle QOC$ (V.O.A)

$\angle OAP = \angle OCQ$ (Alternate interior angles)

By AA criteria,

$\Delta AOP \sim \Delta COQ$

By similarly,

$\frac{AO}{OC} = \frac{AP}{CQ}$ ....(1)

Let AB = CD = 'x' unit

then QC = $\frac{4}{7}x$ and AP = $\frac{2}{7}x$

then, from equation (1)

$\Rightarrow \frac{AO}{OC} = \frac{\frac{2}{7}x}{\frac{4x}{7}} = \frac{1}{2}$

$\Rightarrow OC = 2AO$

$\Rightarrow AO = \frac{1}{2}OC$

Hence proved

23. $\angle A = \angle OPA = \angle OSA = 90°$

Hence, $\angle SOP = 90°$

Also, AP = AS

Hence, OSAP is a square.

AP = AS = 10 cm

CR = CQ = 27 cm

BQ = BC – CQ = 38 – 27 = 11 cm

BP = BQ = 11 cm

x = AB = AP + BP = 10 + 11 = 21 cm

24. LHS = $\frac{\operatorname{cosec}^2 x - \sin^2 x \cot^2 x - \cot^2 x}{\sin^2 x}$

$= \frac{1 - \sin^2 x \cot^2 x}{\sin^2 x}$ ($\because$ $\operatorname{cosec}^2 x - \cot^2 x = 1$)

$= \frac{1}{\sin^2 x} - \cot^2 x = \operatorname{cosec}^2 x - \cot^2 x = 1$

Thus, LHS = RHS

OR

$= \frac{3\left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2 + \left(\frac{2}{1}\right) - 1}{(1)^2}$

$= \frac{1+3+2-1}{1} = 5$

25. Let the measure of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ be θ_1, θ_2, θ_3 and θ_4 respectively

Required area = Area of sector with centre A + Area of sector with centre B + Area of sector with centre C + Area of sector with centre D

$= \frac{\theta_1}{360°} \times \pi \times 7^2 + \frac{\theta_2}{360°} \times \pi \times 7^2 + \frac{\theta_3}{360°} \times \pi \times 7^2 + \frac{\theta_4}{360°} \times \pi \times 7^2$

$= \frac{(\theta_1 + \theta_2 + \theta_3 + \theta_4)}{360°} \times \pi \times 7^2$

$= \frac{(360°)}{360°} \times \frac{22}{7} \times 7 \times 7$

(By angle sum property of a triangle)

$= 154 \text{ cm}^2$

OR

We know that, in 60 minutes, the tip of minute hand moves 360°

In 1 minute, it will move = $\frac{360°}{60} = 6°$

$\therefore$ From 7 : 05 pm to 7 : 40 p. m i.e., 35 min, it will move through = 35 × 6° = 210°

$\therefore$ Area swept by the minute hand in 35 min

= Area of sector with sectorial angle 210° and radius of 6 cm

$= \frac{210}{360} \times \pi \times 6^2$

$= \frac{7}{12} \times \frac{22}{7} \times 6 \times 6$

= 66 cm^2

SECTION-C

26. In order to arrange these books, we have to find HCF of 192, 480 and 672.

Prime factors of 192 = $2^6 \times 3$

Prime factors of 480 = $2^5 \times 3 \times 5$

Prime factors of 672 = $2^5 \times 3 \times 7$

∴ HCF (192, 480 and 672) = $2^5 \times 3$ = 96

∴ There must be 96 books in each stack.

∴ Number of stacks of Science books

$= \frac{192}{96} = 2$

Number of stacks of History books = $\frac{480}{96} = 5$

Number of stacks of Drawing books

$= \frac{672}{96} = 7$

27. $\alpha + \beta = \frac{-b}{a} = -\frac{(-5)}{1}$

$\alpha + \beta = 5$

$\alpha\beta = \frac{c}{a} = \frac{4}{1}$

$\alpha\beta = 4$

(1) $\left(\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta\right)$

$\left(\frac{\beta + \alpha}{\alpha\beta}\right) - 2\alpha\beta$

$\frac{5}{4} - 2(4) = \left(\frac{5}{4} - 8\right) = \frac{-27}{4}$

(2) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{25 - 2(4)}{4}$

$= \frac{17}{4}$

28. Let the digits at units and tens place of the given number be x and y respectively. Then,

Number = 10y + x

Number obtained by reversing the order of the digits = 10x + y

According to the given conditions, we have

(10y + x) + (10x + y) = 165

and, x – y = 3 or, y – x = 3

⇒ 11x + 11y = 165

and, x – y = 3 or, y – x = 3

⇒ x + y = 15

and, x – y = 3 or, y – x = 3

Thus, we obtain the following systems of linear equations.

(i) x + y = 15, x – y = 3

(ii) x + y = 15, y – x = 3

Solving first system of equations, we get

x = 9, y = 6

Solving second system of equations, we get

x = 6, y = 9

Substituting the values of x and y in equation (i), we have

Number = 69 or, 96.

OR

The given system of equations may be written as

$(a - b)x + (a + b)y - (a^2 - 2ab - b^2) = 0$ ...(1)

$(a + b)x + (a + b)y - (a^2 + b^2) = 0$ .....(2)

Subtracting equation (2) from (1)

$(a - b - a - b)x + (a^2 + b^2 - a^2 + 2ab + b^2)$

$-2bx + (2b^2 + 2ab) = 0$

$2bx = 2b(b + a)$

$x = a + b$

Subsitituting x = a + b in equation (1)

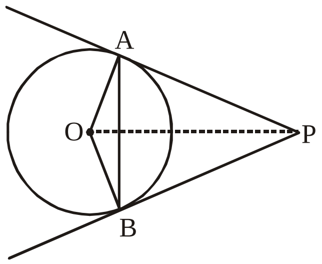
$(a - b)(a + b) + (a + b)y - (a^2 - 2ab - b^2) = 0$

$a^2 - b^2 + (a + b)y = a^2 - 2ab - b^2$

(a = b)y = –2ab

$y = \frac{-2ab}{a+b}$

29. In the given figure



$\angle OAB = 30°$

$\angle OAP = 90°$ [Angle between the tangent and the radius at the point of contact]

$\angle PAB = 90° - 30° = 60°$

AP = BP [Tangents to a circle from an external point]

$\angle PAB = \angle PBA$

[Angles opposite to equal sides of a triangle]

In ΔABP, $\angle PAB + \angle PBA + \angle APB = 180°$

[Angle sum property]

$60° + 60° + \angle APB = 180°$

$\angle APB = 60°$

$\therefore$ ΔABP is an equilateral triangle, where

AP = BP = AB.

PA = 6 cm

In right ΔOAP, $\angle OPA = 30°$

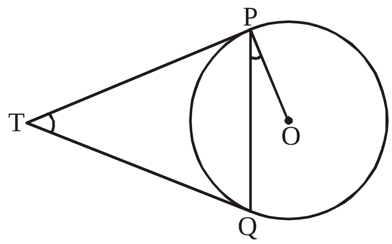
$\tan 30° = \frac{OA}{PA}$

$\frac{1}{\sqrt{3}} = \frac{OA}{6}$

$OA = \frac{6}{\sqrt{3}} = 2\sqrt{3}$ cm

OR

In the given figure



$\angle TPQ = \theta$

$\angle TPO = 90°$ [Angle between the tangent and the radius at the point of contact]

$\angle OPQ = 90° - \theta$

TP = TQ

[Tangents to a circle from an external point]

$\angle TPQ = \angle TQP = \theta$

[Angle opposite to equal sides of a triangle]

In ΔPQT, $\angle PQT + \angle QPT + \angle PTQ = 180°$

[Angle sum property]

$\theta + \theta + \angle PTQ = 180°$

$\angle PTQ = 180° - 2\theta$

$\angle PTQ = 2(90° - \theta)$

$\angle PTQ = 2\angle OPQ$ [using (1)]

Hence proved

30. We have,

$LHS = (1 - \sin\theta + \cos\theta)^2$

$\Rightarrow LHS = 1 + \sin^2\theta + \cos^2\theta - 2\sin\theta + 2\cos\theta - 2\sin\theta\cos\theta$

$\Rightarrow LHS = 2 - 2\sin\theta + 2\cos\theta - 2\sin\theta\cos\theta$

$\Rightarrow LHS = 2(1 - \sin\theta) + 2\cos\theta(1 - \sin\theta)$

$\Rightarrow LHS = 2(1 - \sin\theta)(1 + \cos\theta) = RHS$

31.

Life time	Frequency	x_i	$d_i = x_i - A$	$u_i = \frac{d_i}{h}$	f_iu_i
299.5 – 399.5	14	349.5	–400	–4	–56
399.5 – 499.5	46	449.5	–300	–3	–138
499.5 – 599.5	58	549.5	–200	–2	–116
599.5 – 699.5	76	649.5	–100	–1	–76
699.5 – 799.5	68	749.5	0	0	0
799.5 – 899.5	62	849.5	100	1	62
899.5 – 999.5	48	949.5	200	2	96
999.5 – 1099.5	22	1049.5	300	3	66
1099.5 – 1199.5	6	1149.5	400	4	24
	400				–138

Let the assumed mean be A = 749.5 and h = 100.

We have N = 400, A = 749.5, h = 100 and $\Sigma f_i u_i$ = –138

$$\bar{X} = A + h\left\{\frac{1}{n}\Sigma f_i u_i\right\}$$

$$\bar{X} = 749.5 + 100 \times \left(\frac{-138}{400}\right) = 749.5 - \frac{138}{4}$$

= 749.5 – 34.5 = 715

Hence, the average life time of a tube is 715 hours.

SECTION-D

32. Let the speed of the stream be x km/hr.

∴ Speed of the boat upstream = (5 – x)km/hr.

Speed of the boat downstream = (5 + x) km/hr.

Time taken for going 5.25 km upstream = $\frac{5.25}{5-x}$ hours.

Time taken for going 5.25 downstream = $\frac{5.25}{5+x}$ hours.

Obviously, time taken for going 5.25 km upstream is more than the time taken for going 5.25 km, downstream.

It is given that the time taken for going 5.25 km. upstream is 1 hour more than the time taken for going 5.25 downstream.

$$\therefore \frac{5.25}{5-x} - \frac{5.25}{5+x} = 1$$

$$\Rightarrow 5.25\left\{\frac{1}{5-x} - \frac{1}{5+x}\right\} = 1$$

$$\Rightarrow \frac{21}{4}\left\{\frac{5+x-5+x}{(5-x)(5+x)}\right\} = 1$$

$$\Rightarrow \frac{21}{4} \times \frac{2x}{25-x^2} = 1$$

$$\Rightarrow \frac{21}{2} \times \frac{x}{25-x^2} = 1$$

⇒ $21x = 50 - 2x^2$

⇒ $2x^2 + 21x - 50 = 0$

⇒ $2x^2 + 25x - 4x - 50 = 0$

⇒ $x(2x + 25) - 2(2x + 25) = 0$

⇒ $(2x + 25)(x - 2) = 0$

⇒ $x - 2 = 0, 2x + 25 = 0$

⇒ $x = 2$ $\left[\because x \neq -\frac{25}{2} \text{ as } x > 0\right]$

Hence, the speed of the stream is 2 km/hr.

OR

Let the original duration of the tour be x days.

Total expenditure on tour = Rs.360

∴ Expenditure per day = Rs. $\frac{360}{x}$

Duration of the extended tour = (x + 4) days

∴ Expenditure per day according to new schedule = Rs. $\frac{360}{x+4}$

It is given that the daily expenses are cut down by Rs.3

$$\therefore \frac{360}{x} - \frac{360}{x+4} = 3$$

$$\Rightarrow \frac{360(x+4) - 360x}{x(x+4)} = 3$$

$$\Rightarrow \frac{1440}{x^2 + 4x} = 3$$

⇒ $x^2 + 4x = 480$

⇒ $x^2 + 4x = 480$

$\Rightarrow x^2 + 24x - 20x - 480 = 0$

$\Rightarrow x(x + 24) - 20(x + 24) = 0$

$\Rightarrow (x - 20)(x + 24) = 0$

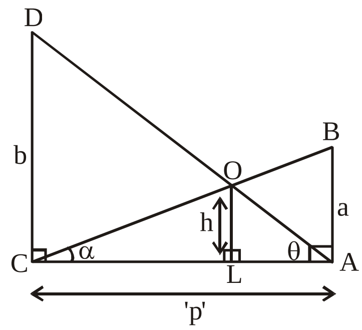
$\Rightarrow x - 20 = 0$ or, $x + 24 = 0 \Rightarrow x = 20$ or, $x = -24$

But, the number of days cannot be negative.

So, x = 20.

Hence, the original duration of the tour was of 20 days.

33.



In ΔOLA and ΔDCA

$\angle OAL = \angle DAC = \theta$

$\angle OLA = \angle DCA = 90°$

By AA criteria

$\angle OLA \sim \Delta DCA$

By similarly $\frac{OL}{DC} = \frac{LA}{AC}$

$\frac{h}{b} = \frac{LA}{p}$ ....(1)

Similarly $\Delta OLC \sim \Delta BAC$ (By similarly)

By similarity, $\frac{OL}{BA} = \frac{CL}{AC}$

$\Rightarrow \frac{h}{a} = \frac{CL}{p}$ ....(2)

By adding equation (1) and equation (2)

$$\frac{h}{b} + \frac{h}{a} = \left(\frac{LA}{p} + \frac{CL}{p}\right) = \frac{AC}{p} = \frac{p}{p}$$

$$h\left(\frac{1}{b} + \frac{1}{a}\right) = \frac{p}{p} = 1$$

$$h = \frac{ab}{a+b}$$

Hence proved

34. We have,

Volume of the water that flows out in one minute

= Volume of the cylinder of diameter 5 mm and length 10 metre

= Volume of the cylinder of radius $\frac{5}{2}$ mm $\left(= \frac{1}{4}\text{cm}\right)$ and length 1000 cm

$$= \frac{22}{7} \times \frac{1}{4} \times \frac{1}{4} \times 1000\,\text{cm}^3$$

Volume of a conical vessel of base radius 20 cm and depth 24 cm

$$= \frac{1}{3} \times \frac{22}{7} \times (20)^2 \times 24\,\text{cm}^3$$

Suppose the conical vessel is filled in x minutes.

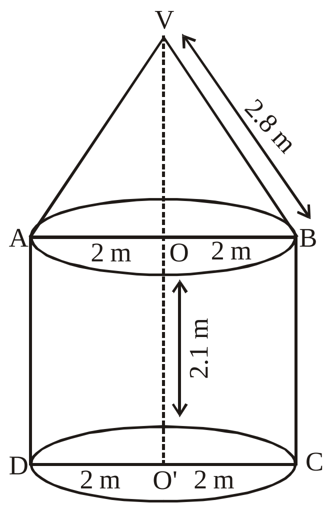
$\therefore$ Volume of the water that flows out in x minutes = Volume of the conical vessel

$$\Rightarrow \frac{22}{7} \times \frac{1}{4} \times \frac{1}{4} \times 1000 \times x = \frac{1}{3} \times \frac{22}{7} \times 20^2 \times 24$$

$$\Rightarrow x = \frac{1}{3} \times \frac{400 \times 24 \times 4 \times 4}{1000} = \frac{512}{10} \text{ minutes}$$

$\Rightarrow$ x = 51 minutes 12 seconds.

OR



For conical portion, we have

r = 2 m and ℓ = 2.8 m

$\therefore$ S_1 = curved surface area of conical portion

$= \pi r\ell$

$= \pi \times 2 \times 2.8\ m^2$

$= 5.6\ \pi m^2$

For cylindrical portion, we have

r = 2 m, h = 2.1 m

$\therefore$ S_2 = curved surface area of cylindrical portion

$= 2\pi rh = 2\pi \times 2 \times 2.1\ m = 8.4\ \pi m^2$

Area of canvas used for making the tent

$= S_1 + S_2$

$= (5.6\pi + 8.4\pi)m^2 = 14 \times \frac{22}{7}\ m^2 = 44\ m^2$

Total cost of the canvas at the rate of

Rs.500 per m^2 = Rs.(500 × 44) = Rs.22000

35.

Class intervals	**Frequency (f)**	**Cumulative frequency (cf)**
0-10	5	5
10-20	x	5 + x
20-30	20	25 + x
30-40	15	40 + x
40-50	y	40 + x + y
50-60	5	45 + x + y
	$\Sigma f_i = 60$	

We have,

Median = 28.5

Clearly, it lies in the class interval 20-30.

So, 20-30 is the median class.

$\ell = 20$, h = 10, f = 20, cf = 5 + x and N = 60

Now,

$$\text{Median} = \ell + \frac{\frac{N}{2} - cf}{f} \times h$$

$$\Rightarrow 28.5 = 20 + \frac{30-(5+x)}{20} \times 10$$

$$\Rightarrow 28.5 = 20 + \frac{25-x}{2}$$

$$\Rightarrow 8.5 = \frac{25-x}{2} \Rightarrow 25 - x = 17 \Rightarrow x = 8$$

We have,

N = 60

$45 + x + y = 60 \Rightarrow x + y = 15$

Putting x = 8 in x + y = 15, we get y = 7

Hence, x = 8 and y = 7.

SECTION-E

36. (i) For the 1st step, the volume of required concrete is $\frac{1}{4} \times \frac{1}{2} \times 50 = \frac{25}{4} m^3$

(ii) For the 2^{nd} step, the volume of required concrete is $\frac{2}{4} \times \frac{1}{2} \times 50 = \frac{50}{4} m^3$

(iii) For the 3^{rd} step, the volume of required concrete is $\frac{3}{4} \times \frac{1}{2} \times 50 = \frac{75}{4} m^3$

OR

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{15} = \frac{15}{2}\left[2 \times \frac{25}{4} + (15-1) \times \frac{25}{4}\right]$$

$$= \frac{15}{2}\left[\frac{50}{4} + \frac{350}{4}\right]$$

$$= \frac{15}{2} \times \frac{400}{4}$$

$= 750\ m^3$.

37. Given : point A lies in x = 3 and y = 4.

$\therefore$ A(3, 4) is the correct position.

Point B lies at x = 6 and y = 7.

So, the correct position of B is (6, 7).

(i) Position of A = (3, 4)

Position of B = (6, 7)

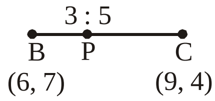
$\therefore$ Distance of AB

$= \left|\sqrt{(3-6)^2+(4-7)^2}\right|$

$= \left|\sqrt{(3)^2+(-3)^2}\right|$

$= \sqrt{18} = 3\sqrt{2}$ unit.

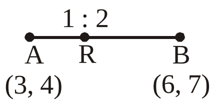
(ii) Position of B = (6, 7) and position of C = (9, 4)



$P = \left(\frac{57}{8}, \frac{47}{8}\right)$

OR

Position of B = (6, 7) and position of A = (3, 4)



R = (4, 5)

(iii) Position of B = (6, 7) and position of C = (9, 4)

$\therefore$ Mid-point of B and C $= \left(\frac{6+9}{2}, \frac{7+4}{2}\right)$

$= \left(\frac{15}{2}, \frac{11}{2}\right)$

38. (i) In ΔPBM, $\tan 45° = \frac{PM}{BM}$

$\Rightarrow 1 = \frac{300}{y}$

$\Rightarrow$ y = 300 meters.

(ii) In ΔAMP, $\tan 60° = \frac{PM}{AM}$

$\Rightarrow \sqrt{3} = \frac{300}{x}$

$\Rightarrow x = \frac{300}{x} = 100\sqrt{3}$

= 100 × 1.732

= 173.2 meters.

(iii) We have proved that

x = 173.2 m and y = 300 m

$\therefore$ AB = x + y

= (173.2 + 300)m

= 473.2 m

(iv) In ΔPMB,

PM = 300, BM = y = 300 m

and $\angle PMB = 90°$

$\therefore (PB)^2 = (PM)^2 + (BM)^2$

[Using Pythogoras theorem]

$= (300)^2 + (300)^2$

= 90000 + 90000

= 180000

$\therefore PB = \sqrt{18 \times 10000}$

$= 100 \times 3\sqrt{2}$

$= 300\sqrt{2}$ m.