

## MATHEMATICS - SOLUTION

Ans :- 1 Fill in the blank and write :-

- a. (iii) Neither symmetric, nor transitive, nor reflexive
- b. (ii) Associative law
- c. (i)  $k = \frac{3}{4}$
- d. (iii) One
- e. (ii)  $\log(\log x)$
- f. (iv) One

Ans :- 2 Fill in the blanks

1.  $\begin{bmatrix} -4 & 2 \\ 3 & -1 \end{bmatrix}$
2.  $-\frac{2}{x^3}$
3. 1
4.  $\sin^{-1}\left(\frac{x}{2}\right)$
5.  $x \sin x.$
6.  $\frac{3}{7}, \frac{-2}{7}, \frac{6}{7}$

Ans :- 3 True / False

- (i) True
- (ii) True
- (iii) True
- (iv) False
- (v) False
- (vi) False

Ans :- 4 Match the column :-

- 1- (iv), 2-(iii), 3-(v), 4-(vi), 5-(i), 6-(ii), 7-(vii)

Ans :- 5 One word

1. 1
2.  $\pi$
3. Skew symmetric
4. 1
5.  $-\frac{(1+\log x)}{(x \log x)^2}.$
6.  $9e^{6x}(3\cos 3x - 4\sin 3x)$
7. -5

Ans - 6 Here  $\int \frac{1-\sin x}{\cos^2 x} dx = \int \frac{1}{\cos^2 x} dx - \int \frac{\sin x}{\cos^2 x} dx$   
 $= \int \sec^2 x dx - \int \tan x \sec x dx$   
 $= \tan x - \sec x + c.$

**Or**

$$\begin{aligned}\int xe^x dx &= x \int e^x dx - x \int \left\{ \frac{d}{dx}(x) \cdot \int e^x dx \right\} dx \\ &= xe^x - \int 1 \cdot e^x dx \\ &= xe^x - e^x = e^x(x - 1) + c.\end{aligned}$$

Ans - 7 Unit vector in the direction of vector  $\vec{a}$  is given by  $\hat{a} = \frac{1}{|\vec{a}|} \vec{a}$

Now,  $|\vec{a}| = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{14}$

$$|\hat{a}| = \frac{1}{\sqrt{14}} (2\hat{i} + 3\hat{j} + \hat{k}) = \frac{2}{\sqrt{14}} \hat{i} + \frac{3}{\sqrt{14}} \hat{j} + \frac{1}{\sqrt{14}} \hat{k}.$$

**Or**

Let  $\vec{a} = x(\hat{i} + \hat{j} + \hat{k})$

$\therefore \vec{a} = x\hat{i} + x\hat{j} + x\hat{k}$

Now  $|\vec{a}| = \sqrt{x^2 + x^2 + x^2}$

$\Rightarrow 1 = \sqrt{3x^2}$  (Q  $\vec{a}$  is a unit vector)

$\therefore x = \pm \frac{1}{\sqrt{3}}$ .

Ans - 8 Here,  $\vec{OP} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

and  $\vec{OQ} = 4\hat{i} + \hat{j} - 2\hat{k}$

$\therefore$  Position vector of the mid-point of  $PQ = \frac{\vec{OP} + \vec{OQ}}{2}$

$$\begin{aligned}&= \frac{(2\hat{i} + 3\hat{j} + 4\hat{k}) + (4\hat{i} + \hat{j} - 2\hat{k})}{2} \\ &= \frac{6\hat{i} + 4\hat{j} + 2\hat{k}}{2} = 3\hat{i} + 2\hat{j} + \hat{k}\end{aligned}$$

**Or**

Projection of the vector  $\vec{a}$  on the vector  $\vec{b}$

$$\frac{1}{|b|} \left( \frac{r \cdot r}{a \cdot b} \right) = \frac{1}{\sqrt{(1)^2 + (2)^2 + (1)^2}} (2.1 + 3.2 + 2.1)$$

$$= \frac{10}{\sqrt{6}} = \frac{5}{3}\sqrt{6}.$$

Ans - 9     $A = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$  and  $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$

Now                           $A + B = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} -3 & 3 \\ 2 & 4 \end{bmatrix}$$

$$(A + B)' = \begin{bmatrix} 3 & 3 \\ 3 & 4 \end{bmatrix}$$

**Or**

$$AB = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Ans - 10    Let                           $y = x^x$   
 $\Rightarrow \log y = x \log x$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = x \cdot \frac{1}{x} + \log x \cdot 1$$

$$\Rightarrow \frac{dy}{dx} = y(1 + \log x)$$

$$\Rightarrow \frac{dy}{dx} = x^x (1 + \log x)$$

**Or**

Here,                           $f(1) = 2(1) + 3 = 5$

Now,                           $\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} (2x + 3)$

$$\Rightarrow \lim_{x \rightarrow 1} f(x) = 2(1) + 3 = 5$$

Since                           $\lim_{x \rightarrow 1} f(x) = f(1) = 5$

Here, function is continuous at  $x = 1$

Ans - 11 The smallest equivalence relation  $R_1$ , containing (1, 2) and (2, 1) is  $\{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$ . Now we are left with only 4 pairs namely (2, 3), (3, 2), (1, 3) and (3, 1). If we add any one, say (2, 3) to  $R_1$ , then for symmetry we must add (3, 2) also and now for transitivity we are forced to add (1, 3) and (3, 1). Thus, the only equivalence relation bigger than  $R_1$  is the universal relation. This shows that the total number of equivalence relations containing (1, 2) and (2, 1) is two.

**Or**

$f$  is not one-one, as  $f(1) = f(2) = 1$ . But  $f$  is onto, as given any  $y \in N$ ,  $y \neq 1$ , we can choose  $x$  as  $y + 1$  such that  $f(y + 1) = y + 1 - 1 = y$ . Also for  $1 \in N$ , we have  $f(1) = 1$ .

Ans - 12 Let  $x = \sec \theta$

$$\text{then, } \sqrt{x^2 - 1} = \sqrt{\sec^2 \theta - 1} = \tan \theta$$

$$\text{Therefore } \cot^{-1} \frac{1}{\sqrt{x^2 - 1}} = \cot^{-1} (\cot \theta) = \theta = \sec^{-1} x$$

which is the simplest form.

**Or**

$$\begin{aligned} \cos(\tan^{-1} x) &= \sin\left(\cot^{-1} \frac{3}{4}\right) \\ \Rightarrow \quad \cos(\tan^{-1} x) &= \sin\left(\tan^{-1} \frac{4}{3}\right) \\ &= \sin\left(\sin^{-1} \frac{4}{\sqrt{3^2 + 4^2}}\right) \\ \Rightarrow \quad \cos(\tan^{-1} x) &= \sin\left(\sin^{-1} \frac{4}{5}\right) = \frac{4}{5} \\ &= \cos(-\tan^{-1} x) = \frac{4}{5} \end{aligned}$$

$$\Rightarrow \quad \tan^{-1} x = -\tan^{-1} x = \cos^{-1} \left(\frac{4}{5}\right)$$

$$\Rightarrow \quad \tan^{-1} x = -\tan^{-1} x = \tan^{-1} \frac{3}{4}$$

$$\Rightarrow \quad x = \frac{3}{4}, -\frac{3}{4}.$$

Ans - 13  $x - y = \pi$

On differentiating w.r.t 'x'

$$1 - \frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = 1$$

**Or**

Let  $y = (e^{\sqrt{x}})^{1/2}$

On differentiating w.r.t 'x',

$$\frac{dy}{dx} = \frac{1}{2}(e^{\sqrt{x}})^{-\frac{1}{2}} e^{\sqrt{x}} \frac{d}{dx}(\sqrt{x})$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2} \frac{e^{\sqrt{x}}}{\sqrt{e^{\sqrt{x}}}} \times \frac{1}{2\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{\sqrt{x}}}{4\sqrt{x}\sqrt{e^{\sqrt{x}}}} = \frac{e^{\sqrt{x}}}{4\sqrt{x}e^{\sqrt{x}}} \text{ or } \frac{1}{4} \sqrt{\frac{e^{\sqrt{x}}}{x}}$$

Ans - 14 Given that,  $f(x) = x^2 + ax + 1$   
 $\Rightarrow f'(x) = 2x + a$   
 in interval (1,2),  $1 < x < 2 \Rightarrow 2 < 2x < 4$   
 $\Rightarrow (2 + a) > (2x + a) > (4 + a)$   
 Hence,  $f(x)$  is strictly increasing, then  $(2 + a) > 0$   
 $\therefore (2 + a) > 0 \Rightarrow a > -2$   
 Therefore, least value of  $a = -2$

**Or**

$$\text{Radius} = r = 3 \text{ cm}$$

$$\text{Area of circle (A)} = \pi r^2$$

$\Rightarrow$  The ratio of change of the area of a circle

$$\frac{dA}{dr} = \frac{d}{dr}(\pi r^2) = 2\pi r$$

$$\frac{dA}{dr} = 2\pi \times 3 = 6\pi \text{ cm}^2 / \text{cm.}$$

Ans - 15 We have  $f(x) = 3x^4 + 4x^3 - 12x^2 + 12$ .  
 $\Rightarrow f'(x) = 12x^3 + 12x^2 - 24x = 12x(x-1)(x+2)$   
 $\Rightarrow f'(x) = 0 \text{ at } x = 0, x = 1 \text{ and } x = -2$   
 Now  $f''(x) = 36x^2 + 24x - 24 = 12(3x^2 + 2x - 2)$

$$\begin{cases} f''(0) = -24 < 0 \\ f''(1) = 36 > 0 \\ f''(-2) = 72 > 0 \end{cases}$$

Therefore, by second derivative test,  $x = 0$  is point of local maxima and local maximum value of  $f$  at  $x = 0$  is  $f(0) = 12$  while  $x = 1$  and  $x = -2$  are the points of local minima and local minimum values of  $f$  at  $x = 1$  and  $-2$  are  $f(1) = 7$  and  $f(-2) = -20$  respectively

**Or**

Let the two positive numbers are  $a$  and  $b$ .

Now, according to the question  $a + b = 15 \Rightarrow b = 15 - a$  .....(i)

If the sum of the squares of both numbers is s, than

$$\begin{aligned}
 & s = a^2 + b^2 \\
 \Rightarrow & s = a^2 + (15 - a)^2 \\
 \Rightarrow & s = 2a^2 - 30a + 225 \\
 \therefore & \frac{ds}{da} = 4a - 30 \text{ and } \frac{d^2s}{da^2} = 4 \quad \dots\dots\text{(ii)}
 \end{aligned}$$

Now, for the maxima and minima of s :  $\frac{ds}{da} = 0$

Therefore,  $4a - 30 = 0 \Rightarrow a = \frac{15}{2}$

Also, since  $\frac{d^2s}{da^2} = 4 > 0$ , therefore s would be minimum for  $a = \frac{15}{2}$  and  $b = 15 - a$

$$\Rightarrow b = 15 - \frac{15}{2} = \frac{15}{2}$$

Hence, required numbers are  $\frac{15}{2}, \frac{15}{2}$ .

Ans - 16 Since E and F are independent, we have

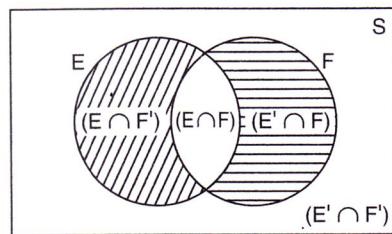
$$P(E \cap F) = P(E).P(F)$$

From the adjoining figure, it is clear that  $E \cap F$  and  $E \cap F'$  are mutually exclusive events and also  $E = (E \cap F) \cup (E \cap F')$

Therefore,  $P(E) = P(E \cap F) + P(E \cap F')$

or  $P(E \cap F') = P(E) - P(E \cap F)$

$$= P(E) - P(E).P(F)$$



$$\Rightarrow P(E \cap F') = P(E)[1 - P(F)] = P(E).P(F')$$

Or

By the definition of conditional probability, we have

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$\text{Therefore, } P(A/A \cup B) = \frac{P[A \cap (A \cup B)]}{P(A \cup B)} \quad \dots\dots(i)$$

Since the events A and B are mutually exclusive, we have

$$P(A \cup B) = P(A) + P(B) \quad \dots\dots(ii)$$

$$\text{Also, by set theory } A \cap (A \cup B) = A \quad \dots\dots(iii)$$

$$\text{Hence, from (i), (ii), (iii), } P(A/A \cup B) = \frac{P(A)}{P(A) + P(B)}$$

**Hence Proved**

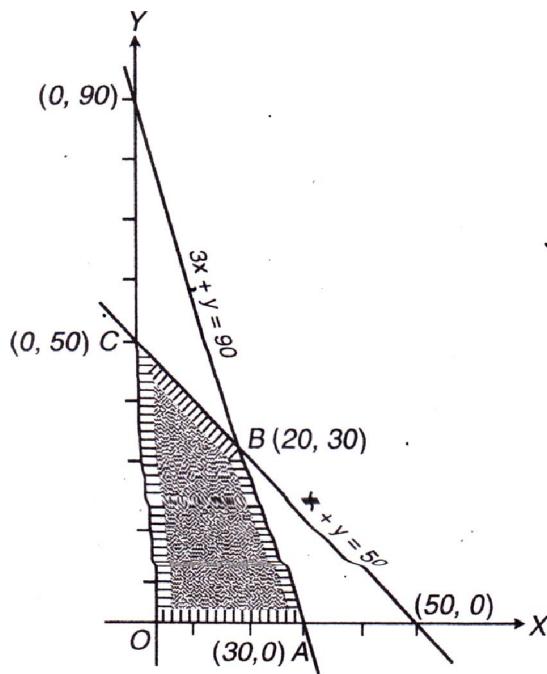
Ans - 17

$$\text{Let, } x + y \leq 50 \quad \dots\dots(i)$$

$$3x + y \leq 90 \quad \dots\dots(ii)$$

$$x \geq 0, y \geq 0 \quad \dots\dots(iii)$$

Let us plot the graph using the inequalities (i) to (iii). The shaded region in the figure is the feasible region determined by the system of constraints (i) to (iii). We observe that the feasible region OABC is bounded. So, we now use corner point method to determine the maximum value of Z.



Corner point
O (0, 0)
A (30, 0)
B (20, 30)
C (0, 50)

Corresponding value ( $Z = 4x + y$ )
0
120 (Maximum)
110
50

Therefore, maximum value of Z is 120 at the point A (30,0).

**Or**

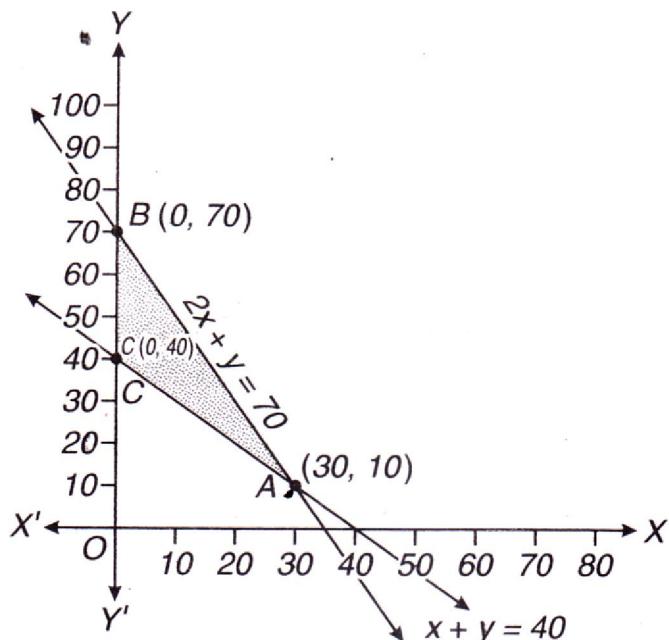
Let,

$$x + y \geq 40 \quad \dots(i)$$

$$2x + y \leq 70 \quad \dots(ii)$$

$$x \geq 0, y \geq 0 \quad \dots(iii)$$

The shaded region in the figure is the feasible region determined by the system of constraints (i) to (iii). We observe that the feasible region ABC is bounded. So, we now use corner point method to determine the maximum value of Z.



**Corner point**

A (30, 10)

B (0, 70)

C (0, 40)

**Corresponding value ( $Z = 5x + 2y$ )**

170 (Maximum)

140

80

Therefore, maximum value of Z is 170 at the point A (30,10).

Ans - 18

$$\frac{dy}{dx} = x \cdot \log x$$

$$\Rightarrow dy = x \log x \, dx$$

$$\Rightarrow \int dy = \int x \log x \, dx$$

$$\Rightarrow y = \log x \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} dx + c$$

$$\Rightarrow y = \frac{x^2}{2} \log x - \int \frac{x}{2} dx + c$$

$$\Rightarrow y = \frac{x^2}{2} \log x - \frac{x^2}{4} + c$$

**Or**

The given differential equation can be written as

$$\frac{dx}{dy} - \frac{x}{y} = 2y$$

This is a linear differential equation of the type.

$$\frac{dx}{dy} + P_1x = Q_1.$$

where  $P_1 = -\frac{1}{y}$  and  $Q_1 = 2y$ .

Therefore I.F. =  $e^{\int -\frac{1}{y} dy} = e^{-\log y}$

$$= e^{\log(y)^{-1}} = \frac{1}{y}$$

Hence, the solution of the given differential equation is

$$x \frac{1}{y} = \int (2y) \left( \frac{1}{y} \right) dy$$

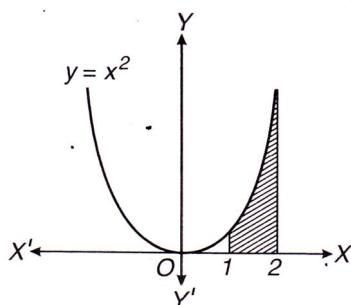
Or  $\frac{x}{y} = \int 2 dy$

or  $\frac{x}{y} = 2y + c$

or  $x = 2y^2 + cy$

Which is a general solution of the given differential equation.

Ans - 19 The given curve  $y = x^2$  is symmetrical about Y-axis whose vertex is  $(0, 0)$ .



$\therefore$  Required area

= Area of region below the curve  $y = x^2$  and area between, X-axis and lines  $x = 1$ ,  $x = 2$

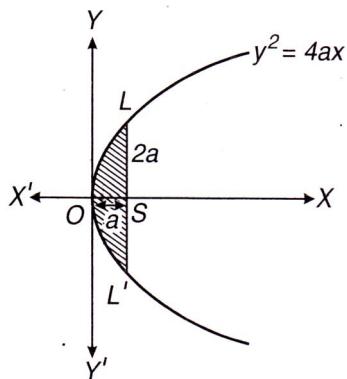
$$= \int_1^2 y dx = \int_1^2 x^2 dx$$

$$= \left| \frac{x^3}{3} \right|_1^2 = \frac{1}{3} (2^3 - 1^3) = \frac{8}{3} - \frac{1}{3}$$

$$= \frac{7}{3} \text{ square unit.}$$

**Or**

Equation of parabola is  $y^2 = 4ax$  and equation of the latus rectum  $x = a$ . Latus rectum meets at points  $L$  and  $L'$  on parabola. Required area is  $OL'SLO$ . If  $A$  is the area, then



$$A = 2 \int_0^a y dx = 2 \int_0^a \sqrt{4ax} dx$$

$$= 2 \int_0^a 2a^{1/2} x^{1/2} dx$$

$$= 2 \times 2a^{1/2} \int_0^a x^{1/2} dx$$

$$= 4a^{1/2} \left[ \frac{x^{3/2}}{3/2} \right]_0^a = \frac{2 \times 4}{3} a^{1/2} \left[ x^{3/2} \right]_0^a$$

$$= \frac{8}{3} a^2 \text{ square unit.}$$

Ans - 20

$$\mathbf{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \mathbf{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

and

$$\mathbf{a}' = 2\hat{i} + 4\hat{j} + 5\hat{k}, \mathbf{b}' = 3\hat{i} + 4\hat{j} + 5\hat{k}$$

∴

$$\mathbf{a} - \mathbf{a}' = (\hat{i} + 2\hat{j} + 3\hat{k}) - (2\hat{i} + 4\hat{j} + 5\hat{k}) = -\hat{i} - 2\hat{j} - 2\hat{k}$$

and

$$\mathbf{b} \times \mathbf{b}' = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$$= \hat{i}(15 - 16) - \hat{j}(10 - 12) + \hat{k}(8 - 9)$$

$$= 2\hat{i} + 2\hat{j} - \hat{k}$$

$$\therefore |\mathbf{b} \times \mathbf{b}'| = \sqrt{(-1)^2 + (2)^2 + (-1)^2} = \sqrt{1+4+1} = \sqrt{6}$$

$$\therefore \text{shortest distance } d = \frac{|(\mathbf{a} - \mathbf{a}') \cdot (\mathbf{b} \times \mathbf{b}')|}{|\mathbf{b} \times \mathbf{b}'|}$$

$$= \frac{|(-\hat{i} - 2\hat{j} - 2\hat{k}) \cdot (-\hat{i} + 2\hat{j} - \hat{k})|}{\sqrt{6}}$$

$$= \frac{|(-1)(-1) + (-2) \times 2 + (-2)(-1)|}{\sqrt{6}}$$

$$= \frac{|1 - 4 + 2|}{\sqrt{6}} = \frac{|-1|}{\sqrt{6}} = \frac{1}{\sqrt{6}}.$$

Ans.

**Or**

$$\text{Here, } \mathbf{a} = \hat{i} + \hat{j} - \hat{k}, \mathbf{b} = 3\hat{i} - \hat{j}; \mathbf{a}' = 4\hat{i} - \hat{k}, \mathbf{b}' = 2\hat{i} + 3\hat{k}$$

$$\therefore \mathbf{a} - \mathbf{a}' = \hat{i} + \hat{j} - \hat{k} - (4\hat{i} - \hat{k}) = -3\hat{i} + \hat{j} + 0\hat{k}$$

The given lines intersect, if  $\begin{bmatrix} \mathbf{a}_1 - \mathbf{a}_2, \mathbf{b}_1, \mathbf{b}_2 \end{bmatrix} = 0$

$$\begin{bmatrix} \mathbf{a}_1 - \mathbf{a}_2, \mathbf{b}_1, \mathbf{b}_2 \end{bmatrix} = \begin{vmatrix} -3 & 1 & 0 \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix} = -\begin{vmatrix} 3 & -1 & 0 \\ 3 & -1 & 0 \\ 3 & 0 & 3 \end{vmatrix} = 0$$

$$Q \begin{bmatrix} \mathbf{a}_1 - \mathbf{a}_2, \mathbf{b}_1, \mathbf{b}_2 \end{bmatrix} = 0$$

Thus, then given lines intersect.

$$\text{Ans - 21} \quad \text{Let,} \quad I = \int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx \quad \dots \dots \text{(i)}$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\sin^4(\pi/2 - x)}{\sin^4(\pi/2 - x) + \cos^4(\pi/2 - x)} dx$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\cos^4 x}{\cos^4 x + \sin^4 x} dx \quad \dots \dots \text{(ii)}$$

By adding eqns. (i) and (ii), we get

$$2I = \int_0^{\pi/2} \frac{\sin^4 x + \cos^4 x}{\sin^4 x + \cos^4 x} dx = \int_0^{\pi/2} dx = (x)_0^{\pi/2} = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}.$$

Ans.

**Or**

We note that  $x^3 - x \geq 0$  on  $[-1, 0]$  and  $x^3 - x \leq 0$  on  $[0, 1]$  and also  $x^3 - x \geq 0$  on  $[1, 2]$ . so we can write.

$$\begin{aligned}
\int_{-1}^2 |x^3 - x| dx &= \int_{-1}^0 (x^3 - x) dx + \int_0^1 -(x^3 - x) dx + \int_1^2 (x^3 - x) dx \\
&= \left[ \frac{x^4}{4} - \frac{x^2}{2} \right]_{-1}^0 + \left[ \frac{x^2}{2} - \frac{x^4}{4} \right]_0^1 + \left[ \frac{x^4}{4} - \frac{x^2}{2} \right]_1^2 \\
&= -\left( \frac{1}{4} - \frac{1}{2} \right) + \left( \frac{1}{2} - \frac{1}{4} \right) + (4-2) - \left( \frac{1}{4} - \frac{1}{2} \right) \\
&= -\frac{1}{4} + \frac{1}{2} + \frac{1}{2} - \frac{1}{4} + 2 - \frac{1}{4} + \frac{1}{2} = \frac{3}{2} - \frac{3}{4} + 2 = \frac{11}{4}.
\end{aligned}$$

Ans - 22 Let  $y = (\sin x)^x + \sin^{-1} \sqrt{x}$  .....(i)  
and,  $u = (\sin x)^x$  .....(ii)  
From eqn. (1)  $y = u + \sin^{-1} \sqrt{x}$  .....(iii)

Taking log on both sides of eqn. (ii)

$$\log u = x \log \sin x$$

On differentiating w.r.t. x in both sides

$$\begin{aligned}
\frac{1}{u} \frac{du}{dx} &= x \frac{d}{dx} (\log \sin x) + \log \sin x \frac{d}{dx} (x) \\
\Rightarrow \frac{du}{dx} &= u \left[ x \times \frac{1}{\sin x} \frac{d}{dx} (\sin x) + \log \sin x (1) \right] \\
\Rightarrow \frac{du}{dx} &= (\sin x)^x \left[ \frac{x}{\sin x} \times \cos x + \log \sin x \right] \\
\Rightarrow \frac{du}{dx} &= (\sin x)^x [x \cot x + \log \sin x] \quad .....(iv)
\end{aligned}$$

On differentiating w.r.t 'x' in both sides of eqn. (iii),

$$\begin{aligned}
\frac{dy}{dx} &= \frac{du}{dx} + \frac{1}{\sqrt{1-(\sqrt{x})^2}} \frac{d}{dx} (\sqrt{x}) \\
\Rightarrow \frac{dy}{dx} &= (\sin x)^x [x \cot x + \log \sin x] + \frac{1}{\sqrt{1-x}} \times \frac{1}{2\sqrt{x}} \\
\Rightarrow \frac{dy}{dx} &= (\sin x)^x [x \cot x + \log \sin x] + \frac{1}{2\sqrt{x}\sqrt{1-x}}
\end{aligned}$$

**Or**



$$y^x \left( \frac{x}{y} \frac{dy}{dx} + \log y \right) + x^y \left( \frac{y}{x} + \log x \frac{dy}{dx} \right) + x^x (1 + \log x) = 0$$

$$\text{or } (x.y^{x-1} + x^y \cdot \log x) \frac{dy}{dx} = -x^x (1 + \log x) - y x^{y-1} - y^x \log y$$

$$\text{Therefore, } \frac{dy}{dx} = -\frac{[y^x \log y + y x^{y-1} + x^x (1 + \log x)]}{x y^{x-1} + x^y \log x}$$

**Ans.**

Ans - 23 The system of equations can be written in the form  $AX = B$ , where

$$A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$$

$$\text{We see that, } |A| = 3(2 - 3) + 2(4 + 4) + 3(-6 - 4) = -17 \neq 0$$

Hence, A is non singular and so its inverse exists. Now,

$$\begin{aligned} A_{11} &= -1, & A_{12} &= -8, & A_{13} &= -10, \\ A_{21} &= -5, & A_{22} &= -6, & A_{23} &= 1, \\ A_{31} &= 1, & A_{32} &= 9, & A_{33} &= 7, \end{aligned}$$

$$\text{Therefore, } A^{-1} = -\frac{1}{17} \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$$

$$\text{So } X = A^{-1} B = -\frac{1}{17} \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix} \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$$

$$\text{i.e., } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{17} \begin{bmatrix} -17 \\ -34 \\ -54 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\text{Hence, } x = 1, y = 2 \text{ and } z = 3$$

**Ans.**

**Or**

On writing the system of equation in the form of  $AX = B$ .

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\text{Hence, } |A| = 2(-4 + 4) + 3(-6 + 4) + 5(3 - 2) = -1 \neq 0$$

So, A is invertible.  $A^{-1}$  exist.

Cofactors of A are following.

$$\begin{array}{lll} A_{11} = -0, & A_{12} = 2, & A_{13} = 1, \\ A_{21} = -1, & A_{22} = -9, & A_{23} = -5, \\ A_{31} = 2, & A_{32} = 23, & A_{33} = 13, \end{array}$$

$$\text{adj } A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{1}{|A|} \text{adj}(A) = \frac{1}{-1} \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

Now

$$X = A^{-1} B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -1 \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence,  $x = 1$ ,  $y = 2$  and  $z = 3$ .

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