Pre Nurture & Career Foundation Division

ANSWER KEY (Paper Code : 44)

NATIONAL STANDARD EXAMINATION IN ASTRONOMY

NSEA-2025 [22-11-2025]

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	b	b	b	a	c	b	a	c	d	d	a	c	b	c	d
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	d	b	a	b	d	a	d	c	b	b	c	a	d	d	d
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	c	d	b	b	b	c	b	c	b	c	d	a	c	b	b
Que.	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	c	a	b	b,c,d	c,d	b,d	c,d	a,c,d	a,d	a,c	a,b	a,b	a,b,c,d	a,b,c	b,c,d

NSEA-2025

Date of Examination : 22nd November, 2025

PAPER CODE - 44

SOLUTIONS

1. Which of the following statement or statements about stars are true?

(a) Among all one solar mass stars, the one with the largest radius is also the hottest.

(b) Type I supernovae are characterized by the absence of Hydrogen in the spectrum.

(c) Cooler stars show less absorption lines in their spectra.

(d) The stars are spectrally classified as O, B, A, F, G, K, M; with O type stars showing dense hydrogen line while M type stars having very less hydrogen lines.

Ans. **(b)**

Sol. Concept

2. As seen from Earth, angular separation between Proxima Centauri and Alpha Centauri is 2.2°. What is the physical separation between the two stars?

(a) 4.25 light years (b) 0.16 light years (c) 2.2 light years (d) 0.33 light years

Ans. **(b)**

Sol. Actual separation S = rθ

r = 4.3 Lt yr

$$\theta = 2.2 \times \frac{\pi}{180} = 0.0384 \text{ radian}$$

S = 4.3 × 0.384 ≃ 0.165 Lt years

3. Two stars in a binary system are separated by 3.0 AU and have mass ratio of 2 : I. Their orbital period is 6.0 years. What are the masses of the stars in terms of solar mass?

(a) 2.0, 1.0 (b) 0.50, 0.25 (c) 2.25, 1.125 (d) 1.78, 0.88

Ans. **(b)**

Sol. $$T = \frac{2\pi d^{3/2}}{\sqrt{GM_{Total}}} = \frac{2\pi d^{3/2}}{\sqrt{G(M_1+M_2)}} = 6 \text{ years} \quad \ldots(1)$$

For sum $$T = \frac{2\pi(1)^{3/2}}{\sqrt{GM_{Sun}}} = 1 \text{ years} \quad \ldots(2)$$

d = 3A.U.

after solving equation (1) & (2)

$M_{Total} = 0.75\ M_{Sun}$

$M_1 + M_2 = 0.75\ M_{Sun}$

$$\frac{M_1}{M_2} = \frac{2}{1}$$

$\Rightarrow$ $M_1 = 0.50\ M_{Sun}$

$M_2 = 0.25\ M_{Sun}$

4. A comet's closest approach to Sun is at I AU. What is the radial component of its velocity at this position?

(a) 0 km/s (b) 21.2 km/s (c) 30 km/s (d) 42.4 km/s

Ans. **(a)**

Sol. At perihelion, the comet's velocity is entirely tangential

So $V_{radical} = 0$

5. A new space station orbits the sun every four and a half years. In a particular year, it is seen on the local meridian at 1:00 am of the 21st of June, 2020. It will be again seen on the local meridian from the Earth, approximately, on

(a) 21 June, 2024, at 1 :00 am
(b) 20 December, 2024, at 4:00 pm
(c) 3 October, 2021, 4:00 pm
(d) 21 March, 2021, 9:00 pm

Ans. (c)

Sol. Synodic period $T = \frac{2\pi}{\omega_{relative}} = \frac{\omega_{earth} - \omega_{station}}{2\pi}$

$$\Rightarrow \frac{1}{T} = \frac{\omega_{earth}}{2\pi} - \frac{\omega_{station}}{2\pi}$$

$$\Rightarrow \frac{1}{T} = \frac{1}{T_{earth}} - \frac{1}{T_{station}} = \frac{1}{1} - \frac{1}{4.5}$$

$$\Rightarrow \frac{1}{T} = \frac{3.5}{4.5}$$

$$\Rightarrow T = \frac{4.5}{3.5} = \frac{9}{7} \text{ years} = 469.607 \text{ days}$$

So It will be again seen on 3 october, 2021 at 4:00 pm.

6. What is the advantage of an equatorial telescope mount over an alt-azimuth mount?

(a) Reduced vibrations and provides a more stable viewing platform.
(b) It allows tracking celestial objects using only one axis of motion.
(c) It eliminates the need for polar alignment before observing.
(d) It is easier to carry and transport due to its lightweight design.

Ans. (b)

Sol. Equatorial mount telescope is specially design to observe the celestial object; and polar alignment is necessary before using.

7. Which of the following statements is correct about constellations?

(a) Any star cannot belong to two constellations simultaneously
(b)Only the bright stars which are imagined as some figure in the sky make constellations.
(c) Brightest star in any constellation has magnitude 1.
(d) There are only 12 constellations along the Ecliptic belt.

Ans. (a)

Sol. Concept.

8. What would be the speed of a comet, on a parabolic orbit around the sun, whose point of closest approach is 1 AU, when at a distance of 4.0 AU?

(a) 42.1 km/s (b) 29.8 km/s (c) 21.1 km/s (d) 84.4 km/s

Ans. (c)

Sol. Parabolic orbit speed at distance r is

$$V = \sqrt{\frac{2GM}{r}} \simeq 21.1 \text{ km/s}$$

here r = 4 AU

9. Globular clusters are typically found in the halo of Milky Way. In which of the following constellation, there are higher chances of seeing globular cluster?

(a) Orion (b) Sagittarius (c) Ursa Minor (d) Virgo

Ans. (d)

Sol. Concept

10. Ecliptic plane makes approximately 60° with the Milky Way plane. One point of intersection lies in the constellation of Sagittarius, in which constellation does the other intersection point lie?

(a) Aquarius (b) Libra (c) Pieces (d) Gemini

Ans. (d)

Sol. Constellation which is dynamically opposite to the Sagittarius on ecliptic is Gemini

11. Which of the following places will have minimum duration between the two zero shadow days in a given calendar year.

(a) Manila (14° 36' N,'120° 59' E) (b) Monteiro (7° 53' S, 37° 7' W)

(c) Kansanshi (12° 6' S, 26° 26' W) (d) Barah (13° 42' N, 30° 22' E)

Ans. (a)

Sol. Duration between the two zero shadow days will be minimum for higher latitude.

12. An observer from Delhi, will see the Sun on the local meridian 365 times in the year 2025. A star, located on the celestial equator, will be seen how many times on the local meridian by the same observer in 2025?

(a) 364 (b) 365 (c) 366 (d) 367

Ans. (c)

Sol. Duration of sidereal is 23.93 hours

Duration of solar is 24 hours

In 1 year $\Rightarrow \dfrac{365.25 \times 24}{23.93} = 366.31$

Star will upper culminate 366 times (passes local meridian)

13. Considering the nuclear reactions that power the energy output of the stars, the correct statements is:

(a) the p-p chain is the dominant process that creates He in very massive stars.

(b) the fusion of H into He is an exothermic process.

(c) the CNO (Carbon-Nitrogen-Oxygen) cycle results in the creation of the elements like Si, S and P following the fusion of the lighter elements.

(d) the fusion process is replaced by the fission process in heavier stars.

Ans. (b)

Sol. Concept

14. Which of the following constellation is broken in two disjoint parts in the sky?

(a) Draco (b) Ursa (c) Serpens (d) Eridanus

Ans. (c)

Sol. Concept

15. In a hypothetical the force experienced by a particle is given by $F = A e^{\frac{px}{t^2}} + BV_t^2$, where $V_t = \sqrt{\frac{2mg}{\rho\eta S}}$ is the expression for terminal velocity. Where m is the mass, g is the acceleration due to gravity, ρ is the density of medium, S is the surface area of the particle, and η is the coefficient of viscosity of the medium. The possible units of A, B and P are

(a) A: newton, B: newton kg m^{-1}, p: ms^2
(b) A: no units, B: newton kg m^{-1}, p: $m^{-1}s^2$
(c) A: newton, B: kg m^{-1}, p: no units
(d) A: newton, B: newton s^2 m^{-2}, p: $m^{-1}s^2$

Ans. (d)

Sol. $F = A e^{\frac{px}{t^2}} + BV_t^2$ By principle of homogeneity

$[A] = [F]$

$[P] = \left[\frac{t^2}{x}\right]$

$[B] = \frac{[F]}{[v_t^2]}$

$A \to$ Newton, $B \to \frac{Ns^2}{m^2}$

$P \to \left(\frac{s^2}{m}\right)$

16. Gravity on the surface of Ganymede, a satellite of Jupiter, is $\left(\frac{1}{7^{th}}\right)$ of that of the Earth, while the gravity on the surface of the Moon is $\left(\frac{1}{6^{th}}\right)$ of that on the Earth. Two identical pendulums are taken one on Ganymede and the other on the Moon. The two pendulums start oscillating together, after how many oscillations on Moon will they come again in the same phase with approximately 1% uncertainty?

(a) 24
(b) 25
(c) 26
(d) 27

Ans. (d)

Sol. On Jupiter $T_J = 2\pi\sqrt{\frac{\ell}{g}}\ \sqrt{7} = \sqrt{7}\ T_0$

On Moon $\to T_M = \sqrt{6}\ T_0$

So $\omega_m : \omega_J = \sqrt{7} : \sqrt{6}$

Let $\omega_m = \sqrt{7}\omega_0$, $\omega_J = \sqrt{6}\omega_0$

Both will be in phase first time after

$T = \frac{2\pi}{\left(\sqrt{7}-\sqrt{6}\right)\omega_0} = \frac{2\pi}{\omega_0}\times\left(\sqrt{7}+\sqrt{6}\right)$

So no. of oscillations on moon = N = $\frac{\omega_m \times nT}{2\pi}$

For n = 2

$$N = \sqrt{7}\omega_0 \times 2 \times \frac{2\pi}{\omega_0} \frac{\left(\sqrt{7}+\sqrt{6}\right)}{2\pi}$$

N = 26.96

17. Consider two telescopes having equal apertures of 400 mm. One has a focal ratio of $\frac{f}{5}$, the other has the focal ratio of $\frac{f}{10}$. What is the relation between their focal lengths?

(a) Focal length of the telescope with $\frac{f}{5}$ ratio is twice as compared to the focal length of telescope $\frac{f}{10}$ ratio.

(b) Focal length of the telescope with $\frac{f}{10}$ ratio is twice as compared to the focal length of telescope $\frac{f}{5}$ ratio.

(c) Focal length of the telescope with $\frac{f}{10}$ ratio is twice as compared to the focal length of telescope with $\frac{f}{5}$ ratio.

(d) Focal lengths of both telescopes are the same

Ans. **(b)**

Sol. focal ratio = $\frac{f}{10}$: f = 10 × (aperture) = 4000 mm

focal ratio = $\frac{f}{5}$: f = 5 × (aperture) = 2000 mm

Focal length of telescope with focal ratio $\frac{f}{10}$ is twice as compared to that of focal ratio $\frac{f}{5}$.

So option (b) is correct.

18. **Statement-I:** The radius vector of a planet sweeps equal area in equal time while revolving around the Sun.

Statement-II: Gravitational force between the Sun and the planet is along the line joining the two.

(a) Both the Statements are true, and Statement II is correct reason of Statement-I.

(b) Both the Statements are true, but Statement II is not the correct reason of Statement I.

(c) The Statement I is true and the Statement II is false.

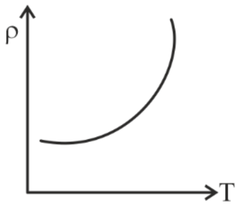
(d) The Statement I is false and the Statement II is true.

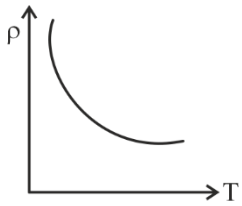
Ans. **(a)**

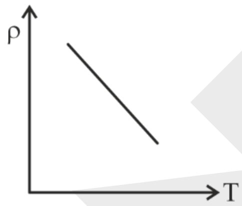
Sol. $\dfrac{dA}{dt} = \dfrac{L}{2m}$

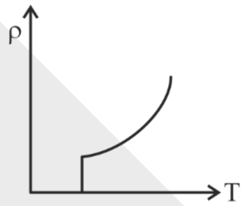
By CoAM $L = \text{Constant}$ So $\rightarrow \dfrac{dA}{dt} = \text{constant}$

19. Which of the following graphs qualitatively depicts the variation in resistivity of a semiconductor with respect to temperature correctly?

(a) 

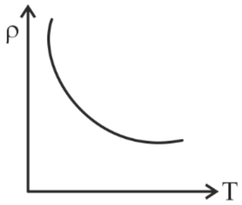
(b) 

(c) 

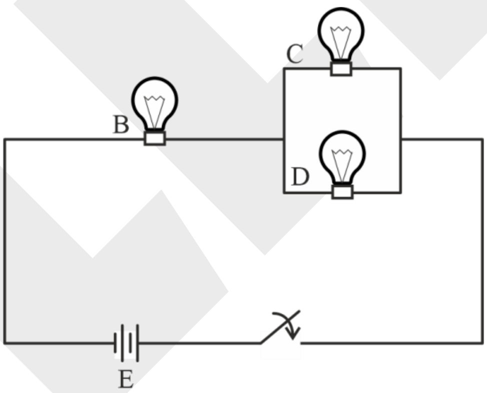
(d) 

Ans. (b)

Sol.



20. Three identical bulbs, B, C and D arc connected in a circuit as shown below. By connecting one more identical bulb



(a) in series with bulb B, the intensity of bulb B will decrease but that of bulb C and D will increase.

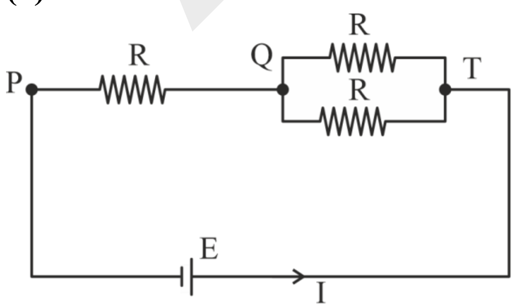
(b) in parallel with bulb B, the intensities of all the three bulbs B.C and D will decrease.

(c) in parallel with bulbs C and D, the intensities of all the three bulbs B, C and D will increase.

(d) in series with bulb C, the intensity of bulb B will decrease and that of bulb D will increase.

Ans. (d)

Sol.

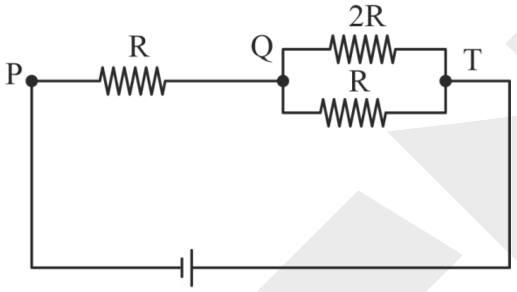


$V_{PQ} : V_{QT} = IR : I\frac{R}{2} = 1 : \frac{1}{2}$

$V_{PQ} = \frac{2E}{3}$; $V_{QT} = \frac{E}{3}$

$P_B = \frac{4E^2}{9R}$; $P_D = \frac{E^2}{9R}$

After connecting identical bulb in series with (c) →



$V_{PQ} = I^1 R : V_{QT} = I^1 \times \frac{2R}{3}$

$V_{PQ} : V_{QT} = 1 : \frac{2}{3} = 3 : 2$

$V_{PQ} = \frac{3E}{5}$; $V_{QT} = \frac{2E}{5}$

$P_B = \frac{9}{25}\frac{E^2}{R}$; $V_D = \frac{4E^2}{25R}$

21. Two cannons are placed on 1000 m high towers at a horizontal distance of 400 m between them along x-axis. Ball A fired at an angle of 45° to the +ve x-axis whereas Ball B is fired at an angle of 45° to the –ve x-axis. Initial velocity of projection given to each ball is u = 40 m/s in magnitude. The point P (x, y) at which the two balls collide is

(a) x = 200 m, y = – 45 m
(b) x = 200 m, y = 0
(c) x = 100 m, y = – 200 m
(d) the two balls will not collide

Ans. (a)

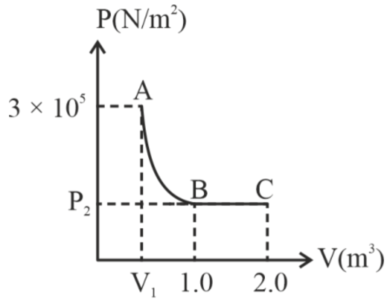
Sol. time of collision = $\frac{d_{rel}}{v_{rel}} = \frac{400}{20\sqrt{2}+20\sqrt{2}}$

along vertical → $\Delta y = 20\sqrt{2} \times 5\sqrt{2} - 5\left(5\sqrt{2}\right)^2$

$= 200 - 250 = -50$ m

along horizontal → $\Delta x = 20\sqrt{2} \times 5\sqrt{2} = 200$ m

22. Sixty moles of Helium gas are initially at 28°C (at A). It undergoes an isothermal process from A to B and then an isobaric process from B to C. Total change in the internal energy of the gas in the complete process from A to C would be approximately,



(a) 206 kJ (b) 450 kJ (c) 431 kJ (d) 225 kJ

Ans. **(d)**

Sol. $A \rightarrow B$

$PV = nRT$

T = constant

no change in internal energy

$B \rightarrow C$

$\Delta U = n\, C_V\, \Delta T$

$= 60 \times \frac{3}{2} R \times (\Delta T)$

$\frac{V_B}{V_C} = \frac{T_B}{T_C}$

$\frac{1}{2} = \frac{301}{T_C}$

$T_C = 602$ K

$\Delta T = 301$ K

$\Delta U = 60 \times \frac{3}{2} \times 8.314 \times 301$

$= 225226.26$ J

$= 225.226 \times 10^3$ J

$\simeq 225$ KJ

23. In an atom, the nucleus consists of 2 protons and 2 neutrons and one electron is revolving around the nucleus. According to Bohr's atomic model, the only visible wavelength corresponds to the transition

(a) 2^{nd} 10 1^{st} orbit (b) 4^{th} to 2^{nd} orbit (c) 4^{th} to 3^{rd} orbit (d) 5^{th} to 3^{rd} orbit

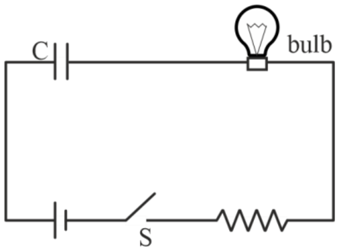
Ans. **(c)**

Sol. $\frac{1}{\lambda} = R_H Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$

$\frac{1}{\lambda} = R_H 4 \left(\frac{1}{3^2} - \frac{1}{4^2} \right)$

$\lambda = 4690$ A° (visible range)

24. Select the statement describing correctly the functioning of the circuit, shown below.



(a) From the time the switch S is closed, the bulb will start glowing with intensity increasing uniformly to a maximum value and will turn off after some time.

(b) As soon as the switch S is closed, the bulb will glow with maximum intensity which will then slowly decrease to zero.

(c) From the time the switch S is closed, the bulb will start glowing with intensity increasing uniformly to a maximum value and will continue glowing thereafter.

(d) If distance between the parallel plates of the capacitor is increased, bulb intensity will attain maximum value in shorter time after the switch is closed.

Ans. (b)

Sol. At the instant of switching capacitor provides zero impedance.

So current is max. After switch is closed $\rightarrow I = I_0\, e^{-t/RC}$

So current decreases.

So power of bulb also decreases.

25. A mechanical spring deviates from Hooke's law as $F \propto -k(e^x - 1)$, where x is the strain. At what value of the strain x, the force deviates from the one obeying Hooke's law with same k and unit length, just by 1%?

(a) x = 1% (b) x = 2% (c) x = 5% (d) x = 20%

Ans. (b)

Sol. $F = K_0\, K(e^x - 1)$

For Hooke's law $\rightarrow$

$F_H = K_0Kx$

$$\frac{F - F_H}{F_H} = \frac{K_0K(e^x - 1) - K_0Kx}{K_0Kx} = \frac{e^x - 1 - x}{x} = 0.01$$

$e^x - 1 = 1.01\, x$

$e^x = 1 + 1.01x$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + ...$$

$$1 + x + \frac{x^2}{2} + \frac{x^3}{6} = 1 + 1.01x$$

$$\frac{x^2}{2} + \frac{x^3}{6} + ... = 0.01\, x$$

$$\frac{x}{2} + \frac{x^2}{6} + ... = 0.01$$

if $x \to$ small so

$\frac{x}{2} \simeq 0.01$

$x \simeq 0.02$

26. Three lenses with focal lengths f, f' and f respectively are kept with common principal axis in that order with successive separation of $\frac{f}{2}$ between each pair. If the point object is placed on the common principal axis to the left of 1st lens at 2f distance from it What should be the value of focal length f' so as to form the final image at distance of 2f from 3rd lens on the right of it?

(a) $-\frac{4f}{3}$ (b) $+\frac{4f}{3}$ (c) $-\frac{3f}{4}$ (d) $-\frac{3f}{5}$

Ans. (c)

Sol.

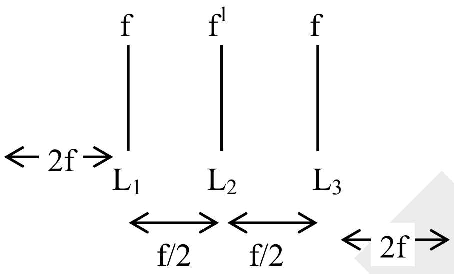


Image of L_1 will be 2f right ward to the L_1 so for L_2 object distance $U = +\frac{3f}{2}$

for L_3 image is at 2f. So for L_3 object should be at 2f to the. Left of L_3, So image of to L_2 should be $\frac{3f}{2}$ left of L_2.

so for $L_2 \to$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f^{1}} \Rightarrow -\frac{1}{\frac{3f}{2}} - \frac{1}{\frac{3f}{2}} = \frac{1}{f_1}$$

$$-\frac{4}{3f} = \frac{1}{f_1}$$

$$f^{1} = -\frac{3f}{4}$$

27. A negatively charged small ball with charge –20 pC and mass 1.00 mg is placed at the centre of a uniformly charged ring of radius 5.00 cm. The negatively charged ball is allowed to move only along the axis of the ring. The ball executes SHM with frequency of 1.00 kHz. The charge on the ring is

(a) 27.4 mC (b) 0.55 mC (c) 13.9 μC (d) 0.69 μC

Ans. (a)

Sol. $F = -\dfrac{KqQx}{\left(R^2+x^2\right)^{3/2}}$

if $x << R$

$F = -\left(\dfrac{KqQ}{R^3}\right)x$

$F = -K_{eff}\,x$

$\omega = 2\pi f = \sqrt{\dfrac{Keff}{m}}$

$2\pi \times 10^3 = \sqrt{\dfrac{KqQ}{R3} \times \dfrac{1}{m}}$

$Q = 27.4$ mc

28. Two hunters approach a prey from opposite directions. One hunter has a probability of $\frac{1}{3}$ for hitting the prey, the other has a probability of $\frac{2}{3}$ of hitting the prey. Both fires a burst of 3 shots each simultaneously. Assume that both the hunters fire one shot each at the same time simultaneously, followed by another shot each at the same time, followed by the third shot each at the same time, each round of shots have negligible time gap between them. The probability that the prey, in question, is hit

(a) after the three rounds is 1
(b) in the first round of shots is 1
(c) after the three rounds is $\frac{7}{9}$
(d) after the three rounds is $\frac{721}{729}$

Ans. (d)

Sol. Let two hunters are A and B.

$P(A) = \frac{1}{3}$ $\quad$ $P(B) = \frac{2}{3}$

$P(\bar{A}) = \frac{2}{3}$ $\quad$ $P(\bar{B}) = \frac{1}{3}$

Probability that they hit the prey in first round

$= P(A).\ P(B) + P(\bar{A}) \cdot (B) + P(A) \cdot P(\bar{B})$

$= \frac{1}{3} \times \frac{2}{3} + \frac{2}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{1}{3} = \frac{7}{9}$

Probability they hit prey after three round.

$= 1 - $ P(Not hit the target in three rounds)

$= 1 - \left(P(\bar{A}) \cdot (\bar{B})\right)^3$

$= 1 - \left(\frac{1}{3} \times \frac{2}{3}\right)^3 = 1 - \frac{8}{729} = \frac{721}{729}$

29. A particle moving with constant velocity along X-axis has angular momentum p = 120 units about a reference point in XY plane. Which of the following statements is correct?

(a) If the reference point is shifted parallel to Y axis, angular momentum need not change necessarily.

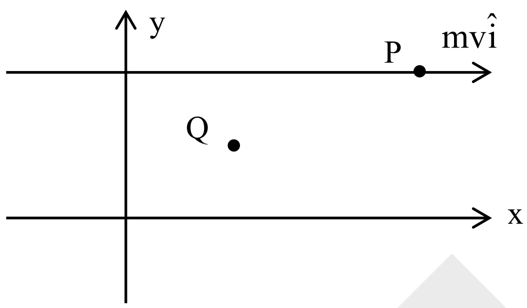
(b) If the reference point is taken to double the present distance from origin, instantaneous angular velocity become half the present value.

(c) It is always possible to increase instantaneous angular velocity to double the present value by shifting reference point parallel to X axis.

(d) If the reference point is shifted parallel to X axis, angular momentum will surely not change about it.

Ans. (d)

Sol.



$\vec{L}_{-P/Q} = \vec{r}_{P/Q} \times mv\hat{i}$

$= \left(x_{P/Q}\hat{i} + y_{P/Q}\hat{j}\right) \times mv\hat{i}$

$= mvy_{P/Q}\ (-\hat{k})$

So if reference point is shifted parallel to x-axis then there will be no change in $y_{P/Q}$.

So $\vec{L}_{P/Q}$ remains constant.

30. An electron moving with speed of 0.1% speed of light, enters a uniform magnetic field of strength 2.5 gauss, at 60 angle. Calculate the number of helical turns it takes in moving 1 km distance.

(a) 20000 to 22000
(b) 22000 to 25000
(c) 30000 to 32000
(d) 45000 to 47000

Ans. (d)

Sol. $T = \dfrac{2\pi m}{qB}$

Pitch length = V cos 60° × T

So no. of turns = $\dfrac{1000}{\text{pitch length}}$

$N \simeq 46638.8$ trun

31. If $a + \dfrac{1}{a} = \sqrt{3}$ then $a^6 + \dfrac{1}{a^6}$

(a) 27
(b) 0
(c) –2
(d) 81

Ans. (c)

Sol. $a+\frac{1}{a}=\sqrt{3}$

by squaring both side

$a^2+\frac{1}{a^2}+2=3$

$a^2+\frac{1}{a^2}=1$

$a^6+\frac{1}{a^6}=\left(a^2+\frac{1}{a^2}\right)^3-3\times a^2\times\frac{1}{a^2}\left(a^2+\frac{1}{a^2}\right)$

$a^6+\frac{1}{a^6}=(1)^3-3(1)=1-3=-2$

32. If $\sin x + \cos x = \sqrt{2}$ then $\sin^4 x + \cos^4 x$ is

(a) 4 (b) –1 (c) 1 (d) $\frac{1}{2}$

Ans. (d)

Sol. $\sin x + \cos x = \sqrt{2}$

by squaring both side:

$\sin^2 x + \cos^2 x + 2\sin x \cos x = 2$

$\sin x \cos x = \frac{1}{2}$

$\sin^4 x + \cos^4 x = (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x$

$= 1-2\times\frac{1}{4}=1-\frac{1}{2}=\frac{1}{2}$

33. If a,b,c are in arithmetic progression and a^2, b^2, c^2 are in geometric progression, then the common ratio is

(a) 0 (b) 1 (c) 2 (d) 3

Ans. (b)

Sol. a,b,c.... are in A.P.

a^2, b^2, c^2.... are in G.P.

Let $a = b - d$, $c = b + d$

$\because$ a^2, b^2, c^2 are in G.P.

$b^4 = a^2c^2$

$b^4 = (b-d)^2 (b+d)^2$

$b^4 = b^4+d^4-2b^2d^2$

$d^4 = 2b^2d^2$

$d^2(d^2-2b^2) = 0$

$\boxed{d=0}$ or $\boxed{d^2=2b^2}$

Case-1 If $d = 0 \Rightarrow a = b = c$

$\Rightarrow$ Common ratio $= \frac{b^2}{a^2} = 1$

Case-2 If $d^2 = 2b^2$

common ratio $= \frac{b^2}{(b-d)^2} = \frac{b^2}{(b-\sqrt{2}b)^2} = \frac{1}{(1-\sqrt{2})^2}$

34. $\sum_{n=1}^{\infty} \frac{1}{n(n+3)} =$

(a) $\frac{1}{3}$ (b) $\frac{11}{18}$ (c) $\frac{1}{2}$ (d) $\frac{4}{17}$

Ans. (b)

Sol. $\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$

$$= \sum_{n=1}^{\infty} \frac{1}{3}\left(\frac{1}{n} - \frac{1}{n+3}\right)$$

$$= \sum_{n=1}^{\infty} \frac{1}{3}\left[\left(\frac{1}{n} - \frac{1}{n+1}\right) + \left(\frac{1}{n+1} - \frac{1}{n+2}\right) + \left(\frac{1}{n+2} - \frac{1}{n+3}\right)\right]$$

$$= \frac{1}{3}\left[\left(\frac{1}{1} - 0\right) + \left(\frac{1}{2} - 0\right) + \left(\frac{1}{3} - 0\right)\right]$$

$$= \frac{1}{3}\left[\frac{1}{1} + \frac{1}{2} + \frac{1}{3}\right] = \frac{1}{3} \times \frac{11}{6} = \frac{11}{18}$$

35. Let f: R $\to$ R be defined as $f(x) = \max\{x^2, 1\}$. Then

(a) f is differentiable everywhere

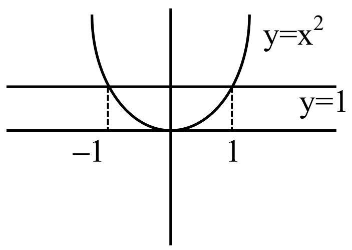
(b) f is continuous everywhere but not differentiable at $x = \pm 1$

(c) f is not continuous at $x = \pm 1$

(d) f is neither continuous nor differentiable.

Ans. (b)

Sol. $f(x) = \max\{x^2, 1\}$



from graph of f(x) it is continuous every where but not differentiable at $x = \pm 1$.

36. Let T_1 denote an equilateral triangle of side length a, T_2 be the triangle whose vertices are midpoints of sides of T_1, T_3 be the triangle whose vertices are midpoints of sides of T_2. We continue in similar manner to obtain the triangles T_4, T_5, ... For each natural number n, let P_n denote the perimeter of the triangle T_n. Then $\sum_{n=1}^{\infty} P_n$

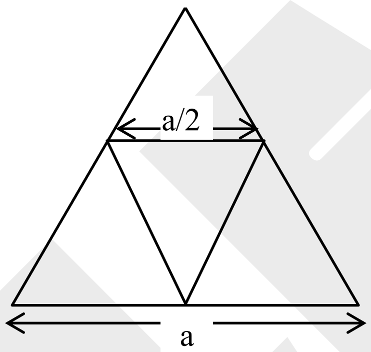
(a) 12a (b) 8a (c) 6a (d) 3a

Ans. **(c)**

Sol. Perimeter of traingle $T_1 = 3a$

Perimeter of triangle $T_2 = \frac{3a}{2}$ (by using midpoint theorem)

perimeter of trinagle $T_3 = \frac{1}{2}\left(\frac{3a}{2}\right)$



$$\sum_{n=1}^{\infty} Pn = P_1 + P_2 + P_3 ....$$

$$= 3a + \frac{1}{2}(3a) + \frac{1}{2^2}(3a)...$$

$$= \frac{3a}{1-\frac{1}{2}} = \frac{3a}{(1/2)} = 6a \qquad \because \left(S_\infty = \frac{a}{1-r}\right)$$

37. $\sqrt{i} + \sqrt[3]{i^2} = x + iy$, where $i = \sqrt{-1}$. (x,y) = ?

(a) (– 0.293, 1.573) (b) (–0.293, 0.707) (c) (–0.207, 1.573) (d) (1.207, 1.573)

Ans. **(b)**

Sol. $\sqrt{i} + \sqrt[3]{i^2} = x + iy$

$$\because \sqrt{z} = \sqrt{a+ib} = \pm\left(\sqrt{\frac{|z|+a}{2}} + i\frac{b}{|b|}\sqrt{\frac{|z|-a}{2}}\right)$$

$$\sqrt{i} = \pm\left(\sqrt{\frac{1+0}{2}} + i\sqrt{\frac{1-0}{2}}\right) = \pm\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)$$

$$\sqrt{i} + \sqrt[3]{-1} = x + iy$$

$$\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}}-1=x+iy \quad \text{or} \quad -\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}-1=x+iy$$

$-0.293 + 0.707i = x+iy$ or $-1.707 + 0.707i = x+iy$

$(-0.293, 0.707)$ or $(-1.707, 0.707)$

38. A standard parabola $x^2 = 36y$ is approximated as an arc of a circle for small values of x. What will be the radius of that circle?

(a) 72 (b) 36 (c) 18 (d) 9

Ans. **(c)**

Sol. The radius of the approximated circle at a particular point is given by radius of curvature:

$$R=\frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

Given curve is $x^2 = 36y$

$$y=\frac{1}{36}x^2 \Rightarrow \frac{dy}{dx}=\frac{x}{18} \quad \text{and} \quad \frac{d^2y}{dx^2}=\frac{1}{18}$$

for small value of x : x = 0

$$\left.\frac{dy}{dx}\right|_{x=0}=0 \quad \left.\frac{d^2y}{dx^2}\right|_{x=0}=\frac{1}{18}$$

$$R=\frac{(1+0)^{3/2}}{\frac{1}{18}}=18$$

39. $\displaystyle\int_0^{\frac{\pi}{2}}\frac{\sin x}{\sin x+\cos x}dx=$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) 0 (d) π

Ans. **(b)**

Sol. $\displaystyle\int_0^{\frac{\pi}{2}}\left(\frac{\sin x}{\sin x+\cos x}\right)dx$

by property ..(4)

$$\int_0^a f(x)\,dx=\int_0^a f(a-x)dx$$

$$I=\int_0^{x/2}\left(\frac{\sin x}{\sin x+\cos x}\right)dx \qquad ...(1)$$

$$= \int_0^{x/2} \frac{\sin\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$$

$$I = \int_0^{x/2} \frac{\cos x}{\sin x + \cos x} dx \qquad ...(2)$$

by adding equation (1) and (2)

$$2I = \int_0^{\pi/2} \left(\frac{\sin x + \cos x}{\sin x + \cos x}\right) dx = \int_0^{\pi/2} dx$$

$$= (x)_0^{\pi/2} = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

40. Let f(x) be continuous on $[0,\pi]$ and $f(x) + f(\pi - x) = \pi$. Then $\int_0^{\pi} f(x)\, dx =$

(a) π^2 (b) $\frac{\pi}{2}$ (c) $\frac{\pi^2}{2}$ (d) $\frac{\pi^2}{4}$

Ans. (c)

Sol. $f(x) + f(\pi - x) = \pi$

Let $I = \int_0^{\pi} f(x) dx$

By property (4) :

$$I = \int_0^{\pi} f(\pi - x) dx = \int_0^{\pi} f(x) dx$$

$$2I = \int_0^{\pi} (f(x) + f(\pi - x)) dx$$

$$2I = \int_0^{\pi} (\pi) dx = \pi(x)_0^{\pi}$$

$$2I = \pi[\pi - 0]$$

$$I = \frac{\pi^2}{2}$$

41. $\lim_{x \to 3} \frac{\sqrt{1 - \cos 2(x-3)}}{x - 3}$

(a) Exists and it equal $\sqrt{2}$

(b) Exists and it equal $-\sqrt{2}$

(c) Does not exist because $(x - 1) \to 0$

(d) Does not exist because left hand limit is not equal to right hand limit.

Ans. (d)

Sol. $\lim\limits_{x\to 3}\dfrac{\sqrt{1-\cos 2(x-3)}}{x-3}$

let $x-3=h$

$\lim\limits_{h\to 0}\dfrac{\sqrt{1-\cos 2h}}{h}$

$\lim\limits_{h\to 0}\dfrac{\sqrt{2\sin^2 h}}{h}\Rightarrow\dfrac{\sqrt{2}\,|\sin h\,|}{h}$

for LHL $=\lim\limits_{h\to 0^-}\dfrac{-\sqrt{2}\sin h}{h}=-\sqrt{2}$

for RHL $\lim\limits_{h\to 0^+}\dfrac{\sqrt{2}\sin h}{h}=\sqrt{2}$

LHL $\neq$ RHL

Limit does not exist because left hand limit is not equal to right hand limit.

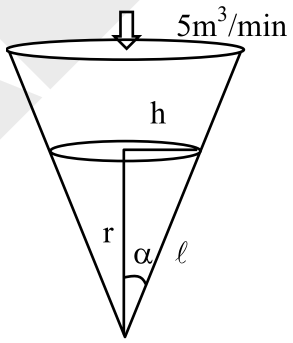
42. A water tank has the shape of an inverted right circular cone, whose semi-vertical angle is $\tan^{-1}\left(\dfrac{3}{4}\right)$. Water is poured into it at a constant rate of 5 m^3/min. Then, the rate (in m/min) at which the water surface moves along the slant surface at the instant when the depth of water in the tank is 10 m is

(a) $\dfrac{1}{9\pi}$ (b) $\dfrac{9}{192\pi}$ (c) $\dfrac{1875}{4\pi}$ (d) $\dfrac{4}{27\pi}$

Ans. (a)

Sol. $\dfrac{dv}{dt}=5m^3/\min$

$v=\dfrac{1}{3}\pi r^2 h$



$\because \tan\alpha=\dfrac{3}{4}$

$\dfrac{h}{r}=\dfrac{4}{3}$ $h=\dfrac{4}{3}r$

at $h=10m$

$$r = \frac{15}{2}\text{m}$$

and $\ell = \frac{25}{2}\text{m}$

$$\Rightarrow \quad v = \frac{1}{3}\pi r^2\left(\frac{4}{3}r\right)$$

$$v = \frac{4}{9}\pi r^3$$

$$\frac{dv}{dt} = \frac{4}{9}\pi \times 3r^2\frac{dr}{dt}$$

$$\Rightarrow \quad 5 = \frac{4}{9}\pi \times 3 \times \frac{225}{4}\frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{15\pi}$$

$$\Rightarrow \quad \ell^2 = r^2 + h^2$$

$$\ell^2 = r^2 + \frac{16}{9}r^2$$

$$\ell^2 = \frac{25}{9} \times r^2$$

$$2\ell\frac{d\ell}{dt} = \frac{25}{9} \times 2r\frac{dr}{dt}$$

$$2 \times \frac{25}{2}\frac{d\ell}{dt} = \frac{25}{9} \times 2 \times \frac{15}{2} \times \frac{1}{15\pi}$$

$$\frac{d\ell}{dt} = \frac{1}{9\pi}\text{m / min}$$

43. How many people should there be in a group such that there is more than half the probability, that two people from the group have their birthday with the same date, irrespective of in which months they were born.

(a) 3 or more (b) 4 or more (c) 7 or more (d) 23 or more

Ans. **(c)**

Sol. According to question at least two person have same birth date irrespective of month.

P(atleast two person has same birth date) = 1 - P(No person has same birth date)

For n = 3

$$P(3) = 1 - \frac{31 \times 30 \times 29}{31 \times 31 \times 31} = 1 - 0.90 \approx 0.10$$

$$P(4) = 1 - \frac{31 \times 30 \times 29 \times 28}{31 \times 31 \times 31 \times 31} \approx 0.2$$

$$P(5) = 1 - \frac{31 \times 30 \times 29 \times 28 \times 27}{31 \times 31 \times 31 \times 31 \times 31}$$

$$= 1 - 0.71 \approx 0.29$$

$$P(6)=1-\frac{31\times30\times29\times28\times27\times26}{31\times31\times31\times31\times31\times31}\approx0.41$$

$$P(7)=1-\frac{31\times30\times29......25}{31^7}\approx0.52>\frac{1}{2}$$

for n = 7,

$P(7) > \frac{1}{2}$

44. A and B alternatively toss a coin. The one who gets a head first wins. If A starts the game, then what is the probability that A wins?

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{3}{5}$

Ans. **(b)**

Sol. Probability of getting head or a tail in one toss = $\frac{1}{2}$

So, probability of winning A if P(H) + $P\left(\bar{H}\bar{T}H\right)+P\left(\bar{H}\bar{T}\bar{H}\bar{T}H\right)+......$

$$P(A)=\frac{1}{2}+\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^5+.........$$

$$=\frac{\frac{1}{2}}{1-\frac{1}{4}}=\frac{\frac{1}{2}}{\frac{3}{4}}=\frac{2}{3}$$

$$\because S_\infty=\frac{a}{1-r}$$

45. Consider the expression Z = 1 + 2^2 + $(3^3)^3$ + $((4^4)^4)^4$ + ... From which term onwards, the total value of the expression Z exceeds 10^{1000}?

(a) 4 (b) 6 (c) 8 (d) 10

Ans. **(b)**

Sol. $z = 1 + 2^2 + (3^3)^3 + ((4^4)^4)^4.......$

$z = 1 + 2^2 + 3^{3^2} + 4^{4^3}.......n^{n^{n-1}}$

A.T.Q.

$n^{n^{n-1}} > 10^{1000}$

$\Rightarrow n^{n-1} \times \log(n) > 1000$

at n = 5

$\Rightarrow 5^4 \times \log5$

$\Rightarrow 625 \times 0.6090 \approx 380.625 < 1000$

at n = 6

$\Rightarrow 6^5 \times \log6 \approx 6050 > 1000$

hence 6^{th} term onwards Z > 10^{1000}

46. Let $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$. Which of the following statements is not true?

(a) A is symmetric
(b) If $a + b + c = 0$ then $\det A = 0$
(c) $\det A = a^3 + b^3 + c^3 - 3abc$
(d) $\det A = \det A^T$

Ans. **(c)**

Sol. $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$

$|A| = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$|A| = -(a^3 + b^3 + c^3 - 3abc)$

Now if $a + b + c = 0$ then $\det A = 0$

Since A is symmetric matrix because $A = A^T$ and $\det A = \det A^T$

Hence option (c) is not true.

47. It has been four hours since Regulus (10h 08m, + 11°58') has crossed the local meridian at Mumbai (19°2'11.11"N, 72°51'34.09"E). Which of the following stars will be closest to the meridian now?

(a) Arcturus (14h 16m 50.75s, 19°02'8")
(b) Sirius (6h 46m 15.1s, –16°44'.6")
(c) Betelgeuse (5h 56m 31.86s, 19°02'8")
(d) Spica (13h 26m 32.96s, –11°17' 44.7")

Ans. **(a)**

Sol. (LST) local sidereal time = 10h08m

now 4 hour later

LST now = 10h08m+4h

= 14h08m

48. A star has an apparent magnitude of 10, and an Absolute magnitude of 10. How many parsecs away from the Earth is it?

(a) 100
(b) 10
(c) 1
(d) 0.1

Ans. **(b)**

Sol. Apparent magnitude m = 10

Absolute magnitude M = 10

Use distance modules formula $m - M = 5 \log_{10} (d) - 5$

d = 10 parsecs.

49. The pole star

(a) can be seen at night from all locations on the Earth.

(b) will be visible during a solar eclipse from nearly the whole of the northern hemisphere.

(c) will be visible during a solar eclipse from the equator.

(d) can, in principle, be observed from India anytime during the day and night.

Ans. **(b,c,d)**

Sol. Pole star is always visible in northern hemisphere.

50. An observer measures the location of the Sun, moon, planets and bright stars like Sirius, very diligently. The correct conclusions that she may reach is/are :

(a) Planets rise and set at the same time as per the sidereal clock but not as per the solar clock for all days of the year

(b) The sun rises and sets at the same time as per the solar clock and not as per the sidereal clock for all days of the year.

(c) The stars rise and set at the same time as per the sidereal clock but not as per the solar clock for all days of the year.

(d) The moon rises and sets at different times as per the sidereal clock and also as per the solar clock for all days of the month.

Ans. **(c,d)**

Sol. Right ascension and declination of star is fixed that why they follow sidereal clock but not solar clock.

Right ascension and declination of planets and moon continuously changes so it will not follow both sidereal and solar clock.

Since sun's orbit is inclined with respect to celestial equator that why sun can't follow solar clock.

51. Which of the following stars is/are circumpolar in Warsaw (52°14'N 21°01'E)

(a) α Cygni (16h 41m, +31°36')
(b) β Bootis (15h 01m, +40°23')
(c) θ Aurigae (5h 59m, +37°12')
(d) γ Draconis (17h 56m, +51°26')

Ans. **(b,d)**

Sol. Star will be circumpolar at latitude ϕ if there declination is greater than $(90 - \phi)$

52. Inside a cylindrical well, at the bottom and touching the wall, a red ball is thrown at an angle of 45° to the horizontal towards the diametrically opposite end of the wall and it hits the wall after a time interval of $\frac{v\sqrt{2}}{g}$, where v is the magnitude of velocity of the red ball and g is acceleration due to gravity. A black ball identical in shape and mass to the red ball is thrown vertically upwards from the bottom of the well with a kinetic energy half of that of the red ball. A green ball having a mass half of the red ball is thrown from the bottom of the well but diametrically opposite to the red ball with a kinetic energy half of that of the red ball. All the balls, if and when they hit the wall, undergo completely elastic collision.

(a) The red ball bounces off the bottom of the well for the first time when it hits the wall the third time.

(b) The black ball will take double the time to hit the bottom of the well to that of the red ball.

(c) The green ball hits the bottom of the well at the same spot as the red ball.

(d) The green ball will take the same time as the red ball to hit the bottom of the well the first time.

Ans. (c,d)

Sol. For red ball →

$$T_R = 2\frac{v}{\sqrt{2}} \times \frac{1}{g} = \frac{\sqrt{2}v}{g}$$

diameter = range of the red ball

it is equal to the diameter of well. so red ball is hitting directly at diametrilally opposite point.

For black ball :-

$$\frac{1}{2}m_B v_B{}^2 = \frac{1}{2}\left(\frac{1}{2}m_R v_R{}^2\right)$$

$$v_B = \frac{v_R}{\sqrt{2}} = \frac{v}{\sqrt{2}}$$

$$T_B = 2\frac{v}{\sqrt{2}} \times \frac{1}{9} = \frac{\sqrt{2}v}{g}$$

$T_R = T_B$

for green ball :-

$$\frac{1}{2}mg\, v_g^2 = \frac{1}{2}\left(\frac{1}{2}m_R v_R^2\right)$$

$$\frac{m_R}{2}v_g^2 = \frac{1}{2}m_R\, v_R^2$$

$$v_g = v_R$$

so red ball and green ball will have same type of motion.

option c,d

53. Two black bodies A and B are emitting in the approximate ratio 2:5. Which of the following statements may be correct from the given information?

(a) Body B is 20% hotter and 10% larger in diameter than body A.

(b) Body B is 50% hotter but half in diameter than body A.

(c) Body B is 10% hotter and 30% larger in diameter than body A.

(d) Body B has double the temperature of A and only 40% diameter of body A.

Ans. (a,c,d)

Sol. $\frac{dQ}{dt} \propto D^2T^4$ (D= diameter)

$$\frac{\left(\frac{dQ}{dt}\right)_A}{\left(\frac{dQ}{dt}\right)_B} = \frac{D_A^2 T_A^4}{D_B^2 T_B^4} = \frac{2}{5} = 0.4$$

option a,c,d

54. Read the two Statements, I and II, about a RLC serics circuit driven by an AC voltage source using an inductor having internal resistance r. Assume that the maximum amplitude of ac signal is vm and frequency is F:

I : when trequency F is cqual to the resonance frequency of the circuit, potential difference across the series combination of L and C has a non-zero amplitude.

II : when frequency F is equal to the resonance frequency of the circuit, current in the circuit has amplitude less than v_m/R.

Select correct statement(s) from the following :

(1) If statement I is correct, statement II has to be correct.
(2) If statement Il is false, statement I cannot be false.
(3) Statement II is false for most of the RLC series circuits with ac voltage source.
(4) Statement I is correct for most of the RLC series circuits with ac voltage source.

Ans. **(a,d)**

Sol. $z = (R + r) + j\,(x_L - x_C)$

at resonance $x_L = x_C$

but due to internal resistance there will be some potential drop across 'r'.

$$I_m = \frac{v_m}{z} = \frac{v_m}{R+r} < \frac{v_m}{R}$$

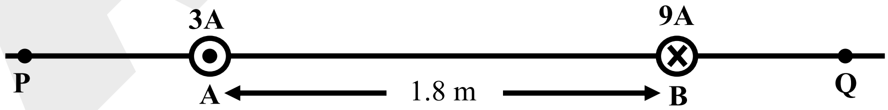
option (a) is correct.

option (d) is also correct.

if statement-II is false then r = 0: so statement-I will be false.

statement-II is false only for ideal RLC circuit. Ans. (a,d)

55. Two current carrying wires A and B are held fixed, parallel to each other, at a distance of 1.8 m. A current of 3A flows through wire A in a direction, coming out of the plane of paper and that through the wire B is 9A going in to the plane of paper as shown.



Which of the following statement(s) is/are true with reference to the given situation?

(a) There can be a point on the left of wire A (PA side) where net magnetic field is zero.
(b) There can be a point on the right of wire B(BQ side) where net magnetic field is zero.
(c) There can be a point between the wires A and B, closer to A, where magnetic field produced by the two wires will be in the same direction having equal magnitude.
(d) There can be a point Q on the right of wire B(BQ side) where net magnetic field is in the upward direction parallel to the plane of paper.

Ans. **(a,c)**

Sol. Magnetic fields due to A and B to the left of A will be cancelled out giving zero resultant.

To the right of B magnetic field due to B always greater than the magnetic field due to A so cannot be cancelled out and always directed downward parallel to paper.

Between A and B $\rightarrow$,

$$\frac{\mu_0 i_1}{2\pi d_1} = \frac{\mu_0 i_2}{2\pi d_2}$$

$$\frac{3}{d_1} = \frac{9}{d_2} \Rightarrow d_2 = 3d_1$$

$d_1 + d_2 = 1.8$

$4d_1 = 1.8$

$$d_1 = \frac{1.8}{4}$$

option (a,c)

56. Let $\delta = \begin{vmatrix} x & x^2 & 1 \\ 1 & x & x^2 \\ x^2 & 1 & x \end{vmatrix}$. Which of the following statements is <u>not</u> true?

(a) δ is an even function.

(b) $\delta = 0$ for all real x

(c) δ is a polynomial of degree 6

(d) If $x = \sqrt[3]{1}$, $\delta = 0$ for all roots

Ans. **(a, b)**

Sol. $\delta = \begin{vmatrix} x & x^2 & 1 \\ 1 & x & x^2 \\ x^2 & 1 & x \end{vmatrix}$

$\delta = x(x^2 - x^2) - x^2(x - x^4) + 1(1 - x^3)$

$\delta = 0 - x^3 + x^6 + 1 - x^3$

$\delta = x^6 - 2x^3 + 1$

$\delta = (x^3 - 1)^2$

$\because$ δ is a polynomial of degree 6

If $x = \sqrt[3]{1}$ then $\delta = 0$ for all roots.

Hence (a) & (b) are not true

57. Which of the following functions are periodic?

(a) sin x + cos x

(b) tan x

(c) x sin x

(d) sinx + sin$\sqrt{2}$x

Ans. **(a, b)**

Sol. (a) $f(x) = \sin x + \cos x$

Periodic with period of 2π.

(b) $f(x) = \tan x$

Periodic with period of π

(c) $f(x) = x \sin x$

Not periodic

(d) $f(x) = \sin x + \sin\sqrt{2}x$

Period of $\sin x = 2\pi$

Period of $\sin\sqrt{2}x = \frac{2\pi}{\sqrt{2}} = \sqrt{2}\pi$

LCM $(2\pi, \sqrt{2}\pi) \rightarrow$ Not exist

$f(x)$ = Not periodic

So option (a) & (b) are true.

58. If $x+\frac{1}{x}=2$ then which of the following are true?

(a) $x^2+\frac{1}{x^2}=2$ (b) $x^3+\frac{1}{x^3}=2$ (c) $x = 1$ (d) $x^4+\frac{1}{x^4}=2$

Ans. (a, b, c, d)

Sol. $x+\frac{1}{x}=2$ … (i)

Squaring on both side

$$\left(x+\frac{1}{x}\right)^2 = 2^2$$

$$x^2+\frac{1}{x^2}+2.x.\frac{1}{x}=4$$

$$x^2+\frac{1}{x^2}=2 \quad \text{… (ii)}$$

Cube on both side of equation (i)

$$\left(x+\frac{1}{x}\right)^3 = 2^3$$

$$x^3+\frac{1}{x^3}+3x.\frac{1}{x}\left(x+\frac{1}{x}\right)=8$$

$$x^3+\frac{1}{x^3}+6=8$$

$$x^3+\frac{1}{x^3}=2 \quad \text{… (iii)}$$

Squaring of equation (ii)

$$x^4+\frac{1}{x^4}+2=4$$

$$x^4+\frac{1}{x^4}=2 \quad \text{… (iv)}$$

From (i)

$x^2 + 1 = 2x$

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$x = 1$ … (v)

$\therefore$ a, b, c, d $\rightarrow$ all are true

59. Which of the following series are not convergent?

(a) $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)$ (b) $\sum_{n=1}^{\infty}\left(3+\left(\frac{2}{3}\right)^n\right)$ (c) $\sum_{n=1}^{\infty} n^{\frac{1}{n}}$ (d) $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$

Ans. (a,b,c)

Sol. (a) $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right) = (1+1+\ldots\ldots\infty\text{ (times)}) + \left(1+\frac{1}{2}+\ldots\infty\right)$

$=$ Not finite $\Rightarrow$ Not convergent

(b) $\sum_{n=1}^{\infty} 3+\left(\frac{2}{3}\right)^n$

$(3+3+\ldots.\infty) + \left(\frac{2}{3}\right) + \left(\frac{2}{3}\right)^2 + \ldots.\left(\frac{2}{3}\right)^{\infty}$

infinite $+ \dfrac{\frac{2}{3}}{1-\frac{2}{3}}$ = Not finite $\Rightarrow$ Not convergent

(c) $\sum_{n=1}^{\infty} n^{1/n}$

if we check graph of $y = x^{1/x}$

$\frac{dy}{dx} = x^{\frac{1}{x}}\left(\frac{1-\ell n x}{x^2}\right)$ it gives critical point at x = e

$\frac{d^2y}{dx^2} = x^{1/x}\left\{\frac{-3+2\ell x}{x^2}\right\} + x^{1/x}\left\{\frac{1-\ln x}{x^2}\right\}^2$

at $x = e \Rightarrow \frac{d^2y}{dx^2} < 0$

So at x = e it gives maximum value of $y = e^{1/e} \approx 1.44$

and at $x \rightarrow \infty$

$y = \lim_{x\to\infty} x^{1/x} = 1$

Hence we can say that it's all term lies between 1 to 1.44.

So sum of sequence is not finite.

Hence it is not convergent.

(d) $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}=\sum_{n=1}^{\infty}\frac{1}{n}-\frac{1}{n+1}$

$= \left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\ldots\ldots\left(\frac{1}{\infty}-\frac{1}{\infty+1}\right)$

= 1 (convergent)

so series in options a,b,c are not convergent.

60. Star A rises half an hour before star B and it sets half an hour after star B from a particular location. Which of the following statement(s) is/are correct?

(a) The location is on equator.

(b) The location is in southern hemisphere and star A is more south than star B.

(c) The location is in northern hemisphere and star A is more north than star B.

(d) Both stars have the same right ascension.

Ans. **(b,c,d)**

Sol. The difference in rise time and set time is same only if their right ascension is same. Star having same right ascension will rise earliar and set later if they are northwards in northern hemisphere and southward in southern hemisphere.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