ALLEN CAREER INSTITUTE
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KOTA (RAJASTHAN)

Pre Nurture & Career Foundation Division

For Class 6th to 10th, Olympiads & Board

ANSWER KEY (Paper Code : 62)

NATIONAL STANDARD EXAMINATION in PHYSICS

NSEP-2025 [23-11-2025]

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	c	b	c	b	a	b	c	b	d	d	c	c	d	c	c
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	a	d	d	b	d	b	a	d	b	d	c	a	c	b	a
Que.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans.	a or d	b	a	b	c	c	c	b	d	c	d	c	d	d	b
Que.	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	a	a	c	a,b,d	a,c,d	a,c	a,b,c	a,b,c,d	b,d	b,c	a,d	a,d	b,c,d	a,b,c	a,b,d

NA = Options are Not Correct

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NSEP-2025 (NSEP STAGE-I)

Date of examination : 23rd Nov, 2025

Date: 23/11/2025

PAPER CODE - 62

Max. Marks: 216 **Time allowed: 2 hours**

SOLUTIONS

1. Two electric charges, +q at the origin O (0, 0) and –2q at the point A (6, 0) are placed on x axis. The locus of the point P in x-y plane where the potential vanishes (V = 0) is

(a) a straight line perpendicular to x axis and passing through (2, 0)

(b) only the point (2, 0)

(c) a circle with center at (–2, 0) and radius 4

(d) an ellipse with foci at O and A

Allen Ans. (c)

Sol. $V_{(x,y)} = 0 = \frac{kq}{\sqrt{x^2+y^2}} - \frac{k2q}{\sqrt{(x-6)^2+y^2}}$

$\Rightarrow (x-6)^2 + y^2 = 4(x^2+y^2)$

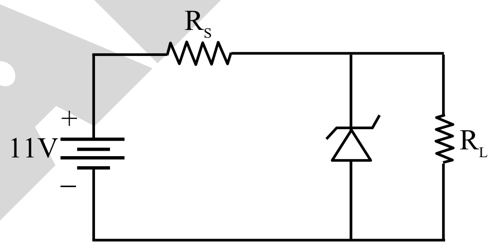
$\Rightarrow x^2 + y^2 + 36 - 12x = 4x^2 + 4y^2$

$\Rightarrow 3x^2 + 3y^2 + 12x - 36 = 0$

$\Rightarrow x^2 + y^2 + 4x - 12 = 0$

$\Rightarrow (x+2)^2 + y^2 = 16$ ……. Equation of circle.

2. In the circuit shown, the Zener diode is an ideal one with breakdown voltage of 5.0 volt. The values of the resistance are R_S = 10 kΩ and R_L = 1 kΩ. The current through the resistances, when the supply voltage is 11.0 V, is



(a) 0.6 mA through R_S and 5.0 mA through R_L

(b) 1.0 mA through R_S and 1.0 mA through R_L

(c) 1.1 mA through R_S and no current through R_L

(d) no current through R_S and 11 mA through R_L

Allen Ans. (b)

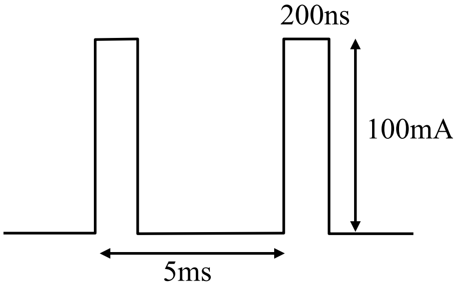
Sol. Let Zener doesn't breakdown

$\therefore\ i = \frac{11}{11}\text{mA} = 1\text{mA}$

$\therefore\ V_{Zener} = 1V < V_{breakdown}$

$\therefore$ Zener diode will behave like open switch & current in R_L = current in R_S = 1mA

3. In an accelerator the electrons are accelerated up to an energy of 50 MeV. The electrons do not emerge continuously from the accelerator rather they come in pulses at time interval of 5.0 milliseconds. Each pulse has a much shorter duration of 200 nanoseconds. Electron current during the pulse is 100 mA, while the current is zero between the two successive pulses (see figure), then



(a) the average current per pulse is 4 mA

(b) the peak value of power delivered by the electron beam is 50 MW

(c) the average power delivered by the electron beam is 200 W

(d) the average power delivered by the electron beam is 2 MW

Allen Ans. (c)

Sol. $V_{acc} = 50$ MV

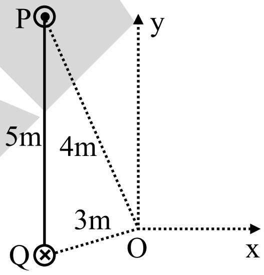
i = 100 mA during the time in which e^- emerges.

$$\therefore i_{avg} = \frac{\Delta Q}{\Delta t} = \frac{200\times10^{-9}\times100\times10^{-3}}{5\times10^{-3}} = 4\mu A$$

$P_{avg} = V_{i\,avg} = 50 \times 10^6 \times 4 \times 10^{-6} = 200$ W

$P_{peak} = V_{i\,max} = 50 \times 10^6 \times 100 \times 10^{-3} = 5$ MW

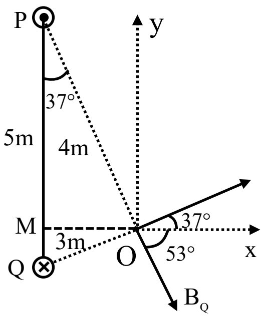
4. Two infinitely long straight parallel wires perpendicular to the plane of the paper are 5 m apart. One of the wires, P carries current I out of the plane of the paper and the other, Q carries the current I into the plane of paper. The magnetic field B at the origin O of the coordinate system with x and y axes as perpendicular and parallel to PQ, respectively, is [Given OP = 4 m and OQ = 3m]



(a) $\frac{\mu_0 I}{2\pi}\left(\hat{i}-\frac{3}{5}\hat{j}\right)$ (b) $\frac{\mu_0 I}{5\pi}\left(\hat{i}-\frac{7}{24}\hat{j}\right)$ (c) $\frac{\mu_0 I}{5\pi}\left(-\hat{i}+\frac{3}{8}\hat{j}\right)$ (d) $\frac{\mu_0 I}{24\pi}\left(2\hat{i}+3\hat{j}\right)$

Allen Ans. (b)

Sol.



$$PM = \frac{16}{5}\text{m} \;\; \& \;\; QM = \frac{9}{5}\text{m}$$

$$\vec{B}_P = \frac{\mu_0 I}{8\pi}\left[\frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}\right]$$

$$\vec{B}_Q = \frac{\mu_0 I}{6\pi}\left[\frac{3}{5}\hat{i} - \frac{4}{5}\hat{j}\right]$$

$$\therefore \vec{B}_{net} = \frac{\mu_0 I}{\pi}\left[\frac{1}{10}\hat{i} + \frac{3}{40}\hat{j} + \frac{1}{10}\hat{i} - \frac{2}{15}\hat{j}\right]$$

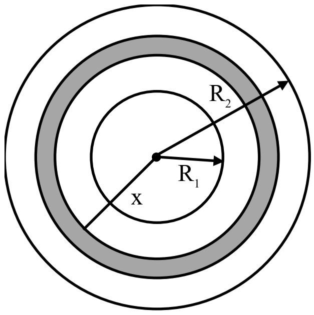
$$= \frac{\mu_0 I}{\pi}\left[\frac{1}{5}\hat{i} - \frac{7}{120}\hat{j}\right] = \frac{\mu_0 I}{5\pi}\left[\hat{i} - \frac{7}{24}\hat{j}\right]$$

5. Charge q is uniformly distributed over the surface of a thin non-conducting annular disc of inner radius R_1 and outer radius R_2. The disc is made to rotate with constant frequency f, about an axis passing through the center of the annular disc and perpendicular to its plane. The magnetic moment of the disc is

(a) $\pi fq\frac{R_2^2 + R_1^2}{2}$ (b) $\pi fq\frac{R_2^2 - R_1^2}{2}$ (c) $\pi fq\frac{R_2^2 - R_1^2}{4}$ (d) $2\pi fq\left(R_2^2 - R_1^2\right)$

Allen Ans. (a)

Sol.



$$\sigma = \frac{q}{\pi\left(R_2^2 - R_1^2\right)}$$

For a small ring of radius x and width 'dx'

dM = IA

= fdq A

dM = f σdA × πx^2

$$= f\frac{q}{\pi\left(R_2^2 - R_1^2\right)} \times 2\pi x dx \cdot \pi x^2$$

$$= \frac{2\pi fq}{R_2^2 - R_1^2}\int_{R_1}^{R_2} x^3 dx$$

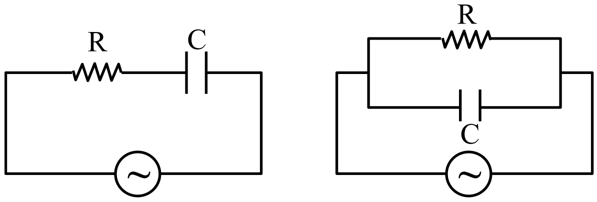
$$= \frac{2\pi fq}{R_2^2 - R_1^2} \times \left(\frac{R_2^4}{4} - \frac{R_1^4}{4}\right)$$

$$= \frac{2\pi fq\left(R_2^2 + R_1^2\right)}{4\left(R_2^2 - R_1^2\right)}\left(R_2^2 - R_1^2\right)$$

$$= \frac{\pi fq\left(R_2^2 + R_1^2\right)}{2}$$

(a) is correct

6. For a resistance R and capacitance C in series, the impedance is twice that of a parallel combination of the same elements when used with an AC voltage frequency f. The frequency f of the applied emf is



(a) $f = 2\pi RC$ (b) $f = \frac{1}{2\pi RC}$ (c) $f = \frac{2\pi}{RC}$ (d) $f = \frac{1}{2\pi\sqrt{R^2 + C^2}}$

Allen Ans. (b)

Sol. in series

$$Z_s = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \qquad \text{..... (i)}$$

In parallel

$$\frac{1}{Z_p} = \sqrt{\frac{1}{R^2} + (\omega C)^2} \qquad \text{..... (i)}$$

here $Z_S = 2Z_P$

$$R^2 + \frac{1}{4\pi f^2 C^2} = 4\frac{1}{\left(\frac{1}{R^2} + 4\pi^2 f^2 C^2\right)}$$

$$\left(R^2 + \frac{1}{4\pi^2 f^2 C^2}\right)\left(\frac{1}{R^2} + 4\pi^2 f^2 C^2\right) = 4$$

$$1 + 1 + 4\pi f^2 R^2 C^2 + \frac{1}{4\pi^2 f^2 C^2 R^2} = 4$$

$$4\pi f^2 R^2 C^2 + \frac{1}{4\pi^2 f^2 C^2 R^2} = 2$$

On solving

$4\pi^2 f^2 R^2 C^2 = 1$

$$f = \frac{1}{2\pi RC}$$

(b) is correct.

7. In an experiment on photoelectric effect on a metal surface, one finds a stopping potential of 1.8 V for the wavelength of 300 nm and a stopping potential of 0.9 V for the wavelength of 400 nm. The cutoff wavelength λ_0 (the maximum wavelength that can produce photoelectric effect) for the metal is

(a) 500 nm (b) 550 nm (c) 600 nm (d) 750 nm

Allen Ans. (c)

Sol. from $eV_0 = hC\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$

$$1.8eV = hC\left(\frac{1}{300nm} - \frac{1}{\lambda_0}\right) \qquad \text{... (i)}$$

$$0.9eV = hC\left(\frac{1}{400nm} - \frac{1}{\lambda_0}\right) \qquad \text{...(ii)}$$

from (i) & (ii)

$$2\left(\frac{1}{400\text{nm}}-\frac{1}{\lambda_0}\right)=\frac{1}{300\text{nm}}-\frac{1}{\lambda_0}$$

$$\frac{1}{200\text{nm}}-\frac{1}{300\text{nm}}=\frac{2}{\lambda_0}-\frac{1}{\lambda_0}$$

$$\frac{3-2}{600}=\frac{1}{\lambda_0}$$

λ_0 = 600 nm

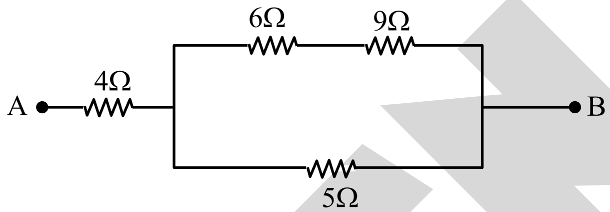
(c) is correct

8. Given that the power dissipated in 5Ω resistance is 7.2 W in the circuit shown.

Statement (1) : Power dissipated in 6Ω resistance is 6W.

Statement (2) : Potential difference V_{AB} between A and B is V_{AB} = 12.4 V

Then



(a) Statement (1) is correct but statement (2) is wrong

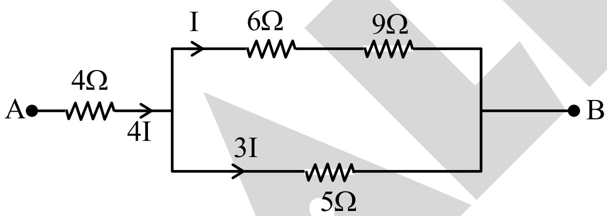
(b) Statement (1) is wrong but statement (2) is correct

(c) Both statements (1) and (2) are wrong

(d) Both statements (1) and (2) are correct

Allen Ans. (b)

Sol.



in parallel $I \propto \frac{1}{R}$ so current

in 5Ω will be 3 times that of current in 15 Ω.

$P_{5\Omega} = (3I)^2 \times 5$

$7.2 = 9 \times 5 \times I^2$

$$I^2=\frac{7.2}{9\times5}=\frac{0.8}{5}=0.16$$

I = 0.4 A

$P_{6\Omega} = I^2 \times 6\Omega = 0.16 \times 6 = 0.96$ W

Statement (1) is wrong.

$V_{AB} = 4 \times 4I + 5 \times 3I$

= 31 I

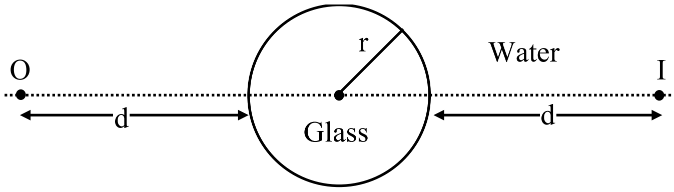
= 31 × 0.4 V

= 12.4 Volt

Statement (2) is correct

(b) is correct.

9. A transparent and homogeneous sphere of glass of radius r is immersed in water (refractive indices of glass and water being ${}_a\mu_g = \frac{3}{2}$ and ${}_a\mu_w = \frac{4}{3}$). The image of a point object O, located at distance d on its axis in front of the sphere, is formed at point I at the same distance d from the sphere on the opposite side as shown.

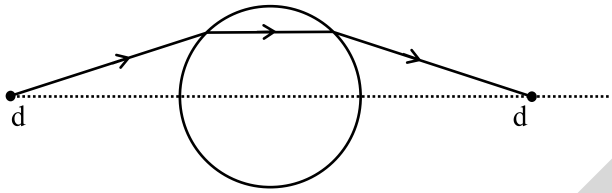


The distance d is equal to

(a) 2r (b) 3r (c) 6r (d) 8r

Allen Ans. (d)

Sol. By symmetry



$$\frac{\mu_r}{v} - \frac{u_i}{v} = \frac{\mu_r - \mu_i}{R}$$

$$\frac{3}{2(\infty)} - \frac{4}{3(-d)} = \frac{\frac{3}{2} - \frac{4}{3}}{r}$$

$$\frac{4}{3d} = \frac{1}{6r}$$

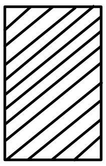
d = 8r

option (d)

10. A certain substance, with a dielectric constant k = 2.5 and the dielectric strength E = 1.8×10^7 N/C, completely fills the space between the plates of a parallel plate capacitor (with circular plates) of capacitance C = 72.0 nF. The minimum diameter of the circular plates, to ensure that the capacitor can withstand a potential difference of V = 4.0 kV, is

(a) 12 cm (b) 24 cm (c) 48 cm (d) 96 cm

Allen Ans. (d)

Sol. 

k = 2.5

E = 1.8×10^7

C = 72 nF

V = 4000 V

$$E = \frac{V}{d}$$

$$d = \frac{4000}{1.8 \times 10^7}$$

$$\frac{k \in_0 A}{d} = 72 \times 10^{-9}$$

$$\frac{k4\pi \in_0 A}{4\pi d} = 72 \times 10^{-9}$$

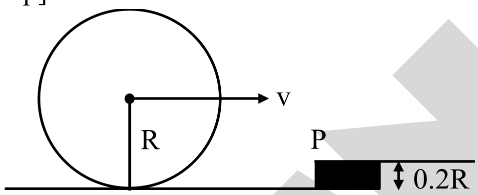
$$\frac{2.5 \times A \times 1.8 \times 10^{7}}{9 \times 10^{9} \times 4\pi \times 4000} = 72 \times 10^{-9}$$

$$\frac{\pi d^2}{4} = \frac{9 \times 4\pi \times 4 \times 4 \times 10^{-3}}{2.5}$$

d = 96 cm

option (d)

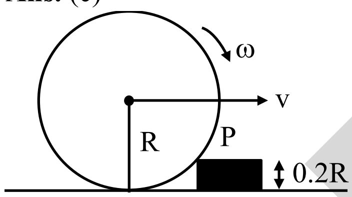
11. A uniform solid sphere of radius R rolls without slipping on a rough horizontal surface with a forward velocity v of its center. On its way, it suddenly encounters a small step of height 0.2 R as shown. The angular velocity of the sphere just after the impact is [given that the sphere does not bounce back, rather it goes ahead up the step]



(a) $\frac{v}{7R}$ (b) $\frac{3}{7}\frac{v}{R}$ (c) $\frac{6v}{7R}$ (d) $\frac{v}{R}$

Allen Ans. (c)

Sol.



About P

$L_i = L_F$

$$MV(0.8R) + \frac{2}{5}MR^2\left(\frac{V}{R}\right) = \left(\frac{2}{5}MR^2 + MR^2\right)\omega$$

$0.8\ VR + 0.4\ VR = 1.4\ R^2\omega$

$$\frac{1.2V}{1.4R} = \omega$$

$$\omega = \frac{6V}{7R}$$

option (c)

12. The magnetic field (B) produced by the current i flowing through the sides of a square loop of side ℓ, at a point P at distance x from the center of the square, on the axis perpendicular to the plane of the square loop and passing through its center, is

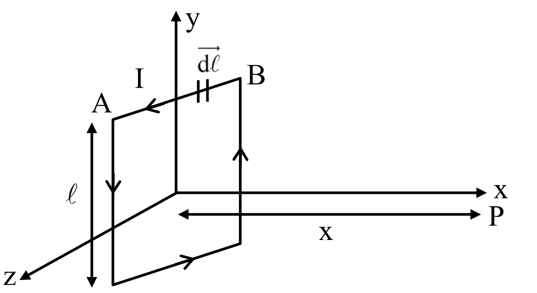
(a) $B = \frac{\mu_0 i}{4\pi}\frac{2\sqrt{2}\ell^2}{(4x^2+\ell^2)\sqrt{2x^2+\ell^2}}$

(b) $B = \frac{\mu_0 i}{4\pi}\frac{4\sqrt{2}\ell x}{(x^2+\ell^2)\sqrt{2x^2+\ell^2}}$

(c) $B = \frac{\mu_0 i}{4\pi}\frac{4 \times 2\sqrt{2}\ell^2}{(4x^2+\ell^2)\sqrt{2x^2+\ell^2}}$

(d) $B = \frac{\mu_0 i}{4\pi}\frac{4\sqrt{2}\ell x}{(4x^2+\ell^2)\sqrt{x^2+\ell^2}}$

Allen Ans. (c)

Sol. 

B due to AB

$$dB = \frac{\mu_0 I}{4\pi}\frac{\vec{d\ell}\times\vec{r}}{r^3}$$

$$\vec{d\ell} = dz\hat{k}$$

Position of element $(0, \frac{\ell}{2}, z)$

$$\vec{r} = \vec{r}_p - \vec{r}_{element}$$

$$\vec{r} = x\hat{i} - \left(\frac{\ell}{2}\hat{j} + z\hat{k}\right)$$

$$\vec{r} = x\hat{i} - \frac{\ell}{2}\hat{j} - z\hat{k}$$

$$|\vec{r}| = \sqrt{x^2 + \frac{\ell^2}{4} + z^2}$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{dz\hat{k}\times\left(x\hat{i} - \frac{\ell}{2}\hat{j} - z\hat{k}\right)}{\left(x^2 + \frac{\ell^2}{4} + z^2\right)^{\frac{3}{4}}}$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{\left[xdz\hat{j} + \frac{\ell}{2}dz\hat{i}\right]}{\left(x^2 + \frac{\ell^2}{4} + z^2\right)^{\frac{3}{4}}}$$

Taking x-component

$$|dB|_x = \frac{\mu_0 I}{4\pi}\frac{\ell}{2}\frac{dz}{\left(x^2 + \frac{\ell^2}{4} + z^2\right)^{\frac{3}{4}}}$$

$$B_x = \frac{\mu_0 I\ell}{8\pi}\int_{-\frac{\ell}{2}}^{\frac{\ell}{2}}\frac{dz}{\left(x^2 + \frac{\ell^2}{4} + z^2\right)^{\frac{3}{4}}}$$

Consider $x^2 + \frac{\ell^2}{4} = r^2$

$$B_x = \frac{\mu_0 I\ell}{8\pi}\int_{-\frac{\ell}{2}}^{\frac{\ell}{2}}\frac{dz}{\left(r^2+z^2\right)^{\frac{3}{4}}}$$

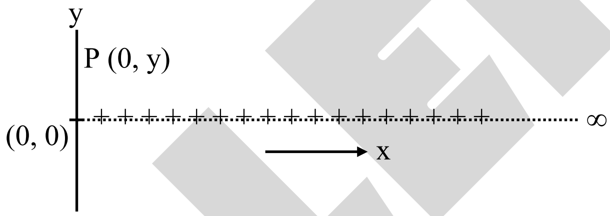
$$B_x = \frac{\mu_0 I\ell^2}{8\pi\left(x^2+\frac{\ell^2}{4}\right)\sqrt{x^2+\frac{\ell^2}{2}}}$$

$$\text{Net } B = 4B_x = \frac{4\times 4\mu_0 I\ell^2\times\sqrt{2}}{8\pi\left(4x^2+\ell^2\right)\sqrt{2x^2+\ell^2}}$$

$$B = \frac{8\sqrt{2}\mu_0 I\ell^2}{4\pi\left(4x^2+\ell^2\right)\sqrt{2x^2+\ell^2}}$$

Option (c)

13. A linear positive charge distribution, with linear charge density λ coulomb per meter, extends along +x-axis from $x = 0$ to $x = \infty$.



The electric field $\vec{E}$ at any point (0, y) on the y-axis

(a) is proportional to $\frac{\lambda}{y^2}$ irrespective of whether y is positive or negative.

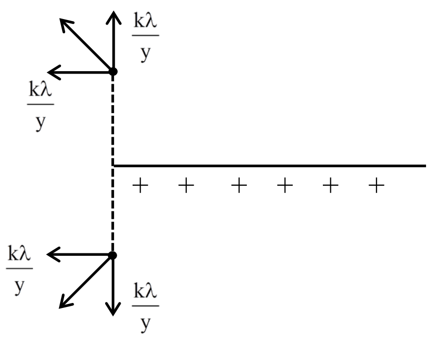
(b) is always directed away and perpendicular to the line of charge.

(c) has a vanishing component parallel to the line of charge.

(d) is directed along a straight line of slope $m = -1$ if y is positive but along a line of slope $m = +1$ if y is negative.

Allen Ans. (d)

Sol.



Option (d)

14. Imagine a situation, in which an infinite sheet with positive charge $+\sigma$ per unit area lies in the xy-plane and a second infinite sheet with negative charge $-\sigma$ per unit area lies in the yz-plane. The net electric field E at any point (x, y, z) [that does not lie on either of these planes xy or yz] can be expressed as

(a) $\vec{E}=\frac{\sigma}{2\in_0}\left(-\hat{i}+\hat{k}\right)$

(b) $\vec{E}=\frac{\sigma}{2\in_0}\hat{j}$

(c) $\vec{E}=\frac{\sigma}{2\in_0}\left[-\frac{x}{|x|}\hat{i}+\frac{z}{|z|}\hat{k}\right]$

(d) $\vec{E}=\frac{\sigma}{\in_0}\left[\frac{x}{|x|}\hat{i}-\frac{z}{|z|}\hat{k}\right]$

Allen Ans. (c)

Sol. E due to ∞ sheet $=\frac{\sigma}{2\in_0}$

due to $+\sigma$: $\vec{E}=\frac{\sigma}{2\in_0}\frac{z}{|z|}\hat{k}$

due to $-\sigma$: $\vec{E}=\frac{-\sigma}{2\in_0}\frac{x}{|x|}\hat{i}$

option (c)

15. For the electric field E, in a region of space where a non-uniform, but spherically symmetric distribution of charge has a charge density $\rho(r)$ as

$$\begin{aligned}&\rho(r)=\rho_0\left(1-\frac{r}{R}\right) && \text{for} \quad r\le R\\ &\rho(r)=0 && \text{for} \quad r\ge R\end{aligned}$$

one can say that

(a) E = 0 : both at r = 0 and r = R

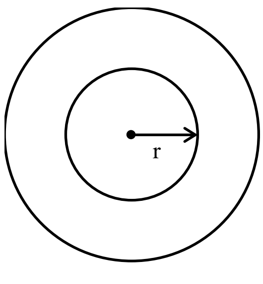
(b) $E\propto r$ for $r<R$ and $E\propto\frac{1}{r^2}$ for $r\ge R$

(c) the magnitude of E increases with r and reaches its maximum at $r=\frac{2R}{3}$

(d) the maximum electric field produced by the given charge distribution is $E_{max}=\frac{\rho_0 R}{3\in_0}$

Allen Ans. (c)

Sol.



$$E_{in}4\pi r^2=\frac{\int_0^r\rho_0\left(1-\frac{r}{R}\right)4\pi r^2dr}{\varepsilon_0}$$

$$E_{in}=\frac{\rho_0}{\varepsilon_0 r^2}\left[\frac{r^3}{3}-\frac{r^4}{4R}\right]$$

$$E_{in}=\frac{\rho}{\varepsilon_0}\left[\frac{r}{3}-\frac{r^2}{4R}\right]$$

$$\frac{dE}{dR}=0 \Rightarrow \frac{1}{3}-\frac{r}{2R} \Rightarrow r=\frac{2R}{3}$$

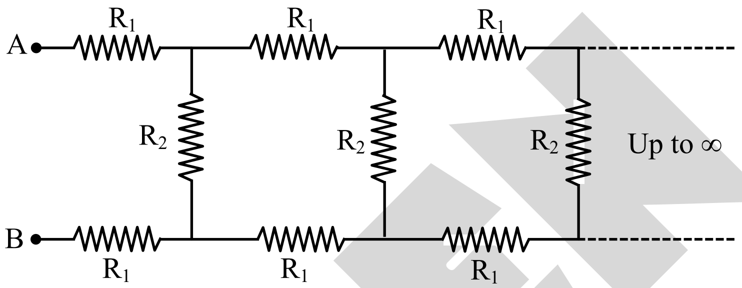
$$\frac{d^2E}{dR^2}<0 \text{ at } r=\frac{2R}{3}$$

$$E_{max}=\frac{\rho}{\varepsilon_0}\left[\frac{2R}{9}-\frac{4R^2}{9\times 4R}\right]$$

$$\frac{\rho}{\varepsilon_0}\frac{R}{9}$$

$\therefore$ option (c)

16. A typical network of resistance R_1 and R_2 shown below extends to infinity towards the right. The total resistance $R_{effective}$ of this network between points A and B is :



(a) $R_{effective}=R_1+\sqrt{R_1^2+2R_1R_2}$

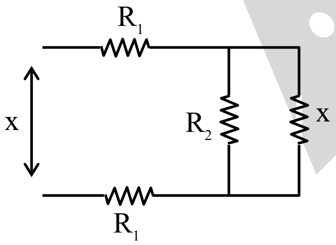
(b) $R_{effective}=R_2+\sqrt{R_1^2+2R_1R_2}$

(c) $R_{effective}=R_1+\sqrt{3R_1R_2}$

(d) $R_{effective}=R_1+\sqrt{R_2^2+2R_1R_2}$

Allen Ans. (a)

Sol.



$$\frac{R_2x}{R_2+x}+2R_1=x$$

$\Rightarrow R_2x + 2R_1R_2 + 2R_1x = R_2x + x^2$

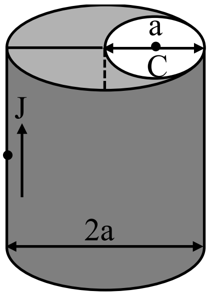
$\Rightarrow x^2 - 2R_1x - 2R_1R_2 = 0$

$$\Rightarrow x=\frac{2R_1\pm\sqrt{4R_1^2+8R_1R_2}}{2}$$

$$x=R_1+\sqrt{R_1^2+2R_1R_2}$$

option (a)

17. A cylindrical cavity of diameter 'a' exists inside a long solid cylinder of diameter '2a' as shown in figure. Both the cylinder and the cavity are taken to be infinitely long. The axis of the cavity is parallel to the axis of the cylinder and is at distance $\frac{a}{2}$ from it. A uniform current of current density J (Am^{-2}) flows through the cylinder along its length and not through the cavity. The magnitude of the magnetic field at a point P on the surface of the cylinder lying farther from the axis of the cavity, is :



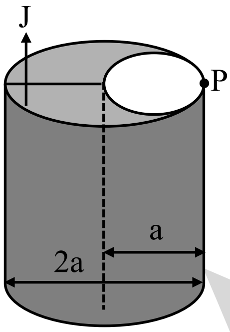
(a) $B = \frac{3}{8}\frac{\mu_0 J}{a}$

(b) $B = \frac{3}{4}\mu_0 Ja$

(c) $B = \frac{3}{8}\mu_0 Ja$

(d) $B = \frac{5}{12}\mu_0 Ja$

Allen Ans. (d)



Sol.

We can treat cavity as superposition of a full solid cylinder carrying current density as +J and a smaller cylinder (the cavity) carrying current density –J.

$$(\vec{B}_{net})_{\text{at point P}} = \vec{B}_{\text{Big cylinder}} + \vec{B}_{\text{small cylinder}}$$

$$= \frac{\mu_0 I_1}{2\pi R} + \frac{\mu_0 I_2}{2\pi r'}$$

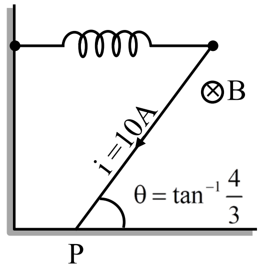
$$= \frac{\mu_0 J\pi a^2}{2\pi a} - \frac{\mu_0 J\left(\frac{\pi a^2}{4}\right)}{2\pi\left(\frac{3a}{2}\right)}$$

$$= \frac{\mu_0 Ja}{2} - \frac{\mu_0 Ja}{12}$$

$$= \frac{5\mu_0 Ja}{12}$$

Option (d) is correct answer

18. A thin uniform rod, length ℓ = 0.200m with negligible mass, is attached to the floor by a frictionless hinge at a fixed point P. A horizontal spring connects the other end of the rod to a vertical wall. The rod is in a uniform magnetic field B = 0.500 tesla directed into the plane of paper. There is a current i = 10.0 A in the rod in the direction shown. Force constant of the spring is 5 N/m. The rod is in equilibrium at $\theta = \tan^{-1}\frac{4}{3}$.



Statement-1: Torque on the rod due to magnetic force is 0.1 Nm clockwise.

Statement-2 : In equilibrium the energy stored in the spring is 0.039 J.

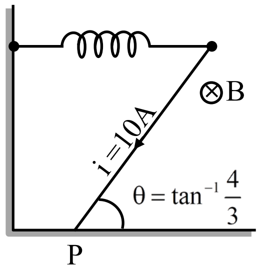
(a) Statement-1 is correct but statement-2 is wrong.

(b) Statement-1 is wrong but statement-2 is correct.

(c) Both statement-1 and statement-2 are wrong.

(d) Both statement-1 and statement-2 are correct.

Allen Ans. (d)



Sol.

(a) Magnetic force on rod

$F_m = I\left(\vec{\ell} \times \vec{B}\right)$ = 10 × 0.2 × 0.5 sin90°

= 1 Newton

Torque due to magnetic field

= (Fm) at centre of mass of rod × $r_\perp$

$= 1 \times \frac{L}{2} = 1 \times \frac{0.2}{2}$

= 0.1 Nm

clockwise

(b) Toque due to spring

$T_{spring} = (F_s)L\sin\theta$

$= kx \times 0.2 \times \frac{4}{5}$

Torque of spring must become torque due to magnetic field

$kx \times 0.2 \times \frac{4}{5} = 0.1$

$$x = \frac{0.1\times5}{4\times0.2\times5} = \frac{1}{8} = 0.125\text{m}$$

$\therefore$ Energy stored in spring = $\frac{1}{2}kx^2$

$$= \frac{1}{2}\times5\times(0.125)^2$$

= 0.039 Joule

(d) option is correct

Both statement (1) and (2) are correct

19. The electric flux though a certain of a dielectric medium is $\phi = (8.00 \times 10^3)\ t^4$ in SI units. The displacement current through that area is 12.5 pA at a time t = 20 ms. The dielectric constant of the dielectric medium is :

(a) 22.1 (b) 5.52 (c) 55.2 (d) 2.76

Allen Ans. (b)

Sol. From formula

$$I_d = \frac{\epsilon_0 \epsilon_r\, d\phi_E}{dt}$$

$$\because \frac{d\phi_E}{dt} = \frac{d}{dt}\left(8\times10^3\times t^4\right)$$

$= 32000\ t^3$

At t = 20×10^{-3} second

$$\frac{d\phi_E}{dt} = 32000\times\left(20\times10^{-3}\right)^3 = 0.256$$

Using all the values in formula of displacement current

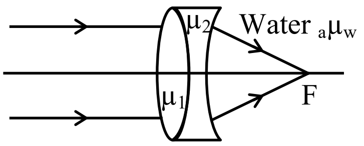
$$I_d = \epsilon_0 \epsilon_r \frac{d\phi_E}{dt}$$

$12.5 \times 10^{-12} = (8.85 \times 10^{-12})\ \epsilon_r \times 0.256$

$\epsilon_r = 5.52$

Correct option is (b) 5.52

20. A thin equi-convex lens of flint glass (refractive index μ_1) is kept coaxially in contact with another thin equi-concave lens of crown glass (refractive index μ_2). The system is completely immersed in water $\left({}_a\mu_W = \frac{4}{3}\right)$.

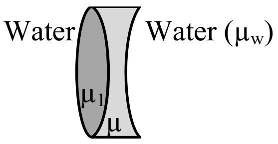


Parallel rays of light incident parallel to the principal axis in water are focused by this system at a distance of 24 cm beyond the system. The thickness of the system is negligible. If the radius of curvature of each surface is R = 20 cm, the difference $(\mu_1 - \mu_2)$ is :

(a) $\frac{2}{9}$ (b) $\frac{3}{9}$ (c) $\frac{4}{9}$ (d) $\frac{5}{9}$

Allen Ans. (d)

Sol.



For equiconvex lens

$$\frac{1}{f_1}=\left(\frac{\mu_1}{\mu_\omega}-1\right)\left(\frac{2}{R}\right)$$

For equiconcave lens

$$\frac{1}{f_2}=\left(\frac{\mu_2}{\mu_\omega}-1\right)\left(-\frac{2}{R}\right)$$

Combined focal length

$$\frac{1}{f_{eq}}=\frac{1}{f_1}+\frac{1}{f_2}$$

$$=\frac{2}{R}\left[\left(\frac{\mu_1}{\mu_w}-1\right)-\left(\frac{\mu_2}{\mu_\omega}-1\right)\right]$$

$$\frac{1}{f_{eq}}=\frac{2}{R\mu_\omega}(\mu_1-\mu_2)$$

$$\frac{1}{24}=\frac{2}{20\times\left(\frac{4}{3}\right)}(\mu_1-\mu_2)$$

$$\mu_1-\mu_2=\frac{80}{144}=\frac{5}{9}$$

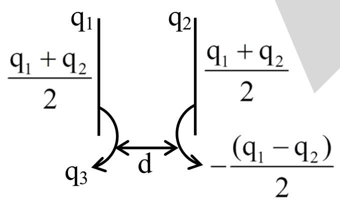
Answer is option (d)

21. Two identical large thin metal plates carrying charges $+q_1$ and $+q_2$ ($q_1 > q_2$), respectively, are kept close at a distance d apart and parallel to each other to form a parallel plate capacitor of capacitance C. The potential difference between the plates is :

(a) $\frac{q_1-q_2}{C}$ (b) $\frac{q_1-q_2}{2C}$ (c) $\frac{q_1-q_2}{4C}$ (d) $\frac{q_1+q_2}{2C}$

Allen Ans. (b)

Sol.



$$q_3=q_1-\frac{q_1+q_2}{2}=\left(\frac{q_1-q_2}{2}\right)$$

$$Q = CV$$

$$V=\frac{Q}{C}$$

$$V=\left(\frac{q_1-q_2}{2}\right)\times\frac{1}{C}$$

$$V=\frac{q_1-q_2}{2C}$$

22. A point mass m moves in a straight line under a retardation kv^2 [where k is a positive constant and v is the instantaneous velocity]. The initial velocity of the point mass is u. The displacement of the point mass at time t is :

(a) $\frac{1}{k}\ell n(1+kut)$ (b) $\frac{1}{k}\ell n\ kut$ (c) $k\ell n\ kut$ (d) $\frac{1}{k}\ell n\ (1-kut)$

Allen Ans. (a)

Sol. $a = -kv^2$

$$\frac{dv}{dt} = -kv^2$$

$$\int_u^v \frac{dv}{v^2} = -k\int_0^t dt$$

$$-\frac{1}{v}\Big|_0^v = -kt$$

$$\frac{1}{v} - \frac{1}{u} = kt$$

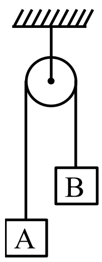
$$\frac{1}{v} = \frac{1}{u} + kt \qquad v = \frac{u}{1+ukt}$$

$$\int_0^x dx = \int_0^t \frac{u}{1+ukt}dt$$

$$x = u\frac{\ell n(1+ukt)\big|_0^t}{uk}$$

$$x = \frac{1}{k}\ln(1+ukt)$$

23. In the arrangement shown in figure, 'a' represent the magnitude of acceleration of small blocks A and B while 'T' is the tension in the massless string passing over the frictionless and massless pulley. The sum of the masses of blocks A and B is constant. For this system, a linear relationship can be obtained between



(a) a and $\frac{1}{T}$ (b) a and T (c) a and T^2 (d) T and a^2

Allen Ans. (d)

Sol. $a = \frac{(m_A - m_B)g}{(m_A + m_B)}$ $\qquad T = \frac{2m_A m_B g}{(m_A + m_B)}$

$$a^2 = \frac{g^2}{(m_A + m_B)^2} \times (m_A^2 + m_B^2 - 2m_A m_B)$$

$$a^2 = \frac{(m_A^2 + m_B^2)g^2}{(m_A + m_B)^2} - \frac{2m_A m_B (g)(g)}{(m_A + m_B)(m_A + m_B)}$$

$$a^2 = \frac{[(m_A^2 + m_B^2) - 2m_A m_B + 2m_A m_B]g^2}{(m_A + m_B)^2} - \frac{Tg}{(m_A + m_B)}$$

$$a^2 = \frac{[(m_A^2 + m_B^2) - 2m_A m_B]g^2}{(m_A + m_B)^2} - \frac{Tg}{(m_A + m_B)}$$

$$a^2 = \left[1 - \frac{2m_A m_B}{(m_A + m_B)^2}\right]g^2 - \frac{Tg}{(m_A + m_B)}$$

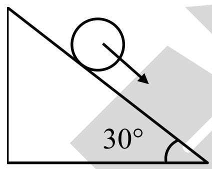
$$a^2 = g^2 - \frac{2m_A m_B g^2}{(m_A + m_B)^2} - \frac{Tg}{(m_A + m_B)}$$

$$a^2 = g^2 - \left[\frac{2g}{(m_A + m_B)^2}\right]T$$

$Y = C - mx$

Linear relation between a^2 & T

24. A thin uniform circular ring of mass m is rolling without slipping down an inclined plane of inclination 30° with the horizontal. The coefficient of friction between the ring and the surface is μ. The correct statement is :



(a) linear acceleration of the centre of the ring along the plane is $a = \frac{g}{2}$.

(b) force of friction between the ring and the inclined plane is $F_{friction} = \frac{mg}{4}$.

(c) the ring keeps rolling for all values of the coefficient of friction $\mu \geq \frac{1}{4}$.

(d) linear acceleration of the center of the ring along the plane is $a = \frac{g}{3}$.

Allen Ans. (b)

Sol. $a = \frac{g\sin\theta}{1 + \frac{I}{mR^2}}$

$$a = \frac{g\left(\frac{1}{2}\right)}{1 + \frac{mR^2}{mR^2}} = \frac{g}{4}$$

$$f = \frac{mg\sin\theta}{1 + \frac{mR^2}{I}}$$

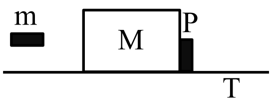
$$f = \frac{mg\left(\frac{1}{2}\right)}{1+1}$$

$$f = \frac{mg}{4} \rightarrow \text{ans (b)}$$

$$\frac{mg}{4} \le \mu mg\frac{\sqrt{3}}{2}$$

$$\mu_{min} \ge \frac{1}{2\sqrt{3}}$$

25. A bullet of mas m can penetrate a target (a heavy block of mass M) up to a distance S, when the target M is held stationary by a stopper P (shown in figure). Up to what distance S' the bullet will penetrate if the block of mass M is free to move (i.e. when the stopper P is removed) on the frictionless surface T.



(a) $S' = S$ (b) $S' = \frac{m}{M}S$ (c) $S' = \frac{m}{m+M}S$ (d) $S' = \frac{M}{M+m}S$

Allen Ans. (d)

Sol. Case-1 : When block is fixed

$\omega_{Net} = \Delta KE$

$\omega_{Res} = K_F - K_I$

$$-F_R.S = O - \frac{1}{2}mv_0^2$$

$$F_R = \frac{1}{2}\frac{mv_0^2}{S} \qquad \ldots(i)$$

F_R = Constant resistance f offered by block.

V_0 = Initial speed of bullet

Case-2 : When block is free to move

Stepe-1 : Linear momentum conservation of bullet + block system, as no consnntant force is acting

$$mv_0 = (m + M)v \Rightarrow v = \frac{mv_0}{m+M} \qquad \ldots(ii)$$

Step-2 : Work energy theorem

$W_{net} = k_F - K_I$

$$-F_R.S' = \frac{1}{2}(m+M)v^2 - \frac{1}{2}mv_0^2 \quad \ldots(iii)$$

On solving equation (i), (ii) and (iii)

$$-F_R.S' = \frac{1}{2}(m+M)\left[\frac{mv_0}{m+M}\right]^2 - \frac{1}{2}mv_0^2$$

$$-\frac{1}{2}\frac{mv_0^2}{S}.S' = \frac{1}{2}\frac{m^2v_0^2}{m+M} - \frac{1}{2}mv_0^2$$

$$-\frac{S'}{S} = \frac{m}{m+M} - 1$$

$$\Rightarrow \frac{S'}{S} = 1 - \frac{m}{m+M} = \frac{M}{m+M}$$

$$\Rightarrow S' = \frac{Ms}{m+M}$$

26. Knowing that the atomic masses of Al and Mg are respectively ${}^{25}_{13}AI$ = 24.990432 u and ${}^{25}_{12}Mg$ = 24.985839 u while electron mass is often expressed as m_e = 0.511 MeV, the Q value (energy liberated) of the β decay nuclear reaction ${}^{25}Al \rightarrow {}^{25}Mg + e^+ + \nu$ in MeV is :

(a) 4.278 (b) 3.767 (c) 3.256 (d) 931.478

Allen Ans. (c)

Sol. B

For β+ decay process

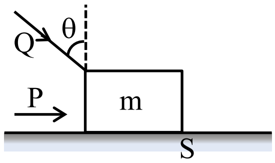
$Q = [m_{parent} - m_{daughter} - 2m_e] \times 931.5$ MeV

$Q = [m_{parent} - m_{daughter}] \times 931.5 - 2\,[m_e \times 931.5$ MeV]

$Q = [24.990432 - 24.985839] \times 931.5 - 2\,[0.511]$

$Q = 3.256$ MeV

27. A block of mass m, lying on a rough horizontal plane, is acted upon by a horizontal force P and and simultaneously by another force Q acting at an angle θ from the vertical as shown. The block will remain in equilibrium if the coefficient of friction between the block and the surface S is :



(a) at least $\dfrac{P+Q\sin\theta}{mg+Q\cos\theta}$

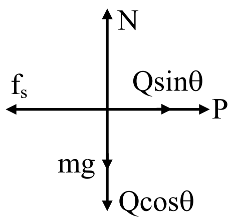
(b) at least $\dfrac{P+Q\cos\theta}{mg+Q\sin\theta}$

(c) equal to $\dfrac{P+Q\sin\theta}{mg-Q\cos\theta}$

(d) equal to $\dfrac{P+Q\cos\theta}{mg-Q\sin\theta}$

Allen Ans. (a)

Sol.



$\Sigma F_y = 0$

$N = mg + Q\cos\theta$ ….(i)

$\Sigma F_x = 0$

$F_s = P + Q\sin\theta$ ….(ii)

$\because f_s \le (f_s)_{max}$

$P + Q\sin\theta \le \mu N$

$P + Q\sin\theta \le \mu\,(mg + Q\cos\theta)$

$\mu \ge \dfrac{P+Q\sin\theta}{mg+Q\cos\theta}$

28. Knowing that the acceleration due to gravity on the Earth surface is g and the radius of the Earth is R, a small body of mass m falls on the Earth from a height $h = \dfrac{R}{5}$ above the Earth's surface. During the freefall, the potential energy of the falling body decreases by:

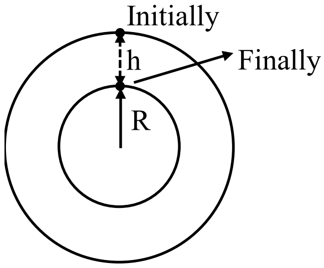
(a) mgh

(b) $\dfrac{4}{5}$mgh

(c) $\dfrac{5}{6}$mgh

(d) $\dfrac{6}{7}$mgh

Allen Ans. (c)



Sol.

$$U_I = -\frac{GM_e m}{R+h}$$

$$U_F = -\frac{GM_e m}{R}$$

$$DU = U_I - U_F$$

$$\Delta U = GM_e m\left[\frac{1}{R} - \frac{1}{R+h}\right]$$

$$\Delta U = GM_e m\left[\frac{1}{R} - \frac{5}{6R}\right]$$

$$\Delta U = GM_e m\left[\frac{1}{6R}\right] = \frac{GMem}{R} \times \frac{1}{6}$$

$$= m\left[\frac{GM_e}{R^2}\right] \times \frac{R}{6}$$

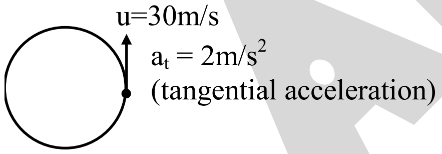
$$\Delta U = \frac{mgR}{6} = \frac{5mgh}{6}$$

29. At some instant, a motor car is moving on a circular path of radius 600 m, with a speed u = 30 ms^{-1}. If its speed is increased at a rate of 2 ms^{-2}, the magnitude of the acceleration of the car at that instant is :

(a) 2.0 ms^{-2} (b) 2.5 ms^{-2} (c) 3.5 ms^{-2} (d) 1.5 ms^{-2}

Allen Ans. (b)

Sol.



R = 600m

$$a_C = a_R = \frac{v^2}{R} = \frac{(30)^2}{600} = \frac{900}{600} = \frac{3}{2} \text{ [Centripital acceleration]}$$

$a_t = 2$

$$a_{Net} = \sqrt{a_t^2 + a_c^2}$$

$$= \sqrt{(2)^2 + (3/2)^2} = \sqrt{\frac{25}{4}} = \frac{5}{2}$$

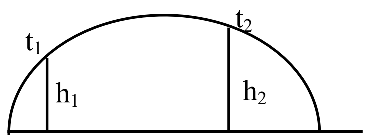
$a_{net} = 2.5\ m/s^2$

30. A cricket ball, thrown across a field, is at heights of h_1 and h_2 above the point of projection, at time t_1 and time t_2 after the throw, respectively. It is then caught by the wicket keeper at the same height as that from which it was thrown. The Time of Flight (T) of the ball is :

(a) $T = \frac{h_1t_2^2 - h_2t_1^2}{h_1t_2 - h_2t_1}$ (b) $T = \frac{h_1t_2^2 + h_2t_1^2}{h_2t_1 + h_1t_2}$ (c) $T = \frac{h_1t_1^2 - h_2t_2^2}{h_1t_1 - h_2t_2}$ (d) $T = \frac{h_1t_1^2 + h_2t_2^2}{h_1t_1 + h_2t_2}$

Allen Ans. (a)

Sol.



$$h_1 = u_y t_1 - \frac{1}{2} g t_1^2$$

$$h_1 + \frac{1}{2} g t_1^2 = u_y t_1 \qquad \text{....(i)}$$

$$h_2 = u_y t_2 - \frac{1}{2} g t_2^2$$

$$h_2 + \frac{1}{2} g t_2^2 = u_y t_2 \qquad \text{....(ii)}$$

Equation (i)/(ii)

$$\frac{h_1 + \frac{1}{2} g t_1^2}{h_2 + \frac{1}{2} g t_2^2} = \frac{t_1}{t_2}$$

$$h_1 t_2 - h_2 t_1 = \frac{g}{2}(t_1 t_2^2 - t_1^2 t_2)$$

$$g = 2\left[\frac{h_1 t_2 - h_2 t_1}{t_1 t_2^2 - t_1^2 t_2}\right]$$

Time of flight

$$T = \frac{2\mu g}{g}$$

$$T = \frac{2}{g}\left[\frac{h_1 + \frac{1}{2} g t_1^2}{t_1}\right]$$

$$T = \frac{2}{t_1}\left[\frac{h_1}{g} + \frac{t_1^2}{2}\right]$$

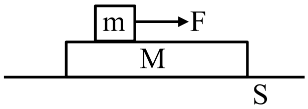
$$T = \frac{2}{t_1}\left[\frac{h_1 (t_1 t_2^2 - t_1^2 t_2)}{2[h_1 t_2 - h_2 t_1]} + \frac{t_1^2}{2}\right]$$

$$T = \left[\frac{h_1 t_1 t_2 (t_2 - t_1)}{t_1 (h_1 t_2 - h_2 t_1)} + t_1\right]$$

$$T = \frac{h_1 t_2 (t_2 - t_1) + t_1 h_1 t_2 - t_1^2 h_2}{h_1 t_2 - h_2 t_1}$$

$$T = \frac{h_1 t_2^2 - h_2 t_1^2}{h_1 t_2 - h_2 t_1}$$

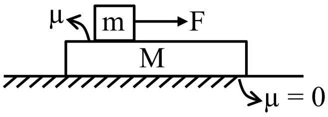
31. A plate of mass M is placed on a horizontal frictionless surface S. A block of mass m is placed on the plate. The coefficient of dynamic friction between the block and the plate is μ. If a horizontal force $F = 2\mu mg$ is applied to the block (as shown), the acceleration of the plate will be :-



(a) $\frac{\mu mg}{M}$ (b) $\frac{\mu mg}{m+M}$ (c) $\frac{2\mu mg}{M}$ (d) $\frac{2\mu mg}{m+M}$

Allen Ans. (a or d)

Sol.



$F = 2\mu mg$

Suppose together :

$$a = \left(\frac{F}{m+M}\right) = \frac{2\mu mg}{m+M}$$

$$f = M.a = \frac{2Mm\mu g}{m+M}$$

$$f = \mu mg\left[\frac{2M}{m+M}\right]$$

$(f_s)_{max} = \mu mg$

If $m > M$:

then $f \leq (f_s)_{max}$

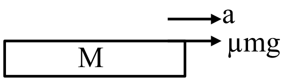
block move together

$$a = \frac{2\mu mg}{m+M}$$

If $m < M$:

then $f > (f_s)_{max}$

our assumption is wrong.



$$a = \frac{\mu mg}{M}$$

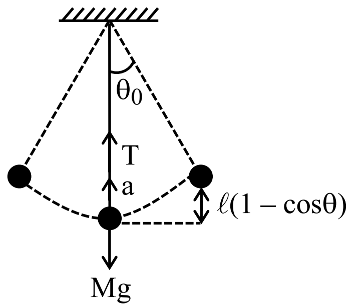
32. A simple pendulum, with a bob of mass m, oscillates in a vertical plane, with an angular amplitude θ_0. The tension in its string when it passes through the mean position is 2mg. Neglecting the effect of air friction and the viscosity of air, the angular amplitude θ_0 is :-

(a) 30° (b) 60°

(c) 90° (d) 120°

Allen Ans. (b)

Sol.



$$T - mg = \frac{mv^2}{\ell}$$

$$2mg - mg = \frac{mv^2}{\ell} \qquad ...(1)$$

$$mg\ell = mv^2$$

$$h = \ell(1 - \cos\theta)$$

Work done by gravity = ΔK

$$mg\ell(1 - \cos\theta) = \frac{1}{2}mv^2 - 0$$

From (1) :

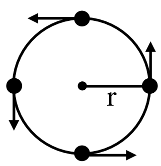
$$mg\ell(1 - \cos\theta) = \frac{1}{2}(mg\ell)$$

$$\frac{mg\ell}{2} = mg\ell\cos\theta$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = 60°$$

33. Because of their mutual gravitational attraction, four identical planets each of mass m are orbiting in a circular path of radius r in the same sense (angular direction). The magnitude of the velocity of each planet is :-



(a) $\left[\frac{Gm}{r}\left(\frac{1+2\sqrt{2}}{4}\right)\right]^{1/2}$

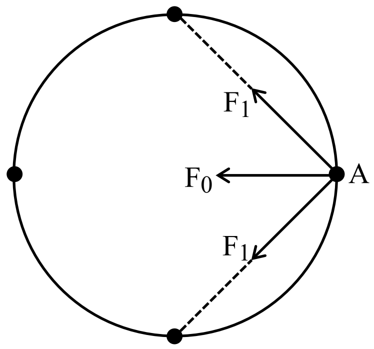
(b) $3\sqrt{\frac{Gm}{r}}$

(c) $\sqrt{\frac{Gm}{r}(1+2\sqrt{2})}$

(d) $\left[\frac{1}{2}\frac{Gm}{r}\left(\frac{1+\sqrt{2}}{2}\right)\right]^{1/2}$

Allen Ans. (a)

Sol.



Net force on planet A is :

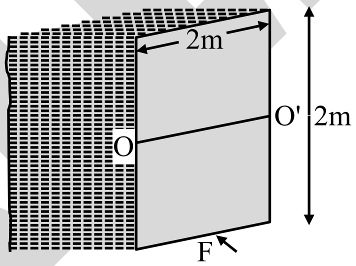
$$F = F_1\sqrt{2} + F_0$$

$$F = \frac{Gm^2}{(r\sqrt{2})^2}\sqrt{2} + \frac{Gm^2}{(2r)^2}$$

$$F = \frac{Gm^2}{r^2}\left[\frac{1}{\sqrt{2}} + \frac{1}{4}\right] = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{Gm}{r}\frac{(1+2\sqrt{2})}{4}}$$

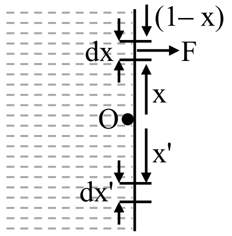
34. A rigid square sheet of size 2 m × 2 m is hinged at the middle of the vertical edges to serve as a door which can turn about the horizontal axis OO'. A fluid of density ρ fills the space to the left of the sheet up to its top. The horizontal force F required (to be applied at the lower edge) to hold the sheet vertical is :-



(a) $\frac{2}{3}\rho g$ (b) $\frac{4}{3}\rho g$ (c) $\frac{8}{3}\rho g$ (d) $\frac{1}{3}\rho g$

Allen Ans. (b)

Sol.



$$\tau_{upper} = \int_0^1 \rho g(1-x)(2dx)(x)$$

$$= \rho g 2\left[\frac{1}{2} - \frac{1}{3}\right] = \frac{\rho g}{3} \text{ (cw)}$$

$$\tau_{lower} = \int_0^1 \rho g(1+x')(2dx')(x')$$

$$= \rho g 2\left[\frac{1}{2}+\frac{1}{3}\right] = \rho g \times \frac{5}{3} \text{ (Acw.)}$$

$$\tau_{net} = \rho g\left(\frac{5}{3}-\frac{1}{3}\right)$$

$$= \frac{4}{3}\rho g = F \times 1$$

$$F = \frac{4}{3}\rho g$$

35. A major artery in human body, with radius 0.4 cm, carries blood at a flow rate of 5.0 cubic centimeters per second. The pressure difference of blood per meter length of the artery is nearly :- [Given that the coefficient of viscosity (η) of blood at body temperature is 4.0×10^{-3} Pa.s and the density of mercury is 13.6 g/cm^3]

(a) 9.6 mm of Hg (b) 3.2 mm of Hg (c) 1.5 mm of Hg (d) 6.0 mm of Hg

Allen Ans. (c)

Sol. Rate of liquid flow is given by poiseuilles equation

$$\frac{dQ}{dt} = \left(\frac{\pi}{8}\right)\frac{(\Delta P)r^4}{\eta L}$$

$$5 \times 10^{-6} = \frac{\pi}{8} \times \frac{\Delta P \times (4 \times 10^{-3})^4}{4 \times 10^{-3} \times \ell}$$

$$\frac{\Delta P}{\ell} = \frac{5 \times 10^{-6} \times 4 \times 10^{-3}}{(4 \times 10^{-3})^4} \times \frac{8}{\pi}$$

$= 0.2 \times 10^3$ Pa/m

= 1.5 mm of Hg/m

36. If P represents radiation pressure, E represents radiation energy striking per unit area per unit time and c represents speed of light then the possible values of non-zero integers x, y and z such that $P^x E^y c^z$ is dimensionless, may be :-

(a) $x = 1, y = 1, z = 1$ (b) $x = -1, y = 1, z = 1$

(c) $x = 1, y = -1, z = 1$ (d) $x = 1, y = 1, z = -1$

Allen Ans. (c)

Sol. Dimension of

$P = [ML^{-1}T^{-2}]$

$E = [MT^{-3}]$

$c = [LT^{-1}]$

$P^xE^yc^z = [M^{x+y} L^{-x+z} T^{-2x-3y-z}]$

$[M^0L^0T^0] = [M^{x+y} L^{-x+z} T^{-2x-3y-z}]$

$\Rightarrow$ $x + y = 0$; $x = z$ & $2x + 3y + z = 0$

$2x + 3(-x) + x = 0$

$x = 0$; $y = 0$; $z = 0$

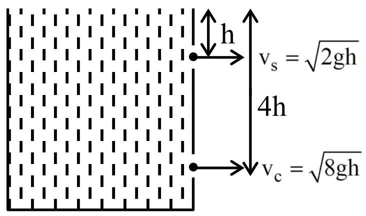
But question says x, y & z must be non-zero, so only option 'c' satisfy this.

37. A large tank, open at the top, has two small holes in the vertical wall. One is a square hole of side 's' at a depth h below the top and the other is a circular hole of radius r at a depth 4h below the top (given that s << h; r << h). When the tank is completely filled up to the brim with water, the quantity of water flowing out per second from each hole is the same, then r is equal to :-

(a) $2\pi s$ (b) $\frac{s}{2\pi}$ (c) $\frac{s}{\sqrt{2\pi}}$ (d) $\frac{s}{2\sqrt{\pi}}$

Allen Ans. (c)

Sol.



$v_s = \sqrt{2gh}$ $v_c = \sqrt{2g(4h)}$

Q_s = Area × velocity $v_c = \sqrt{8gh}$

$Q_s = s^2\sqrt{2gh}$ $Q_c = (\pi r^2)\sqrt{8gh}$

According to given condition

$Q_s = Q_c$

$s^2\sqrt{2gh} = \pi r^2\sqrt{8gh}$

$s^4(2gh) = \pi^2 r^4(8gh)$

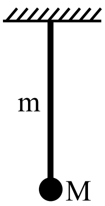
$s^4 = 4\pi^2 r^4$

$r^4 = \frac{s^4}{4\pi^2}$

$r = \frac{s}{(4\pi^2)^{1/4}}$

$r = \frac{s}{\sqrt{2\pi}}$

38. A pendulum consists of a heavy but very small bob of mass M suspended at the end of a rigid rod of mass m and length L. The time period of small oscillations in the vertical plane, about a horizontal axis through the upper end of the rod is :-

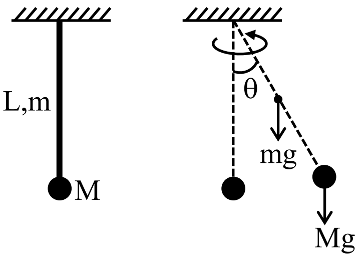


(a) $2\pi\sqrt{\left(\frac{m+2M}{m+3M}\right)\times\left(\frac{3L}{2g}\right)}$ (b) $2\pi\sqrt{\left(\frac{m+3M}{m+2M}\right)\times\left(\frac{2L}{3g}\right)}$

(c) $2\pi\sqrt{\left(\frac{3L}{2g}\right)}$ (d) $2\pi\sqrt{\left(\frac{2m+M}{3m+M}\right)\times\left(\frac{3L}{2g}\right)}$

Allen Ans. (b)

Sol.



$\tau = I\alpha$

$$-\left\{(mg\sin\theta)\frac{L}{2}+(Mg\sin\theta)L\right\}=\left(ML^2+\frac{mL^2}{3}\right)\alpha$$

θ is very small

So, $\sin\theta \approx \theta$

$$-\left(\frac{m}{2}+M\right)g\theta=\left(\frac{3M+m}{3}\right)L\alpha$$

$$-\left(\frac{m+2M}{2}\right)g\theta=\left(\frac{3M+m}{3}\right)L\alpha$$

$$\alpha=-\left(\frac{m+2M}{m+3M}\right)\frac{3g}{2L}\theta$$

By comparing standard equation

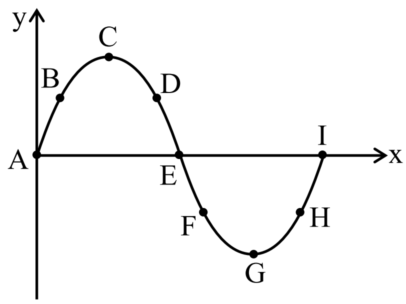
$\alpha = -\,\omega^2\theta$

$$\omega=\sqrt{\left(\frac{m+2M}{m+3M}\right)\frac{3g}{2L}}$$

$$T=\frac{2\pi}{\omega}$$

$$T=2\pi\sqrt{\left(\frac{m+3M}{m+2M}\right)\frac{2L}{3g}}$$

39. A transverse wave is travelling along a long stretched string from left to right (along +ve x direction). The snapshot of a small part of the string at any moment t is shown in the figure. At this particular instant :-



(a) A and E are at rest for a moment while C and G have maximum speed

(b) B and D have upward velocity whereas F and H have downward

(c) D, E, F are moving downward at that moment

(d) B and H are moving downward at that moment

Allen Ans. (d)

Sol. $v_P = -v_\omega$ (slope)

Wave is in +x direction, so v_ω = +ve

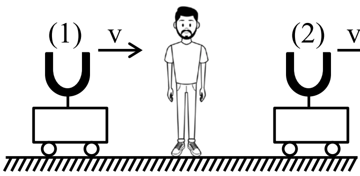
Positive slope : A, B, H, I

Negative slope : D, E, F

Zero slope : C, G

From given option slope of B and H are positive, so at the given moment B & H are moving downward.

40. Two tuning forks, with natural frequency 700 Hz each, move relative to a stationary observer. Fork (1) moves towards the observer while the fork (2) moves away from the observer. Both the forks move with same velocity v on the same line. The observer, standing between the two forks, hears 4 beats per sec. Using the speed of sound in air as $v_s = 350\ ms^{-1}$, the speed of each tuning fork is :-



(a) $2.0\ ms^{-1}$ (b) $1.5\ ms^{-1}$ (c) $1.0\ ms^{-1}$ (d) $0.5\ ms^{-1}$

Allen Ans. (c)

Sol. f_1 = frequency heared by observer due to tunning fork 1

f = frequency heared by observer due to tunning fork 2

$$f_1 = f_0\left[\frac{v_\omega \pm v_0}{v_\omega \pm v_s}\right]$$

$$\Rightarrow\ f_1 = 700\left[\frac{350+0}{350-v}\right]$$

$$f_2 = 700\left[\frac{350}{350+v}\right]$$

According to given condition,

$f_{beat} = f_1 + f_2$

$$4 = 700\left[\frac{350}{350-v}\right] - 700\left[\frac{350}{350+v}\right]$$

$$4 = 700 \times 350\left[\frac{(350+v)-(350-v)}{(350)^2 - v^2}\right]$$

$$(350)^2 - v^2 = \frac{1}{2} \times 350 \times 350 \times 2v$$

$(350)^2 - v^2 = (350)^2 v$

$v^2 + (350)^2 v - (350)^2 = 0$

$v^2 + 122500\ v - 122500 = 0$

From the given option the most appropriate choice for speed of tunning fork will be

$v \approx 1$ m/s

41. The speed of sound in a mixture of 1 mole of Helium (molar mass = 4 g) and 2 moles of oxygen (molar mass = 32 g) at 27 °C is nearly :-

(a) $318\ ms^{-1}$ (b) $332\ ms^{-1}$ (c) $381\ ms^{-1}$ (d) $401\ ms^{-1}$

Allen Ans. (d)

Sol. $V = \sqrt{\frac{\gamma RT}{M}}$

$$\gamma_{mix} = \frac{C_{P_{mix}}}{C_{V_{mix}}}$$

$$= \frac{n_1 C_{P_1} + n_2 C_{P_2}}{n_1 C_{V_1} + n_2 C_{V_2}}$$

$$= \frac{1\left(\frac{5R}{2}\right) + 2\left(\frac{7R}{2}\right)}{1\left(\frac{3R}{2}\right) + 2\left(\frac{5R}{2}\right)}$$

$$= \frac{19}{13} \approx 1.46$$

$$M_{mix} = \frac{n_1 M_1 + n_2 M_2}{n_1 + n_2}$$

$$= \frac{1(4) + 2(32)}{1+2} = \frac{68}{3} \text{ gm/mol}$$

$$= \frac{68 \times 10^{-3}}{3} \text{ kg/mol}$$

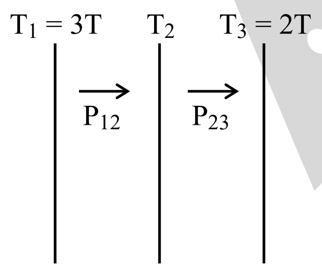
$$\Rightarrow \quad V = \sqrt{\frac{\gamma_{mix} \times RT}{M_{mix}}} = 401 \text{ m/s}$$

42. Three identical large metal plates are kept parallel and close to each other. Each plate can be treated as an ideal black body and has very high thermal conductivity. The first and third plates are maintained at high temperature $T_1 = 3T$ and $T_3 = 2T$. The temperature T_2 of the middle (i.e. second) plate under steady state condition is :-

(a) $\frac{5T}{2}$ (b) $\left(\frac{65}{2}\right)^{1/4} T$ (c) $\left(\frac{97}{2}\right)^{1/4} T$ (d) $\left(\frac{65}{4}\right)^{1/4} T$

Allen Ans. (c)

Sol.



For equilibrium,

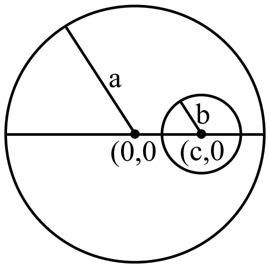
$P_{12} = P_{23}$

$$\sigma e A\left(T_1^4 - T_2^4\right) = \sigma e A\left(T_2^4 - T_3^4\right)$$

$$\Rightarrow \quad T_2^4 = \frac{1}{2}\left(T_1^4 + T_3^4\right)$$

$$\Rightarrow \quad T_2 = \left(\frac{97}{2}\right)^{1/4} T$$

43. A thin uniform circular disc of radius 'a' is placed in XY plane with its center at origin (0,0). A small circular disc of radius b with center at (c, 0) is cut and taken out to create a hole. The center of mass of the remaining disc is at :-



(a) $-\frac{b^2}{a^2}c, 0$ (b) $-\frac{b^2}{a^2-c^2}c, 0$ (c) $-\frac{b^2}{a^2+b^2}c, 0$ (d) $-\frac{b^2}{a^2-b^2}c, 0$

Allen Ans. (d)

Sol. $x_{cm} = \frac{M_1x_1 - M_2x_2}{M_1 - M_2}$

$M = \sigma A = \sigma\pi r^2$

$M_1 = \sigma\pi a^2$ at $x_1 = 0$ & $M_2 = \sigma\pi b^2$ at $x_2 = c$

$$x_{cm} = \frac{(\sigma A_1)(x_1) - (\sigma A_2)(x_2)}{\sigma A_1 - \sigma A_2}$$

$$= \frac{(\pi a^2)(0) - (\pi b^2)(c)}{\pi a^2 - \pi b^2}$$

$$= -\left(\frac{b^2 c}{a^2 - b^2}\right)$$

$y_{cm} = 0$, because of symmetry along x-axis.

44. One mole of an ideal monoatomic gas, contained in a cylinder fitted with movable piston, is originally at P_1, V_1 and $T_1 = 27$ °C. The gas is slowly heated. Initially 8.31 watt-hour of energy is added to it; at the same time it is allowed to expand at constant pressure to a new state P_1, V_2 and T_2. The correct option is :-

(a) Value of T_2 is 1740 °C

(b) Work done by the gas is 2160 R joule

(c) Internal energy of the gas increases by 1440 R joule

(d) $\frac{V_2}{V_1} = 5.8$

Allen Ans. (d)

Sol. $n = 1$, $T_1 = 27$ °C = 300 K

Heat added, Q = 8.31 watt-hr

= 8.31 × 3600 J

$C_P = \frac{5R}{2}$; $C_V = \frac{3R}{2}$

For an isobaric process

$Q = nC_P\Delta T$

$$8.31 \times 3600 = (1)\left(\frac{5R}{2}\right)\Delta T$$

$R = 8.31$

$\Rightarrow$ $\Delta T = 1440$ K

$\Rightarrow$ Final temperature

$T_2 = T_1 + \Delta T = 1740$ K not 1740 °C

Work done $= P\Delta V = nR\Delta T$

$= 1440$ R

Change in internal energy

$\Delta U = nC_V\Delta T$

$$= (1)\left(\frac{3R}{2}\right)(1440)$$

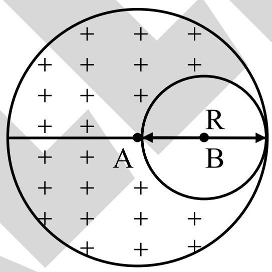
$= 2160$ R

At constant pressure

$V \propto T$

$$\frac{V_2}{V_1} = \frac{T_2}{T_1} = \frac{1740}{300} = 5.8$$

45. A non-conducting solid sphere, of radius R, with its center at A, has a spherical cavity of diameter R with center at B as shown. There is no charge in the cavity while the solid part has a uniform volume charge density ρ. Electric potential at the center of the sphere (at point A) is $V = \frac{k\rho R^2}{12\varepsilon_0}$ (in SI units) where the value of k is :-



(a) 3 (b) 5 (c) 7 (d) 9

Allen Ans. (b)

Sol. $Q_{total} = \rho \times \frac{4}{3}\pi R^3$

$$Q_{removed} = \rho \times \frac{4}{3}\pi\left(\frac{R}{2}\right)^3$$

$$V_A = \frac{3}{2} \times \frac{1}{4\pi\varepsilon_0}\frac{Q_T}{R} - \frac{1}{4\pi\varepsilon_0}\frac{Q_R}{R/2}$$

$$= \frac{1}{4\pi\varepsilon_0 R}\left[\frac{3}{2}\rho \times \frac{4}{3}\pi R^3 - 2\rho \times \frac{4}{3}\pi\frac{R^3}{8}\right]$$

$$= \frac{5\rho R^2}{12\varepsilon_0}$$

K = 5

46. Energy from the Sun falls on the Earth surface at the rate of 1400 W/m^2, which is known as solar constant. The respective rms values E_{rms} and B_{rms} of electric and magnetic fields in the sunlight (electromagnetic radiation) reaching Earth surface are (Take speed of light c = 3 × 10^8 ms^{-1}) :-

(a) E_{rms} = 726.5 V/m, B_{rms} = 2.42 μT

(b) E_{rms} = 7260 V/m, B_{rms} = 242 nT

(c) E_{rms} = 1030 V/m, B_{rms} = 3.42 μT

(d) E_{rms} = 10300 V/m, B_{rms} = 342 nT

Allen Ans. (a)

Sol. Intensity of light

$$I = \frac{1}{2}\varepsilon_0 E_0^2 c$$

$$E_0 = \sqrt{\frac{2\times1400}{3\times10^8\times8.85\times10^{-12}}}$$

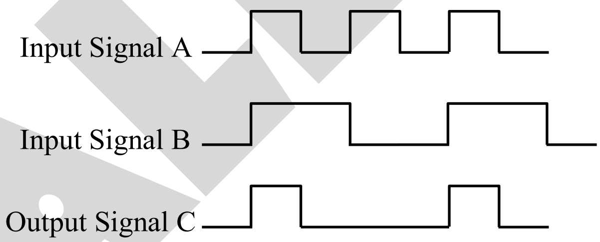
$$= E_{rms}\sqrt{2}$$

E_{rms} = 726 v/m

$$B_{rms} = \frac{E_{rms}}{c}$$

$$= 2.42\ \mu T$$

47. The figure below depicts the voltage wave forms of binary input signals A and B and the output signal C of a certain logic gate.



The logic gate is

(a) AND (b) NAND (c) OR (d) XOR

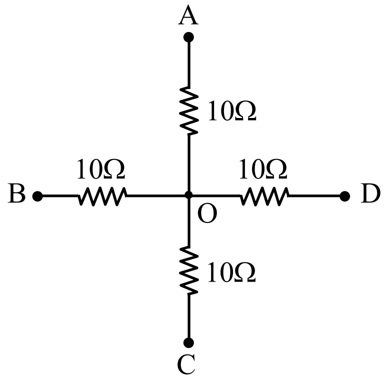
Allen Ans. (a)

Sol. Truth table of given signal

A	B	C(Output)
0	0	0
1	1	1
0	1	0
1	0	0
0	0	0
1	1	1

Given gate is AND gate

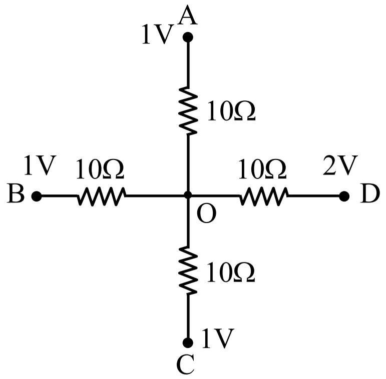
48. In a certain electrical network, the three nodes A, B and C are each at a potential of 1.0 volt while the node D is at a potential 2.0 volt. The potential at the Node O in volt is



(a) $\frac{3}{2}$ (b) $\frac{4}{3}$ (c) $\frac{5}{4}$ (d) $\frac{6}{5}$

Allen Ans. (c)

Sol.

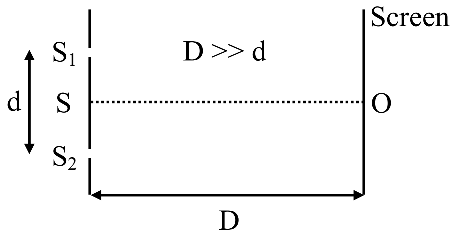


Apply KCL at point O

$$\frac{V_A - V_O}{10} + \frac{V_B - V_O}{10} + \frac{V_C - V_O}{10} + \frac{V_D - V_O}{10} = 0$$

$$V_O = \frac{V_A + V_D + V_C + V_D}{4} = \frac{5}{4}\text{volt}$$

49. In young's double slit experiment, a fine beam of coherent monochromatic light of wavelength λ = 600 nm is incident on identical slits S_1 and S_2 at separation d. The intensity at the central maximum formed at O is I_{max} and the angular fringe width is $\beta = 0.1°$. When a thin transparent film is placed in front of the slit S_2, the intensity at O changes. It is found that the smallest thickness of the film. for which the intensity at O becomes half the maximum intensity $\left(\text{i.e.} \frac{I_{max}}{2}\right)$, is 250 nm. Neglecting the absorption of light by the film, the zero order fringe earlier at O now forms at O' where OO' = 0.5 mm. Choose correct option(s) :-



(a) The refractive index of the film is 1.6

(b) The fringe width near O is 2mm

(c) On the screen, O' is above O

(d) The distance D of the screen from the double slit is nearly 1.15 m.

Allen Ans. (a,b,d)

Sol. $\lambda = 600$ nm $I_{max} = 4I_0$ $\beta = \dfrac{\lambda}{d} = 0.1°$

$I = 4I_0 \cos^2\dfrac{\theta}{2}$

$2I_0 = 4I_0 \cos^2\dfrac{\theta}{2}$

$\cos\dfrac{\theta}{2} = \pm\dfrac{1}{\sqrt{2}}$

$\therefore \ \theta = \dfrac{\pi}{2} = \dfrac{2\pi}{\lambda}\Delta x$

$\Delta x = \dfrac{\lambda}{4}$

$(\mu - 1)t = \dfrac{\lambda}{4}$

$\mu - 1 = \dfrac{1}{4} \times \dfrac{600 \times 10^{-9}}{250 \times 10^{-9}} = 0.6$

$\therefore \ \mu = 1.6$ Option (a)

$OO' = d\sin\theta \approx d\tan\theta = \dfrac{\lambda}{4}$

$d = \dfrac{0.5 \times 10^{-3}}{D} = \dfrac{\lambda}{4}$

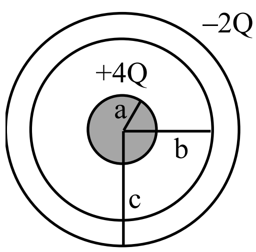
$\omega = \dfrac{\lambda D}{d}$ = fringe width $= 4 \times 0.5 \times 10^{-3} = 2$ mm

$\omega = 2$mm Option (b)

$\omega = \beta D$

$D = \dfrac{\omega}{\beta} = \dfrac{2 \times 10^{-3}}{0.1 \times \pi} \times 180 \approx 1.15$ m Option (d)

50. An insulated non-conducting solid sphere of radius 'a', carrying a positive charge +4Q uniformly distributed throughout its volume, is surrounded by a concentric thick conducting spherical shell of inner radius b and outer radius c. This thick shell carries a negative charge –2Q (see figure). The correct option (s) is/are :-



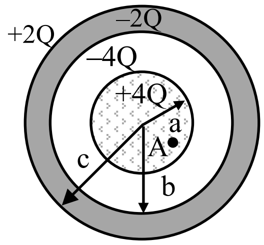
(a) Electric field strength at distance r (r < a) from the center is $\vec{E} = \dfrac{1}{4\pi \in} \dfrac{4Q}{a^2} \vec{r}$

(b) Charge on the inner surface of the conducting spherical shell is +2Q

(c) Charge on the outer surface of the conducting spherical shell is +2Q

(d) Electrical energy stored in region 0 < r < a [i.e. in the inner sphere] is $\dfrac{2Q^2}{5\pi \in_0 a}$

Allen Ans. (a,c,d)



Sol.

$$\vec{E}_A = \frac{k(4Q)\vec{r}}{R^3} = \frac{1}{4\pi\in}.\frac{4Q}{a^2}.\vec{r} \qquad \text{Option (a)}$$

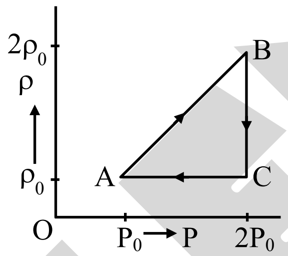
Option (b) is incorrect

Option (c) is correct

Energy stored inside volume of a solid non conducting sphere is

$$U = \frac{1}{10}\frac{kQ^2}{R} = \frac{1}{10}\times\frac{1}{4\pi\in}\times\frac{(4Q)}{a} = \frac{2}{5}\frac{Q^2}{\pi\in a} \qquad \text{Option (d)}$$

51. One mole of an ideal monoatomic gas of molecular mass M undergoes a cyclic process (ABCA) shown in the figure as a density (ρ) versus pressure (P) curve. The correct option (s) is/are :-



(a) Work done on the gas in going from A to B is $W_{AB} = \frac{MP_0}{\rho_0}\ell n2$

(b) Work done by the gas in the process BC is $W_{BC} = \frac{MP_0}{2\rho_0}$

(c) Efficiency (η) of the complete cycle ABCA is $\eta = \frac{2}{5}(1-\ell n2)$

(d) Heat rejected by the gas in the complete cycle ABCA is $Q_{ABCA} = \frac{MP_0}{\rho_0}(1-\ell n2)$

Allen Ans. (a,c)

Sol. $A \to B$

$P \propto \rho$

$\therefore$ PV = Constant \ Isothermal pressure

$$W_{AB} = nRT\ln\frac{v_2}{v_1} = P_0\frac{M}{\rho_0}\ell n\frac{\rho_0}{2\rho_0} = -\frac{P_0M}{\rho_0}\ell n2$$

$$W_{AB} = -\frac{\rho_0 M}{\rho_0}\ell n2 \qquad \text{Option (a)}$$

$B \to C$ Isobaric pressure

$$W = P\Delta V = 2P_0\left(\frac{M}{\rho_0}-\frac{M}{2\rho_0}\right) = \frac{P_0M}{\rho_0}$$

$$Q_{AB} = -\frac{P_0M}{\rho_0}\ell n2$$

$Q_{BC} = nC_P\Delta T = \frac{5}{2}(P_2V_2 - P_1V_1) = \frac{5}{2}\left[2P_0\frac{M}{\rho_0} - 2P_0\frac{M}{2\rho_0}\right]$

$= \frac{5}{2}P_0\frac{M}{\rho_0}$

$Q_{CA} = nC_V\Delta T = \frac{3}{2}(P_2V_2 - P_1V_1) = \frac{3}{2}\left(P_0\frac{M}{\rho_0} - 2P_0\frac{M}{\rho_0}\right)$

$= -\frac{3}{2}\frac{P_0M}{\rho_0}$

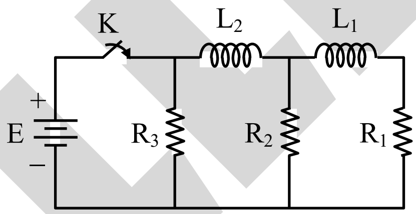
$Q_a = Q_{BC} = \frac{5}{2}\frac{P_0M}{\rho_0}$

$W = \Sigma W = \Sigma Q = \frac{P_0M}{\rho_0}(1 - \ell n2)$

$\eta = \frac{W}{Q_a} = \frac{2}{5}(1 - \ell n2)$ Option (c)

$Q_{rej} = Q_{AB} + Q_{CA} = -\frac{P_0M}{\rho_0}\left(\ell n2 + \frac{3}{2}\right)$

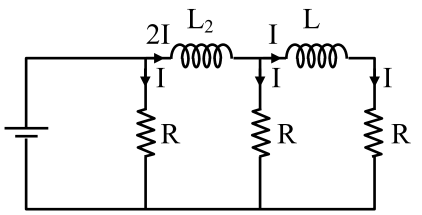
52. Two ideal inductors $L_1 = L_2 = L$ and the three identical resistors $R_1 = R_2 = R_3 = R$ have been connected to a DC source of emf E as shown in the circuit. When the key K is kept pressed (closed) for a long time, the current through the resistance R_1 on the extreme right is measured to be I. Immediately after releasing (switching off) the key, the current through the resistors is :-



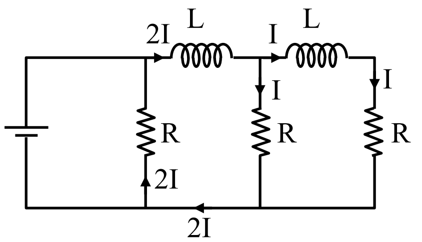
(a) I downwards in R_1
(b) I downwards in R_2
(c) 2I upwards in R_3
(d) zero in each R_1, R_2 and R_3

Allen Ans. (a,b,c)

Sol. At steady state : inductor behave as an ideal inductor



Just after opening switch inductor do not allow its correct to change



53. A particle of mass m moves along x axis with its potential energy as $U(x) = \frac{\alpha}{x^2} - \frac{\beta}{x}$ where α and β are positive constants. The particle is released from rest at $x_0 = \frac{\alpha}{\beta}$. Then

(a) U(x) can be expressed as $U(x) = \frac{\alpha}{x_0^2}\left[\left(\frac{x_0}{x}\right)^2 - \frac{x_0}{x}\right]$

(b) velocity of the particle v(x) as a function of x can be expressed as $v(x) = \left[\frac{2\alpha}{mx_0^2}\left\{\frac{x_0}{x} - \left(\frac{x_0}{x}\right)^2\right\}\right]^{\frac{1}{2}}$

(c) the maximum speed of the particle is $v_{max} = \sqrt{\frac{\alpha}{2mx_0^2}}$.

(d) the total energy of the particle KE(x) + U(x) is zero.

Allen Ans. (a,b,c,d)

Sol. (a) $U(x) = \frac{\alpha}{x^2} - \frac{\beta}{x} = \frac{\alpha}{x_0^2}\left[\frac{x_0^2}{x^2} - \frac{\beta x_0^2}{x\alpha}\right]$

$$U(x) = \frac{\alpha}{x_0^2}\left[\left(\frac{x_0}{x}\right)^2 - \frac{x_0}{x}\right] \quad \because \ x_0 = \frac{\alpha}{\beta}$$

(b) $K_i + U_i = K_f + U_f$

$$O + \left(\frac{\alpha}{x_0^2} - \frac{\beta}{x_0}\right) = \frac{1}{2}mv^2 + \frac{\alpha}{x_0^2}\left(\left(\frac{x_0}{x}\right)^2 - \frac{x_0}{x}\right)$$

$$\frac{1}{2}mv^2 = \frac{\alpha}{x_0^2}\left[\frac{x_0}{x} - \left(\frac{x_0}{x}\right)^2\right] + \left(\frac{\alpha}{x_0^2} - \frac{\beta}{x_0}\right)$$

$$= \frac{\alpha}{x_0^2}\left[\frac{x_0}{x} - \left(\frac{x_0}{x}\right)^2\right] + \left(\frac{\alpha}{x_0^2} - \frac{\alpha}{x_0^2}\right) \quad \because \ \beta = \frac{\alpha}{x_0}$$

$$v = \left[\frac{2\alpha}{mx_0^2}\left\{\frac{x_0}{x} - \left(\frac{x_0}{x}\right)^2\right\}\right]^{1/2}$$

(c) for $v_{max} \rightarrow U_{min}$

$$\frac{dU}{dx} = 0 = \alpha\frac{(-2)}{x^3} + \frac{\beta}{x^2} = 0$$

$$\Rightarrow -\frac{2\alpha}{x} + \beta = 0$$

$$x = \frac{2\alpha}{\beta} = 2x_0$$

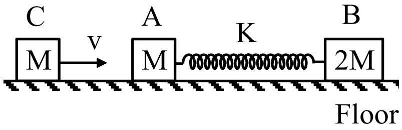
$$\text{so } v_{max} = \left\{\frac{2\alpha}{mx_0^2}\left[\frac{x_0}{2x_0} - \left(\frac{x_0}{2x_0}\right)^2\right]\right\}^{1/2}$$

$$v_{max} = \left\{\frac{2\alpha}{mx_0^2}\left[\frac{1}{2} - \frac{1}{4}\right]\right\}^{1/2} = \sqrt{\frac{\alpha}{2mx_0^2}}$$

(d) T.E. = $K_i + U_i = 0 + \left(\frac{\alpha}{x_0^2} - \frac{\beta}{x_0}\right)$

$= 0 + \frac{\alpha}{x_0^2} - \frac{\alpha}{x_0^2} = 0$

54. Two blocks A and B, of masses M and 2M, respectively, are connected by a massless spring of natural length L_0 and spring constant K. The blocks are initially at rest on a smooth horizontal floor with spring at its natural length L_0. A third block C of mass M, identical to that of block A, moves on the floor with speed v along the line joining A and B and collides with a elastically. In the subsequent motion :-



(a) the spring will be compressed to a maximum when at a length of $v\sqrt{\frac{M}{3K}}$

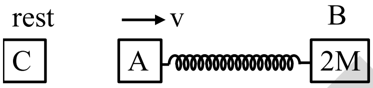
(b) the kinetic energy of A and B together, when the spring is compressed to the maximum, is $\frac{Mv^2}{6}$

(c) the blocks A and B stop for a moment when the spring is at the maximum compression.

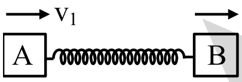
(d) the time required to reach the maximum compression from the normal length is $\frac{\pi}{2}\sqrt{\frac{2M}{3K}}$

Allen Ans. (b,d)

Sol. After collision between C & A



(a) at maximum compression A, B move together



Energy conservation $K_i + U_i = K_f + U_f$

$\frac{1}{2}Mv^2 + 0 = \frac{1}{2}(M + 2M)v_1^2 + \frac{1}{2}kx_{max}^2$ ....(1)

momentum conservation

$P_i = P_f$

$MV = (M + 2M)V_i$

$V_i = \frac{V}{3}$ ......(2)

Using (1) and (3)

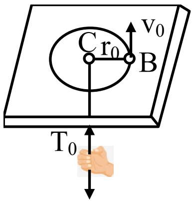
$\frac{1}{2}MV^2 = \frac{1}{2}3M\left(\frac{v^2}{9}\right) + \frac{1}{2}Kx_{max}^2$

$x_{max} = V\sqrt{\frac{2}{3}\frac{M}{K}}$

(b) K.E. = $\frac{1}{2}(M + 2M)v_1^2 = \frac{1}{2}(3M)\frac{v^2}{9} = \frac{1}{6}Mv^2$

(d) $\frac{T}{4} = \frac{2\pi}{4}\sqrt{\frac{\mu}{k}} = \frac{2\pi}{4}\sqrt{\frac{M(2M)}{3Mk}} = \frac{\pi}{2}\sqrt{\frac{2M}{3k}}$

55. A small block B of mass m = 0.25 kg, lying on a frictionless horizontal table, is attached to a massless cord (breaking strength 40 N) passing through a narrow hole C at the centre of the table. Initially when the block is revolving in a circle of radius r_0 = 0.80 m about a vertical axis through the hole, with a tangential speed of v_0 = 4.00 m/s ; the tension in the string is T_0 and the kinetic energy of the block is K_0. The string is then pulled down slowly from below, decreasing the radius of circular path from r_0 to r so that the kinetic energy of the block is now K and the tension in the string is T. As a result :-



(a) the tension $T = T_0 \frac{r_0^4}{r^4}$

(b) the kinetic energy $K = K_0 \frac{r_0^2}{r^2}$

(c) the radius r of the circular path just when the string breaks is 0.40 m.

(d) the work done by the tension in the string in reducing the radius of circle from r_0 to $\frac{r_0}{2}$ is $4K_0$.

Allen Ans. (b,c)

Sol. (a) angular momentum conservation

$mv_0r_0 = mvr$

$v = \frac{v_0r_0}{r}$

$T_0 = \frac{mv_0^2}{r_0}$

$T = \frac{mv^2}{r} = \frac{mv_0^2r_0^2}{r^3}$

$\Rightarrow T = \frac{T_0r_0^3}{r^3}$

(b) $k = \frac{1}{2}mv^2 = \frac{1}{2}m\frac{v_0^2r_0^2}{r^2} = \frac{k_0r_0^2}{r^2}$

(c) $T = \frac{T_0r_0^3}{r^3} = 40$

$\Rightarrow \frac{mv_0^2r_0^2}{r^3} = 40$

$\Rightarrow \frac{0.25 \times 16 \times 0.64}{40} = r^3$

$\Rightarrow r = 0.4$ m

(d) $W = \Delta K = k_f - k_i$

$= \frac{k_0r_0^2}{\left(\frac{r_0}{2}\right)^2} - k_0 = 3k_0$

56. A thin uniform metallic rod, of length $\ell = 1.0$ m and area of cross section A = 2 mm^2, is made to rotate with angular velocity $\omega = 400$ rad/s in a horizontal plane about a vertical axis through one of its ends. The density and the Young's modulus of the material of the rod are $\rho = 10^4$ kg m^{-3} and $Y = 2.0 \times 10^{11}$ Nm^{-2}. Taking r as the distance of a point on the rod from the axis of rotation, the

(a) tension at midpoint of the rod is T = 1200 N.

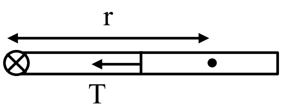
(b) tension in the rod varies with distance r from the axis of rotation as $T = 1600\ r^2$N

(c) stress in the rod at r = 0.5 m is 3.0×10^8 Nm^{-2}

(d) elongation of the rod is $\frac{8}{3}$mm.

Allen Ans. (a,d)

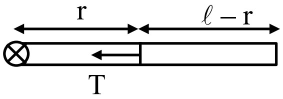
Sol. **(a)**



$$T = \frac{m}{2}\omega^2 r = \frac{m}{2}\omega^2\left(\frac{\ell}{2}+\frac{\ell}{4}\right)$$

$$= \frac{3m\omega^2\ell}{8} = \frac{3\left[\rho(A\ell)\omega^2\ell\right]}{8} = \frac{3\times10^4\times2\times10^{-6}\times1\times16\times10^4}{8} = 1200\text{ N}$$

(b)



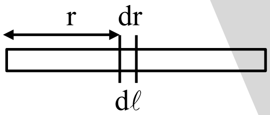
$$T = \left[\frac{m}{\ell}(\ell - r)\right]\omega^2\left[r + \frac{\ell - r}{2}\right]$$

$$= \frac{m(\ell - r)}{\ell}\omega^2\left(\frac{r+\ell}{2}\right) = \frac{m\omega^2(\ell^2 - r^2)}{2\ell} = \frac{\rho A\ell\omega^2(\ell^2 - r^2)}{2\ell}$$

$$T = \frac{10^4\times2\times10^{-6}\times16\times10^4(1-r^2)}{2} = 1600\ (1 - r^2)$$

(c) stress = $\frac{T}{A} = \frac{1200}{2\times10^{-6}} = 6\times10^8$

(d)



$$\frac{T}{A} = Y\frac{d\ell}{dr}$$

$$\int\frac{T}{AY}dr = \int d\ell$$

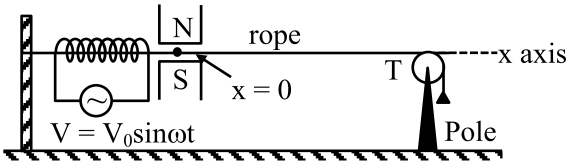
$$\int_0^\ell \frac{1600(1-r^2)}{2\times10^{-6}\times2\times10^{11}}dr = \Delta\ell$$

(a) & (d)

$$4\times10^{-3}\left(\ell - \frac{\ell^3}{3}\right) = \Delta\ell$$

$$\Delta\ell = \frac{8}{3}\times10^{-3}\text{ m} = \frac{8}{3}\text{mm}$$

57. One end of a long and thin rope, stretched horizontally with a tension T = 8N, along x axis, is supporting a weight after passing over a pulley fixed on a vertical pole (see figure). At the other end, a simple harmonic oscillator (a clamped iron rod along the axis of a solenoid fed with AC voltage and oscillating between north and south poles) at x = 0, generates a transverse wave of frequency 100 Hz and an amplitude of 2cm, in the rope. The wave propagates along the rope. The mass per unit length of the rope is 20 g/m. Ignoring the effect of gravity (on the rope), the correct option (s) is/are :-



(a) Wavelength of the transverse wave is 20cm.

(b) Maximum magnitude of transverse acceleration of any point on the rope is nearly 800 ms^{-2}

(c) If the oscillator produces maximum negative displacement at x = 0 at time t = 0, the equation of the wave can be expressed as $y(x, t) = -0.02 \sin[10 \pi x - 100 \pi t]$ in SI units.

(d) Tension in the given rope remaining unchanged, if a harmonic oscillator of frequency 200 Hz is used (instead of earlier frequency 100 Hz), the wavelength will be 10 cm.

Allen Ans. (a,d)

Sol. Velocity of wave in rope = $\sqrt{T/\mu} = \sqrt{\frac{8}{20\times10^{-3}}} = 20\text{m}/\text{s}$

(a) Wavelength = $\frac{v}{f} = \frac{20}{100}\text{m} = 20$ cm

(b) Max. transverse acceleration = $\omega^2 A$

$$= (2\pi \times 100)^2 \times \frac{2}{100} = 800\,\pi^2\ \text{m/s}^2$$

(c) Equation of wave = $y = A \sin(\omega t - kx + \phi) = 0.02 \sin\left(200\,\pi - \frac{2\pi}{0.2}x + \phi\right)$

at t = 0, x = 0 y = – A

$-A = A\sin\phi$

$\phi = -90°$

so equation is : $y = -0.02 \cos(200\,\pi - 10\,\pi x)$

(d) if f = 200 Hz $\lambda = \frac{v}{f} = \frac{20}{200} = 10\text{cm}$

58. Nuclei of a radioactive element A are being produced at a constant rate α. The element A has a decay constant λ. If there are N_0 nuclei at t = 0, then :-

(a) number of nuclei N(t), at time t, is $N(t) = \frac{1}{\lambda}\left[\left(\alpha - \lambda N_0\right)e^{-\lambda t}\right]$

(b) if $\alpha = \lambda N_0$, the number of nuclei N(t) at any time t will remain constant.

(c) if $\alpha = 2\lambda N_0$ then $N(t) = 2N_0$ as $t \to \infty$

(d) if $\alpha = 2\lambda N_0$ the number of nuclei N(t) after one half-life of A is $N\left(\frac{T}{2}\right) = \frac{3}{2}N_0$

Allen Ans. (b,c,d)

Sol. **(a)** $\frac{dN}{dt} = +\alpha - \lambda N$

$$\int \frac{dN}{\alpha - \lambda N} = \int dt$$

$$\left.\frac{\ell n\left(\alpha - \lambda N\right)}{-\lambda}\right|_{N_0}^{N} = t$$

$\alpha - \lambda N = (a - \lambda N_0)e^{-\lambda t}$

$$N = \frac{\alpha}{\lambda}\left(1 - e^{-\lambda t}\right) + N_0 e^{-\lambda t}$$

(b) $\alpha = \lambda N_0$

$N = N_0(1 - e^{-\lambda t}) + N_0 e^{-\lambda t}$

$N = N_0 =$ constant

(c) $\alpha = 2\lambda N_0 \quad t \to \infty \Rightarrow e^{-\lambda t} = 0$

$N = 2N_0$

(d) $\alpha = 2\lambda N_0 \quad t = \frac{\ell n 2}{\lambda} \Rightarrow e^{-\lambda t} = \frac{1}{2}$

$$N = 2N_0\left(1 - \frac{1}{2}\right) + N_0\frac{1}{2} = \frac{3N_0}{2}$$

59. A single electron orbits around a stationary nucleus of charge +Ze in a hydrogen-like atom, where Z is the atomic number and e is the magnitude of the charge on an electron. It requires 47.25 eV to excite the electron from second Bohr orbit to the third Bohr orbit. Ionization energy of hydrogen atom is 13.6 eV. Then

(a) the value of Z is 5

(b) the energy required to excite the electron from the 3rd orbit to the 4th orbit is 16.53 eV (nearly)

(c) the wavelength of electromagnetic radiation required to liberate the electron completely when in the first Bohr orbit is 36.56 Å.

(d) the angular momentum of an electron in the second Bohr orbit is 1.056×10^{-33} Js

Allen Ans. (a,b,c)

Sol. **(a)** $E_3 - E_2 = 47.25$ eV

$$\Rightarrow -13.6\ Z^2\left(\frac{1}{9} - \frac{1}{4}\right) = 47.25$$

$\Rightarrow Z^2 = 25$

$\Rightarrow Z = 5$

(b) $E_4 - E_3 = -13.6 \times 25 \times \left(\frac{1}{16} - \frac{1}{9}\right)$

$= 16.53$ eV

(c) $n = 1 \to n = \infty$

$$E = -13.6 \times 25\left(\frac{1}{\infty} - \frac{1}{1}\right) = 340 \text{ eV}$$

Wavelength required $= \frac{12400}{340}\text{Å} = 36.56\text{Å}$

(d) $L = \frac{nh}{2\pi} = \frac{2 \times 6.62 \times 10^{-34}}{2 \times 3.14} = 2.19 \times 10^{-34}$

60. A circular coil of thin insulated copper wire (N = 2000 turns), wrapped around an iron cylinder of cross-section area $\Delta S = 0.001\ m^2$, is connected to a suspended type moving coil ballistic galvanometer. The suspended rectangular coil of the galvanometer is of mass m = 80 g, length $\ell = 5$ cm, breadth b = 3 cm and has n = 100 turns of fine copper wire wound on a non-metallic frame of ivory. This rectangular coil of the galvanometer is free to execute torsional oscillations in a radial magnetic field B = 0.1 tesla.The galvanometer is being used to measure the charge by employing the formula $q = \frac{T}{2\pi}\frac{c}{nAB}\theta$ [Given that the moment of inertia of the oscillating coil about the vertical axis is $I = 2.7 \times 10^{-6}$ kgm^2 and the torsional constant (torsional rigidity) of the suspension fiber is $c = 3.0 \times 10^{-3}$ Nm/radian : $A = \ell \times b$ is the area of the coil]

When the magnetic induction of 1.0 weber per meter^2, perpendicular to the plane of the circular coil, is reversed (in opposite direction), a deflection of 40 mm is observed on a scale placed 1.0 meter away in front of the reflecting mirror attached with the suspension fiber of the rectangular coil. The correct statement(s) is/are

(a) the time period of the oscillating rectangular coil is T = 0.19 s.

(b) the net change in flux through the circular coil wrapped on the iron cylinder is 4.0 weber.

(c) the induced charge in the circular coil wrapped on the iron cylinder is $q_{ind} = 240\ \mu C$.

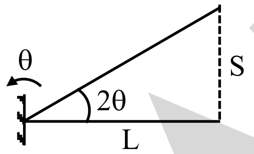
(d) total resistance of the circuit containing the circular coil is $R = 33.3\ k\Omega$.

Allen Ans. (a,b,d)

Sol. (a) $T = 2\pi\sqrt{\frac{T}{C}} = 2\pi\sqrt{\frac{2.7\times10^{-6}}{3\times10^{-3}}} = 0.19s$

(b) $\Delta\phi = N(BA\cos0° - BA\cos180°) = 2BAN$

$= 2 \times 1 \times 0.001 \times 200 = 4wb$



(c)

Mirror doubles the beam deflection so $\theta = \frac{S}{2L} = 0.02$ rad

T = 0.19 s

so $q = \frac{T}{2\pi}\frac{C}{nAB}.\theta$

$= 0.03 \times \frac{0.003}{100\times15\times10^{-4}\times0.1}\times0.02 = 120\mu C$

(d) $R = \frac{N\Delta\phi}{q} = \frac{4}{120\times10^{-6}} = 33.3k\Omega$