

PAPER # 02 - MATHEMATICS

1. (d) The mid-point of line segment joining (0, 0) and (-4, -2) is $\left(\frac{0-4}{2}, \frac{0-2}{2}\right)$ i.e. (-2, -1).
2. (c) $\tan 45^\circ \cos 60^\circ + \sin 60^\circ \cot 60^\circ$
 $1 \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} = \frac{1}{2} + \frac{1}{2} = 1$
3. (a) Given equation, $2x^2 - \sqrt{5}x + 1 = 0$
 On comparing it with $ax^2 + bx + c = 0$, we get
 $a = 2, b = \sqrt{5}$ and $c = 1$
 $\therefore D = (\sqrt{5})^2 - 4(2)(1)$ [$\because D = b^2 - 4ac$]
 $= 5 - 8 = -3$
4. (b) We have, $\sqrt{3} \sin \theta = \cos \theta$
 $\Rightarrow \frac{\sin \theta}{\cos \theta} = \frac{1}{\sqrt{3}} \Rightarrow \tan \theta = \frac{1}{\sqrt{3}}$
 $\Rightarrow \tan \theta = \tan 30^\circ \quad \theta = 30^\circ$
5. (d) Given, $AB \parallel EW$
 $\therefore \frac{DA}{AE} = \frac{DB}{BW}$ [by Thales theorem]
 $\Rightarrow \frac{DA}{DE - DA} = \frac{DB}{DW - DB}$
 $\Rightarrow \frac{4}{12 - 4} = \frac{DB}{24 - DB}$
 $\Rightarrow \frac{4}{8} = \frac{DB}{24 - DB}$
 $\Rightarrow 24 - DB = 2DB$
 $\Rightarrow 24 = 3DB$
 $\Rightarrow DB = \frac{24}{3} = 8 \text{ cm}$
6. (b) Let 4 be the event 'getting an even number.'
 Clearly, event A occurs, if we obtain anyone of 2, 4, 6 as an outcome.
 \therefore Number of outcomes favourable to A = 3
 Hence, $P(A) = \frac{3}{6} = \frac{1}{2}$
7. (a) Length of the arc $= \frac{\theta}{360} \times 2\pi r$
 $\Rightarrow 4.4 = \frac{30^\circ}{360} \times 2 \times \frac{22}{7} \times r$
 $\Rightarrow 4.4 = \frac{1}{12} \times \frac{44}{7} \times r$
 $\Rightarrow r = \frac{4.4 \times 12 \times 7}{44} = 8.4 \text{ cm}$
8. (c) We know that
 Product of zeroes $= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$
 $\therefore \alpha\beta = \frac{7}{4}$
9. (c) Total number of cards = 52
 Kings which are red in colour = 2
 $P(\text{king of red colour}) = \frac{2}{52} = \frac{1}{26}$
10. (a) If point P lies inside the circle then no tangent can be drawn.
11. (b) Let α and β be the zeros of the polynomial $f(x) = ax^2 + bx + c$. Then,
 $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$
 Let S and P denote respectively the sum and product of the zeros of a polynomial whose zeros are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$. Then,
 $S = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{-\frac{b}{a}}{\frac{c}{a}} = -\frac{b}{c}$ and
 $P = \frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{\frac{c}{a}} = \frac{a}{c}$
 Hence, the required polynomial $g(x)$ is given by
 $g(x) = k\left(x^2 - Sx + P\right) = k\left(x^2 + \frac{bx}{c} + \frac{a}{c}\right)$
 where k is any non-zero constant.

12. (a) Class mark, frequency of the class

$$x = \frac{\sum fx}{\sum f} = \frac{\sum (A \times B)}{\sum A}$$

where B is the class mark.

$$\text{Class mark} = \frac{1}{2} (\text{upper limit} + \text{lower limit})$$

and A is the frequency of the class.

13. (a) Let d be the common difference of the AR According to the question,

$$a_{17} - a_{10} = 7$$

$$\Rightarrow (a + 16d) - (a + 9d) = 7 \Rightarrow 7d = 7 \Rightarrow d = 1$$

14. (b) The mode is the most frequent observation. Here, the mode is 14 with a frequency of 15.

15. (d) $(x + 2)(3x - 5) = 0$

$$\Rightarrow x + 2 = 0 \text{ or } 3x - 5 = 0$$

$$\Rightarrow x = -2 \text{ or } x = \frac{5}{3}$$

Hence, the roots of the given equation are -2 and $\frac{5}{3}$.

16. (d) Given, equation $2x^2 - 6x + 7 = 0$

On comparing it with $ax^2 + bx + c = 0$, we get

$$a = 2, b = -6 \text{ and } c = 7$$

$$\therefore D = b^2 - 4ac = (-6)^2 - 4(2)(7)$$

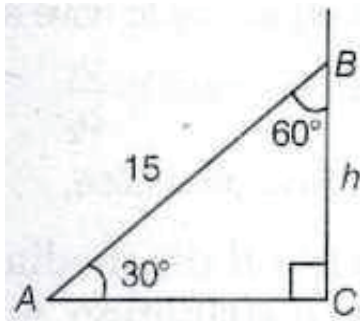
$$= 36 - 56 = -20 < 0$$

So, the roots are imaginary.

17. (c) Given, $\angle ABC = 60^\circ$

In $\triangle ABC$, $\angle BAC + \angle ABC + \angle ACB = 180^\circ$

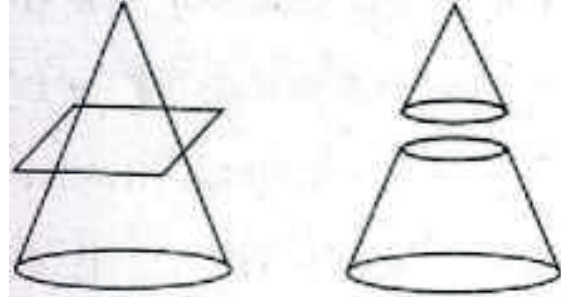
$$\Rightarrow \angle BAC = 180^\circ - 90^\circ - 60^\circ = 30^\circ$$



$$\text{So, } \sin 30^\circ = \frac{BC}{AB} = \frac{h}{15}$$

$$\Rightarrow \frac{1}{2} = \frac{h}{15} \Rightarrow h = \frac{15}{2} \text{ m}$$

18. (c) Circle



19. (c). Assertion Given $x + y - 8 = 0$ and $x - y - 2 = 0$

$$\text{Here, } a_1 = 1, b_1 = 1, c_1 = 8$$

$$\text{and } a_2 = 1, b_2 = -1, c_2 = -2$$

$$\text{So, } \frac{a_1}{a_2} = \frac{1}{1}, \frac{b_1}{b_2} = \frac{1}{-1} \text{ and } \frac{c_1}{c_2} = \frac{-8}{-2} = 4$$

$$\therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

So, the system of equations has a unique solution and the Assertion is true.

Reason For equations to have a unique solution,

$$\frac{a_1}{a_2} \text{ should not be equal to } \frac{b_1}{b_2}.$$

\therefore The given Reason is false.

20. (a) Reason is clearly true.

Using the relation given in reason, we have

$$2 \text{ Mean} = 3 \text{ Median} - \text{Mode}$$

$$= 3 \times 150 - 154$$

$$= 296$$

$$\therefore \text{Mean} = \frac{296}{2} = 148, \text{ which is true.}$$

Thus, both Assertion and Reason are true and Reason is the correct explanation of Assertion.

21. Given, $x = a \cos \theta$ and $y = b \sin \theta$

$$\begin{aligned} \therefore b^2 x^2 + a^2 y^2 &= b^2 (a \cos \theta)^2 + a^2 (b \sin \theta)^2 \quad (1) \\ &= a^2 b^2 \cos^2 \theta + a^2 b^2 \sin^2 \theta \\ &= a^2 b^2 (\cos^2 \theta + \sin^2 \theta) \\ &= a^2 b^2 (1) \quad [\because \cos^2 A + \sin^2 A = 1] \\ &= a^2 b^2 (1) \end{aligned}$$

22. Let us assume that $\frac{2}{5\sqrt{3}}$ is a rational number.

$\therefore \frac{2}{5\sqrt{3}} = \frac{p}{q}$, where p, q ($q \neq 0$) are integers and p, q are coprimes. (1)

$$\Rightarrow \frac{2q}{5p} = \sqrt{3}$$

Since, 2, 5, p and q are integers.

$\therefore \frac{2q}{5p}$ is rational, so $\sqrt{3}$ is rational.

But this contradicts the fact that $\sqrt{3}$ is irrational.

Hence, $\frac{2}{5\sqrt{3}}$ is an irrational number.

Hence proved(1)

OR

Let us assume that $6 - 2\sqrt{3}$ is rational number.

Then, it will be of the form $\frac{a}{b}$, where a, b are coprime integers and $b \neq 0$.

$$\text{Now, } 6 - 2\sqrt{3} = \frac{a}{b}$$

On rearranging, we get

$$6 - \frac{a}{b} = 2\sqrt{3} \quad (1)$$

Since, 6 and $\frac{a}{b}$ are rational. So, their difference will be rational.

$\therefore 2\sqrt{3}$ is rational.

But we know that, $\sqrt{3}$ is irrational.

So, this contradicts the fact that $\sqrt{3}$ is irrational.

Therefore, our assumption is wrong.

Hence, $6 - 2\sqrt{3}$ is irrational.

Hence proved(1)

23. We have, $p(x) = 5x^2 - 7x + 1$, whose zeroes are α and β .

$$\therefore \text{Sum of zeroes, } \alpha + \beta = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$= -\frac{(-7)}{5} = \frac{7}{5} \dots(i) \quad (1)$$

and product of zeroes, $\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$$= \frac{1}{5} \dots(ii)$$

$$\text{Now, } \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{7/5}{1/5}$$

[from Eqs. (i) and (ii)]

$$= 7 \quad (1)$$

24. We have, $\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}$

$$\Rightarrow \frac{(x+2) + 2(x+1)}{(x+1)(x+2)} = \frac{4}{x+4} \quad (1)$$

$$\Rightarrow (x+4)(3x+4) = 4(x^2 + 3x + 2)$$

$$\Rightarrow x^2 - 4x - 8 = 0$$

On comparing it with $ax^2 + bx + c = 0$, we get

$$a = 1, b = -4 \text{ and } c = -8$$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-(-4) \pm \sqrt{16 - 4(1)(-8)}}{2} = \frac{4 \pm \sqrt{48}}{2}$$

$$\Rightarrow x = 2 \pm 2\sqrt{3} \quad (1)$$

25. $\therefore \triangle AGF \sim \triangle DBG \dots(i)$

[by AA similarity criterion]

Now, in $\triangle AGF$ and $\triangle EFC$, we get

$$\angle FAG = \angle CEF \text{ [each } 90^\circ]$$

and $\angle AFG = \angle ECF$ [corresponding angles because $GF \parallel BC$ and AC is the transversal]

$$\therefore \triangle AGF \sim \triangle EFC \dots(ii) \quad (1)$$

From Eqs. (i) and (ii), we get

$$\triangle DBG \sim \triangle EFC$$

$$\Rightarrow \frac{BD}{EF} = \frac{DG}{EC}$$

$$\Rightarrow \frac{BD}{DE} = \frac{DE}{EC} \text{ [}\because \text{ DEFG is a square]}$$

$$\therefore DE^2 = BD \times EC \text{ Hence proved. (1)}$$

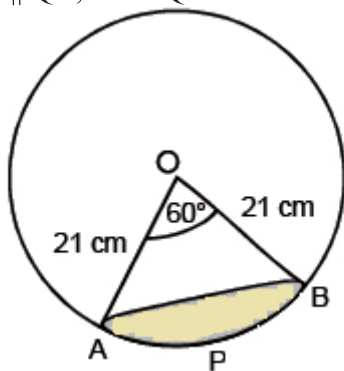
OR

Given,

In $\triangle PQO$, $DE \parallel OQ$

So by using Basic Proportionality Theorem,
 $PD/DO = PE/EQ$ **(i)**
 Again given, in ΔPOR , $DF \parallel OR$,
 So by using Basic Proportionality Theorem,
 $PD/DO = PF/FR$ **(ii)**
 From equation **(i)** and **(ii)**, we get,
 $PE/EQ = PF/FR$
 Therefore, by converse of Basic Proportionality Theorem,
 $EF \parallel QR$, in ΔPQR .

26.



Given,
 Radius = 21 cm
 $\theta = 60^\circ$

(i) Length of an arc = $\theta/360^\circ \times \text{Circumference}$ ($2\pi r$)

$$\begin{aligned} \therefore \text{Length of an arc AB} &= (60^\circ/360^\circ) \times 2 \times (22/7) \times 21 \\ &= (1/6) \times 2 \times (22/7) \times 21 \end{aligned}$$

Or Arc AB Length = 22cm

(ii) It is given that the angle subtended by the arc = 60°

So, the area of the sector making an angle of 60°

$$\begin{aligned} &= (60^\circ/360^\circ) \times \pi r^2 \text{ cm}^2 \\ &= 441/6 \times 22/7 \text{ cm}^2 \end{aligned}$$

Or, the area of the sector formed by the arc APB is 231 cm^2

(iii) Area of segment APB = Area of sector OAPB – Area of ΔOAB

Since the two arms of the triangle are the radii of the circle and thus are equal, and one angle is 60° , ΔOAB is an equilateral triangle. So, its area will be $\sqrt{3}/4 \times a^2$ sq.

Units.

The area of segment APB = $231 - (\sqrt{3}/4) \times (OA)^2$

$$= 231 - (\sqrt{3}/4) \times 21^2$$

Or, the area of segment

$$APB = [231 - (441 \times \sqrt{3})/4] \text{ cm}^2$$

27. Given A circle inscribed in a ΔPQR such that

$$PQ = PR$$

To prove $QT = TR$

Proof We know that the tangents from an external points to a circle are equal in length.

$$PS = PU \text{ [tangents from P] ... (i)}$$

$$QS = QT \text{ [tangents from O] ... (ii)}$$

$$RT = RU \text{ [tangents from R] ... (iii)}$$

$$\text{Now, } PQ = PR \text{ [given] } (1 \frac{1}{2})$$

$$\Rightarrow PQ - PS = PR - PS$$

[subtracting PS from both sides]

$$\Rightarrow PQ - PS = PR - PU \text{ [from Eq. (i)]}$$

$$\Rightarrow QS = RU$$

$$\Rightarrow QT = RU \text{ [from Eq. (ii)]}$$

$$\Rightarrow QT = RT \text{ [from Eq. (iii)]}$$

Hence proved. (1 $\frac{1}{2}$)

28. There are 6 possible outcomes (1, 2, 3, 4, 5 and 6) in a single throw of a die.

(i) We know that even prime number is only 2.

So, number of favourable outcomes = 1

$$\therefore P(\text{getting an even prime number}) = \frac{1}{6} \text{ (1 } \frac{1}{2}\text{)}$$

(ii) The numbers divisible by 2 are 2, 4 and 6. So, number of favourable outcomes = 3

$\therefore P(\text{getting a number divisible by 2})$

$$= \frac{3}{6} = \frac{1}{2}$$

OR

Number of red cards = 26

Number of queens = 4

But, out of these 4 queens, 2 are red.

\therefore Number of queens which are not red = 2

Now, number of cards which are red or queen

$$= 26 + 2 = 28 \text{ (1)}$$

$\therefore P(\text{getting either red card or queen})$

$$= \frac{\text{Number of card which are red or queen}}{\text{Total number of cards}}$$

$$= \frac{28}{52} = \frac{7}{13} \text{ (1)}$$

Now, P (not getting either red card or queen)
 = 1 - P (getting either red card or queen)
 $= 1 - \frac{7}{13} = \frac{13-7}{13} = \frac{6}{13}$

29. Here, class intervals are not in inclusive form.

So, we first convert them in inclusive form by subtracting $h/2$ from the lower limit and adding $h/2$ to the upper limit of each class, where h is the difference between the lower limit of a class and the upper limit of the preceding class.

The given frequency distribution in inclusive form is as follows.

Age(in yr)	Number of cases
4.5-14.5	6
14.5-24.5	11
24.5-34.5	21
34.5-44.5	23
44.5-54.5	14
54.5-64.5	5

(1)

We observe that the class 34.5-44.5 has the maximum frequency.

So, it is the modal class such that $l = 34.5$, $h = 10$, $f_1 = 23$, $f_0 = 21$ and $f_2 = 14$

$$\therefore \text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h,$$

$$\Rightarrow \text{Mode} = 34.5 + \frac{23-21}{46-21-14} \times 10$$

$$= 34.5 + \frac{2}{11} \times 10 = 36.31 \quad (1)$$

30. The given equations are

$$10x + 3y = 75 \dots(i)$$

$$6x - 5y = 11 \dots (ii)$$

Multiplying Eq. (i) by 5 and Eq. (ii) by 3, we get

$$50x + 15y = 375 \dots(iii)$$

$$18x - 15y = 33 \dots(iv) (1)$$

Adding Eqs. (iii) and (iv), we get

$$68x = 408$$

$$\Rightarrow x = \frac{408}{68} \Rightarrow x = 6 (1)$$

Putting $x = 6$ in Eq.(i), we get

$$(10 \times 6) + 3y = 75$$

$$\Rightarrow 60 + 3y = 75$$

$$\Rightarrow 3y = 75 - 60$$

$$\Rightarrow 3y = 15$$

$$\Rightarrow y = 5$$

$$\therefore x = 6 \text{ and } y = 5 (1)$$

OR

The given equations are

$$11x + 15y + 23 = 0 \dots (i)$$

$$7x - 2y - 20 = 0 \dots(ii)$$

Multiplying Eq. (i) by 2 and Eq. (ii) by 15 and adding the results, we get

$$22x + 105x = -46 + 300$$

$$\Rightarrow 127x = 254$$

$$\Rightarrow x = \frac{254}{127} = 2 (1)$$

Putting $x = 2$ in Eq. (i), we get

$$22 + 15y = -23$$

$$\Rightarrow 15y = -23 - 22$$

$$\Rightarrow 15y = -45$$

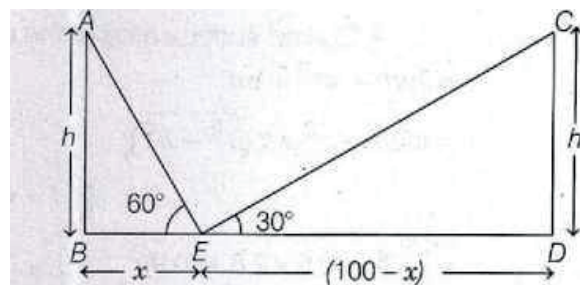
$$\Rightarrow y = \frac{-45}{15} \Rightarrow y = -3$$

Hence, $x = 2$ and $y = -3 (2)$

31. Let AB and CD be two pillars of equal height h and distance between them be $BD = 100$ m.

Let E be a point on the road such that $BE = x$,

$$DE = (100 - x), \angle AEB = 60^\circ \text{ and } \angle CED = 30^\circ.$$



In right angled $\triangle ABE$,

$$\frac{AB}{BE} = \tan 60^\circ$$

$$\Rightarrow \frac{h}{x} = \sqrt{3} \quad [\because \tan 60^\circ = \sqrt{3}]$$

$$\Rightarrow h = \sqrt{3} x \dots (i) (1)$$

In right angled $\triangle CDE$,

$$\frac{CD}{DE} = \tan 30^\circ$$

$$\Rightarrow \frac{h}{100-x} = \frac{1}{\sqrt{3}} \dots(ii) (1)$$

From Eqs. (i) and (ii); we get

$$\sqrt{3}x = \frac{100-x}{\sqrt{3}}$$

$$\Rightarrow 3x = 100 - x \quad \Rightarrow 4x = 100$$

$$\therefore x = 25$$

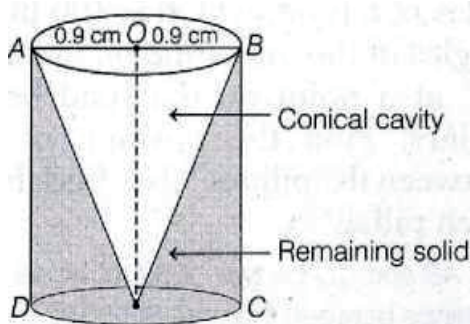
On putting $x = 25$ in Eq. (i), we get

$$h = \sqrt{3} \times 25$$

$$= 25 \times 1.732 = 43.3 \text{ m}$$

Hence, height of each pillar is 43.3 m and position of the point from pillar making an angle of 60° is 25 m. (1)

32. Let's be the total surface area of the remaining solid.



Then, S = Curved surface area of the cylinder + Area of the base of the cylinder + Curved surface area of the cone

$$= 2\pi rh + \pi r^2 + \pi rl \quad (1)$$

$$= \pi[2rh + r^2 + r\sqrt{r^2 + h^2}]$$

$$[\because l = \sqrt{r^2 + h^2}]$$

$$= \frac{22}{7} [5.04 + 0.81 + 0.9\sqrt{0.81 + 7.84}]$$

$$= \frac{22}{7} [5.85 + 0.9\sqrt{8.65}]$$

$$= \frac{22}{7} [5.85 + 0.9 \times 2.94]$$

$$= \frac{22}{7} \times [5.85 + 2.64] = \frac{186.78}{7}$$

$$= 26.68 \text{ cm}^2 \quad (2)$$

OR

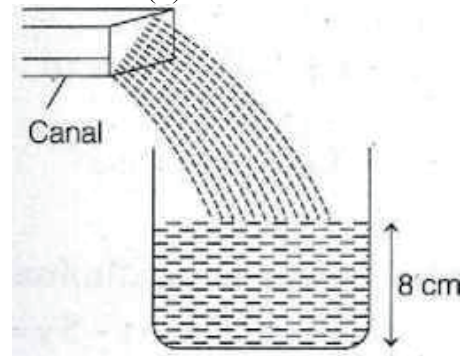
Given, speed of flow of water = 10 km/h
 $= 10 \times 1000 \text{ m/h}$ [$\because 1 \text{ km} = 1000 \text{ m}$]

\Rightarrow Length of water flow in 1 h = $10 \times 1000 \text{ m}$

\Rightarrow Length of water flow in 30 min (i.e. in $\frac{1}{2}$ h)

$$= \frac{1}{2} \times 10 \times 1000$$

$$= 5000 \text{ m} (1)$$



(1)

Now, volume of water flowing in 30 min
 $=$ Volume of cuboid of length 5000 m, width 6 m and depth 1.5 m

$$= 5000 \times 6 \times 1.5 \text{ m}^3 = 45000 \text{ m}^3 (1)$$

Hence, the required area covered for irrigation with 8 cm or m of standing water

$$= \frac{4500}{8} \times 100 = 562500 \text{ m}^2$$

$$= \frac{562500}{1000} \text{ hec} [\because 1 \text{ hec} = 10000 \text{ m}^2]$$

$$= 56.25 \text{ hec} (2)$$

33. Given, equations are $5x - y = 5 \dots(i)$

and $3x - y = 3 \dots(ii)$

Table for $5x - y = 5$ or $y = 5x - 5$ is

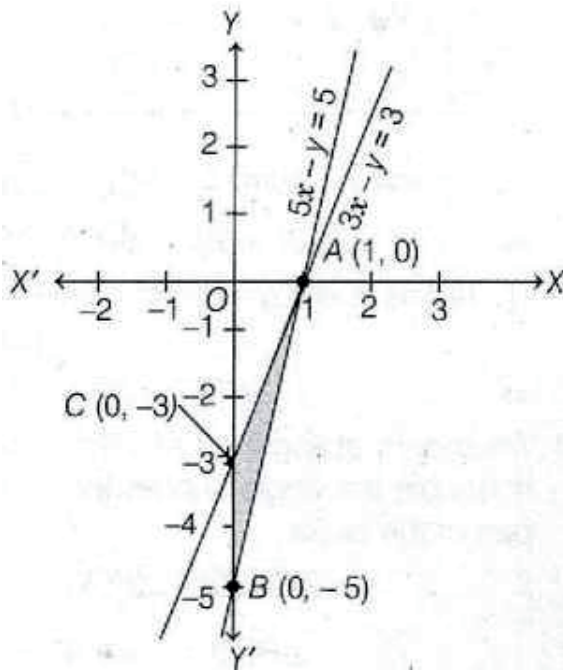
X	1	0
Y	0	-5
Points	A (1,0)	B (0, -5)

Plot the points A(1, 0) and B(0, -5) on a graph paper and join these points to form line AB. (1)

Table for $3x - y = 3$ or $y = 3x - 3$ is

X	1	0
Y	0	-3
Points	A (1, 0)	C (0, -3)

Plot the points A (1, 0) and C (0, -3) on the same graph paper and join these points to form line AC. (1)

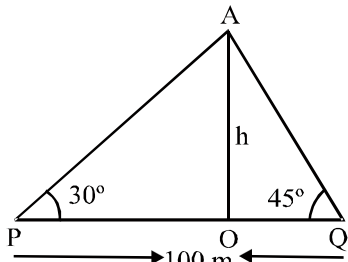


(2)

Hence, the triangle formed by given lines is ΔABC whose vertices are $A(1, 0)$, $B(0, -5)$ and $C(0, -3)$.

(1)

34. Let OA be the tree of height h metre. In triangles POA and QOA , we have $\tan 30^\circ = \frac{OA}{OP}$ and $\tan 45^\circ = \frac{OA}{OQ}$



$$\begin{aligned} \Rightarrow \frac{1}{\sqrt{3}} &= \frac{h}{OP} \text{ and } 1 = \frac{h}{OQ} \\ \Rightarrow OP &= \sqrt{3} h \text{ and } OQ = h \\ \Rightarrow OP + OQ &= \sqrt{3} h + h \\ \Rightarrow PQ &= (\sqrt{3} + 1)h \\ \Rightarrow 100 &= (\sqrt{3} + 1)h [\because PQ = 100 \text{ m}] \\ \Rightarrow h &= \frac{100}{\sqrt{3} + 1} \text{ m} \\ \Rightarrow h &= \frac{100(\sqrt{3} - 1)}{2} \text{ m} \\ \Rightarrow h &= 50(1.732 - 1) \text{ m} = 36.6 \text{ m} \end{aligned}$$

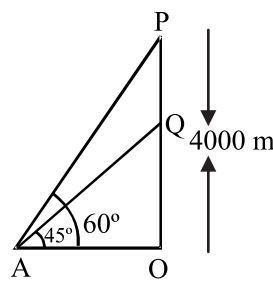
Hence, the height of the tree is 36.6 m

OR

Let P and Q be the positions of two aeroplanes when Q is vertically below P and $OP = 4000$ m. Let the angles of elevation of P and Q at a point A on the ground be 60° and 45° respectively.

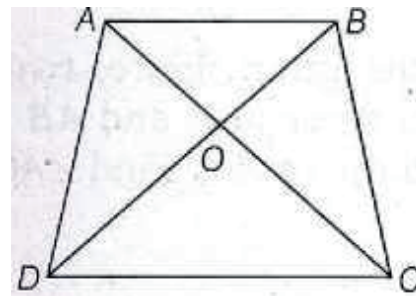
In triangles AOP and AOQ , we have

$$\begin{aligned} \tan 60^\circ &= \frac{OP}{OA} \text{ and } \tan 45^\circ = \frac{OQ}{OA} \\ \Rightarrow \sqrt{3} &= \frac{4000}{OA} \text{ and } 1 = \frac{OQ}{OA} \\ \Rightarrow OA &= \frac{4000}{\sqrt{3}} \text{ and } OQ = OA \\ \Rightarrow OQ &= \frac{4000}{\sqrt{3}} \text{ m} \end{aligned}$$



$$\begin{aligned} \therefore \text{Vertical distance between the aeroplanes} &= PQ = OP - OQ \\ &= \left(4000 - \frac{4000}{\sqrt{3}} \right) \text{ m} = 4000 \frac{(\sqrt{3} - 1)}{\sqrt{3}} \text{ m} \\ &= 1690.53 \text{ m} \end{aligned}$$

35. Given $ABCD$ is a trapezium in which $AB \parallel DC$.



To prove $\frac{OA}{OC} = \frac{OB}{OD}$ (2)

Proof In ΔOAB and ΔODC , we have $AB \parallel DC$

Then, $\angle OAB = \angle OCD$ [alternate interior angles]

$\angle AOB = \angle DOC$ [vertically opposite angles]

and $\angle ABO = \angle CDO$ [alternate interior angles]

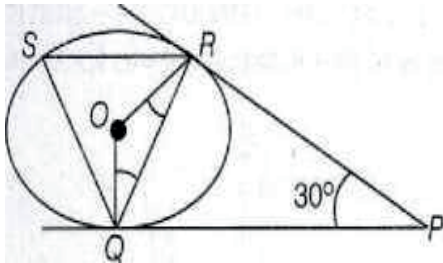
$\therefore \Delta OAB \sim \Delta OCD$ [by AAA similarity criterion]

Hence, $\frac{OA}{OC} = \frac{OB}{OD}$

[if two triangles are similar, then their corresponding sides are proportional]

Hence proved (1)

36. (i) In quadrilateral POOR, we have



$\angle QPR + \angle PRO + \angle PQO + \angle ROQ = 360^\circ$
 $\Rightarrow 30^\circ + 90^\circ + 90^\circ + \angle ROQ = 360^\circ$

[\because radius is always perpendicular to the tangent at point of contact]

$\Rightarrow \angle ROQ = 360^\circ - 210^\circ = 150^\circ$

(ii) We know that angle subtended by an arc at centre is double the angle subtended by it at any other part of the circle.

$2 \angle RSQ = \angle ROQ$

$\angle RSQ = \frac{1}{2} \times 150^\circ = 75^\circ$

(iii) In ΔQOR , $OQ = OR$ [radii]

$\angle ORQ = \angle OQR$

Now, $\angle ROQ + \angle ORQ + \angle OQR = 180^\circ$

$\Rightarrow 2 \angle OQR = 180^\circ - 150^\circ$

$\Rightarrow 2 \angle OQR = 30^\circ$

$\Rightarrow \angle OQR = 15^\circ$

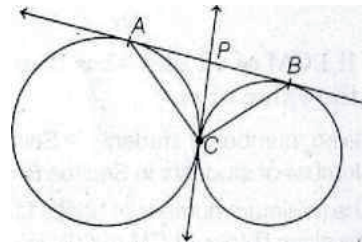
Again, $\angle OQP = 90^\circ$ [$\because OQ \perp QP$]

$\Rightarrow \angle OQR + \angle RQP = 90^\circ$

$\Rightarrow \angle RQP = 90^\circ - 15^\circ = 75^\circ$

OR

Draw a tangent to the circles at point C. Let it meets AB at P



Then, $PA = PC$ and $PB = PC$

[the tangents from an external points to a circle are equal in length]

$PA = PC \Rightarrow \angle PAC = \angle PCA$

$PB = PC \Rightarrow \angle PBC = \angle PCB$

$\therefore \angle PAC + \angle PBC = \angle PCA + \angle PCB = \angle ACB$

$\Rightarrow \angle PAC + \angle PBC + \angle ACB = 2 \angle ACB$

$\Rightarrow 180^\circ = 2 \angle ACB$

$\Rightarrow \angle ACB = 90^\circ$

37. (i) For first metre, the charge is Rs. 100

i.e. first term, $a = 100$

As, there is increasing of Rs. 25 for each subsequent metres, therefore common difference, $d = 25$

So, the AP thus formed is

100, 125, 150, ...

(ii) Labour charge to dig the well is the 15th term of AP.

We know, $a_n = a + (n - 1)d$

$\therefore a_{15} = 100 + (15 - 1)25$

$= 100 + 14 \times 25 = 450$

\therefore Labour charge = Rs. 450

(iii) Money saved by Ram = Rs. 450 - Rs. 400 = Rs. 50

OR

We know that $S_n = \frac{n}{2} [2a + (n - 1)d]$

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$$\text{Sum of 15 terms, } S_{15} = \frac{15}{2} [2 \times 100 + 14 \times 25]$$

$$= \frac{15}{2} [200 + 350] = \frac{15}{2} \times 550 = 4125$$

38. (i) Given, number of students in Section A = 32

Number of students in Section B = 36

The minimum number of books to be acquired for the class library = LCM of (32, 36)

$$= 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$$

$$= 2^5 \times 3^2$$

$$= 32 \times 9 = 288$$

- (ii) The prime factors of 36 are

$$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$$

- (iii) HCF (867, 255) = 51

OR

Given, LCM (12, 42) = 10m + 4

Factors of 12 = 2 × 2 × 3

and factors of 42 = 2 × 3 × 7

Now, LCM (12, 42) = 2 × 2 × 3 × 7 = 84

$$\therefore 84 = 10m + 4$$

$$\Rightarrow 84 - 4 = 10m$$

$$\therefore m = \frac{80}{10} = 8$$