SOLUTION

THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

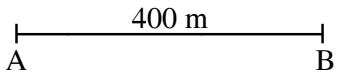
57th NMTC - SCREENING TEST - KAPREKAR CONTEST

SUB-JUNIOR LEVEL - VII & VIII GRADES

1. AB is a straight road of length 400 metres. From A, Samrud runs at a speed of 6m/s towards B and at the same time Saket starts from B and runs towards A at a speed of 5m/s. After reaching their destinations, they return with the same speeds. They repeat it again and again. How many times do they meet each other in 15 minutes?

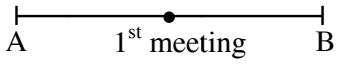
(A) 25 (B) 23 (C) 24 (D) 20

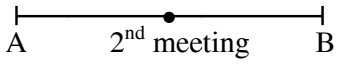
Ans. Bonus

Sol. 

1^{st} Meeting time $t_1 = \frac{400}{11}\text{sec} = 36.36 \text{ sec}$

For second meeting





Total distance covered = $\frac{800}{11} \approx 72.72 \text{ sec}$

So, according to question

For 1^{st} meeting, relative distance covered = 400 m

For 2^{nd} and upcoming meeting, relative distance covered = 800 m

So, According to question

Distance covered in 15 min (900 sec)

= 900 × 11 = 9900 m (relative speed = 11 m/s)

So, $\underbrace{400}_{1^{st}} + \underbrace{800+800+.....n^{\text{times}}}_{\text{'n' meetings}} \le 9900$

$\Rightarrow 800n \le 9900 - 400$

$n \le \frac{9500}{800}$

$n \le 11.875$

$n = 11$

Total meetings = 1 + n

= 1 + 11

= 12

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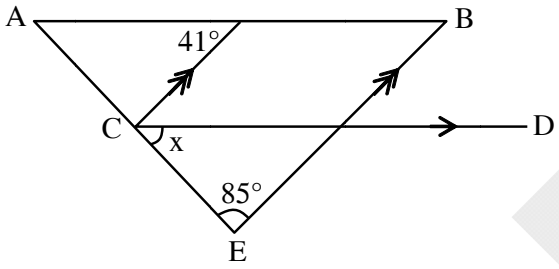
SUB-JUNIOR LEVEL - VII & VIII GRADES

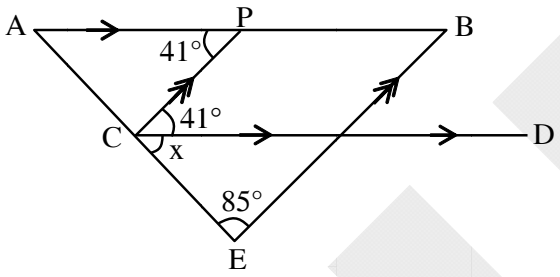
2. In the adjoining figure, the measure of the angle x is



(A) 84° (B) 44° (C) 64° (D) 54°

Ans. (D)

Sol.



As, AP || CD

$\angle APC = \angle PCD = 41°$

As, CP || EB

$\Rightarrow (x + 41°) + 85 = 180°$ (co-interior angles)

$\Rightarrow x = 95 - 41°$

$\Rightarrow x = 54°$

3. The value of x which satisfies $\frac{1}{x+a}+\frac{1}{x+b}=\frac{1}{x+a+b}+\frac{1}{x}$ is

(A) $\frac{a+b}{2}$ (B) $\frac{a-b}{2}$ (C) $\frac{b-a}{2}$ (D) $\frac{-(a+b)}{2}$

Ans. (D)

Sol. $\frac{1}{x+a}+\frac{1}{x+b}=\frac{1}{x+a+b}+\frac{1}{x}$

$\frac{1}{x+a}-\frac{1}{x}=\frac{1}{x+a+b}-\frac{1}{x+b}$

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SUB-JUNIOR LEVEL - VII & VIII GRADES

$$\frac{x-(x+a)}{x(x+a)}=\frac{(x+b)-(x+a+b)}{(x+b)(x+a+b)}$$

$$\Rightarrow \frac{-a}{x(x+a)}=\frac{-a}{(x+b)(x+a+b)}$$

$\Rightarrow x^2 + x(a + 2b) + b(a + b) = x^2 + ax$

$\Rightarrow ax + 2bx + b(a + b) = ax$

$\Rightarrow b[2x + (a + b)] = 0$

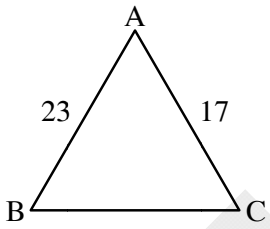
$$x=-\frac{(a+b)}{2}$$

4. Two sides of an isosceles triangle are 23 cm and 17 cm respectively. The perimeter of the triangle (in cm) is

(A) 63 (B) 57 (C) 63 or 57 (D) 40

Ans. (C)

Sol.



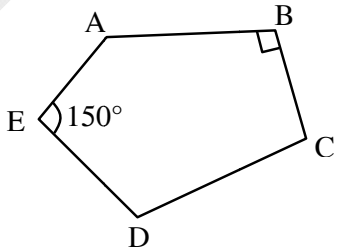
If ΔABC is isosceles Δ

$\Rightarrow$ BC = 23 or 17

Then perimeter = 23 + 17 + 23 = 63

= 23 + 17 + 17 = 57

5. ABCDE is a pentagon with $\angle B = 90°$ and $\angle E = 150°$. If $\angle C + \angle D = 180°$ and $\angle A + \angle D = 180°$, then the external angle $\angle D$ is



(A) 120° (B) 110° (C) 105° (D) 115°

Ans. (A)

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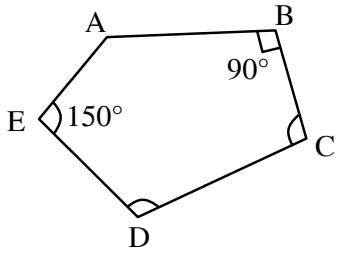
SUB-JUNIOR LEVEL - VII & VIII GRADES

Sol.



$\Rightarrow \angle A + \angle B + \angle C + \angle D + \angle E = 540°$ (Angle sum property)

$\Rightarrow \angle A + 90° + 180° + 150° = 540°$

$\Rightarrow \angle A = 120°$

As, $\angle A + \angle D = 180°$

$\angle D = 180° - 120° = 60°$

$\Rightarrow$ ext. $\angle D = 120°$

6. The unit's digit of the product $3^{2025} \times 7^{2024}$ is

(A) 1 (B) 2 (C) 3 (D) 6

Ans. (C)

Sol. $3^{2025} \times 7^{2024}$

By cyclicity of 3 & 7

(_____3) × (_____1)

= (_____3)

7. The smallest positive integer n for which 18900 × n is a perfect cube is

(A) 189 (B) 18900 (C) 21 (D) 490

Ans. (D)

Sol. $18900 \times n \rightarrow$ perfect cube

$\Rightarrow n \times 3^3 \times 7^1 \times 2^2 \times 5^2 \Rightarrow$ To convert into perfect cube, all power should be multiple of 3

$\Rightarrow n = 7^2 \times 2^1 \times 5^1$

$n = 490$

8. Two numbers a and b are respectively 20% and 50% more of a third number c. The percentage of a to b is

(A) 120% (B) 80% (C) 75% (D) 110%

Ans. (B)

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SUB-JUNIOR LEVEL - VII & VIII GRADES

Sol. According to questions

$a = \frac{120c}{100}$ ....(1)

$b = \frac{150c}{100}$ ....(2)

Divide

$\frac{eq.(1)}{eq.(2)} \Rightarrow \frac{a}{b} = \frac{4}{5}$

Then, $\frac{a}{b} \times 100 = \frac{4}{5} \times 100 = 80\%$

9. If $a + b = 2, \frac{1}{a} + \frac{1}{b} = 18$, then $a^3 + b^3$ lies between

(A) 7 and 8 (B) 6 and 7 (C) 8 and 9 (D) 5 and 6

Ans. (A)

Sol. $a + b = 2$

$\frac{1}{a} + \frac{1}{b} = 18 \Rightarrow \frac{a+b}{ab} = 18$

$\Rightarrow \frac{2}{ab} = 18$

$\Rightarrow ab = \frac{1}{9}$

$\Rightarrow a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

$= (2)^3 - 3\left(\frac{1}{9}\right)(2)$

$= 8 - \frac{2}{3} = \frac{22}{3} = 7\frac{1}{3}$

10. If $\sqrt{12 + \sqrt[3]{x}} = \frac{7}{2}$ and $x = \frac{p}{q}$, p, q are natural numbers with G.C.D. (p. q) = 1, then p + q is

(A) 65 (B) 56 (C) 45 (D) 54

Ans. (A)

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Sol. $\sqrt{12+\sqrt[3]{x}}=\frac{7}{2}$

By squaring

$12+\sqrt[3]{x}=\frac{49}{4}$

$\sqrt[3]{x}=\left(\frac{49}{4}-12\right)=\frac{1}{4}$

By Cubing

$x=\frac{1}{64}=\frac{p}{q}$

Then, p + q = 65

11. The smallest number of 4-digits leaving a remainder 1 when divided by 2 or 3 or 4 or 6 has

(A) 5 as its unit digit (B) Only one zero as one of the digits

(C) Exactly two zeroes as its digits (D) 7 as its unit digit

Ans. (C)

Sol. According to question

Smallest, x = LCM(2, 3, 4, 6) + 1

= 12 + 1

Then smallest 4-digit number is 1008 i.e. divisible by 2, 3, 4, 6

Required number x

$\Rightarrow$ x = 1008 + 1

x = 1009

According to question

Exactly two zeroes as its digits

12. If a : b = 2 : 3, b : c = 4 : 5 and a + c = 736, then the value of b is

(A) 392 (B) 378 (C) 384 (D) 386

Ans. (C)

Sol. $\frac{a}{b}=\frac{2}{3}=\frac{8}{12}$

$\left.\begin{matrix}a:b:c\\8:12:15\end{matrix}\right\}\Rightarrow\frac{a}{c}=\frac{8}{15}$

Let a = 8k

c = 15k

a + c = 736

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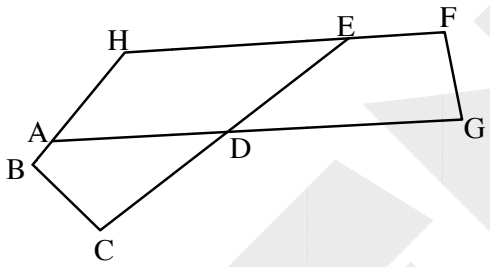
$23k = 736$

$\Rightarrow k = 32$

Then, $b = 12k = 12 \times 32 = 384$

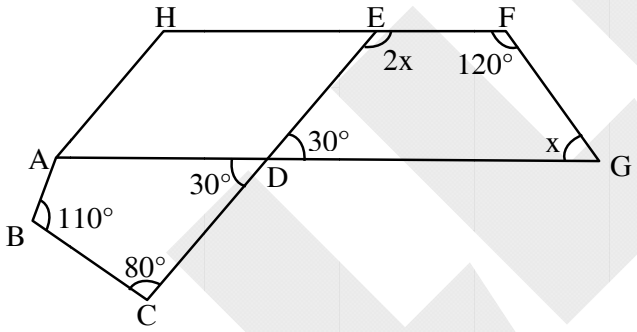
13. In the given figure, $\angle B = 110°$; $\angle C = 80°$; $\angle F = 120°$; $\angle ADC = 30°$; $2\angle DGF = \angle DEF$. The measure of $\angle BHF$ is



(A) 115° (B) 135° (C) 100° (D) 130°

Ans. (D)

Sol.



In quadrilateral □ABCD

$\angle BAD + 110° + 80° + 30° = 360°$

$\angle BAD = 360° - 220°$

$\angle BAD = 140°$

$\angle HAD = 180° - 140°$ (Linear pair)

$2\angle DGF = \angle DEF$

Let, $\angle DGF = x$

$\angle DEF = 2x$

$\angle EDG = \angle ADC = 30°$ (Vertically opposite angles)

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$\angle F = 120°$

In quadrilateral □EDGF

$2x + 120° + x + 30° = 360°$

$3x + 150° = 360°$

$3x = 360° - 150°$

$3x = 210°$

$x = 70°$

Now, In quadrilateral □HAGF

$\angle AHF + \angle HAG + \angle AGF + \angle GFH = 360°$

$\angle AHF + 40° + x + 120° = 360°$

$\angle AHF = 360° - 120° - 40° - x$ (use $x = 70°$)

$= 360° - 120° - 40° - 70°$

$= 360° - 230°$

$\angle AHF = 130°$

14. If $\frac{1}{b+c}+\frac{1}{c+a}=\frac{2}{a+b'}$ then the value of $\frac{a^2+b^2}{c^2}$ is

(A) 2 (B) 1 (C) $\frac{1}{2}$ (D) 3

Ans. (A)

Sol. $\frac{2c+a+b}{(b+c)(c+a)}=\frac{2}{a+b}$

$(a + b + c + c)(a + b) = 2(b + c)(c + a)$

$a^2 + ab + ab + b^2 + ac + bc + ac + bc = 2[bc + ab + c^2 + ac]$

$a^2 + b^2 = 2c^2$

$\frac{a^2+b^2}{c^2}=2$

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15. If 3 men or 4 women can do a job in 43 days, the number of days the same job is done by 7 men and 5 women is

(A) 12 (B) 10 (C) 11 (D) 13

Ans. (A)

Sol. $3m = 4w$

$1m = \frac{4w}{3}$

Total $= 7m + 5w$

$= 7 \times \frac{4w}{3} + 5w$

$= \frac{43w}{3}$

$M_1D_1 = M_2D_2$

$M_1 = 4w$

$D_1 = 43$ days

$M_2 = \frac{43w}{3}$

$D_2 = ?$

$4w \times 43 = \frac{43w}{3} \times D_2$

$D_2 = 12$ days

Section B (Fill in the Blanks)

16. The expression $49(a + b)^2 - 46(a - b)^2$ is factorized into $(\ell a + mb)(na + pb)$, then the numerical value of $(\ell + m + n + p)$ is _____.

Ans. 28

Sol. $\left[7(a+b)\right]^2 - \left(\sqrt{46}(a-b)\right)^2$

$\left[7(a+b)+\sqrt{46}(a-b)\right]\left[7(a+b)-\sqrt{46}(a-b)\right]$

$\left[\left(7+\sqrt{46}\right)a+\left(7-\sqrt{46}\right)b\right]\left[\left(7-\sqrt{46}\right)a+\left(7+\sqrt{46}\right)b\right]$

$\ell + m + n + p = 7+\sqrt{46}+7-\sqrt{46}+7-\sqrt{46}+7+\sqrt{46}$

$= 28$

SOLUTION

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SUB-JUNIOR LEVEL - VII & VIII GRADES

17. The integer part of the solution of the equation in x,

$\frac{1}{3}(x-3)-\frac{1}{4}(x-8)=\frac{1}{5}(x-5)$ is _____.

Ans. 17

Sol. $\frac{x-3}{3}-\frac{x-8}{4}=\frac{x-5}{5}$

$\frac{4x-12-3x+24}{12}=\frac{x-5}{5}$

$(x + 12) \times 5 = (x - 5) \times 12$

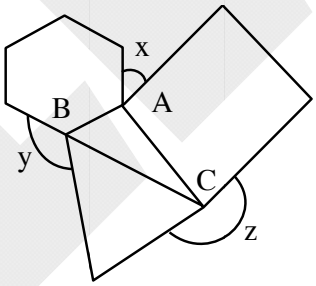
$5x + 60 = 12x - 60$

$7x = 120$

$x=\frac{120}{7}=17.14$

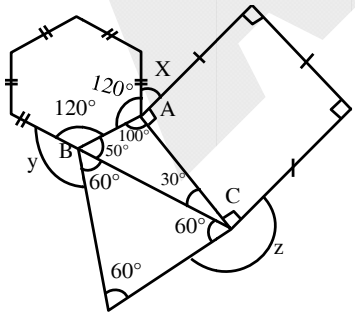
Integer part = 17

18. In the adjoining figure, ABC is a triangle in which $\angle BAC = 100°$, $\angle ACB = 30°$. An equilateral triangle, a square and a regular hexagon are drawn as shown in the figure. The measure (in degrees) of $(x + y + z)$ is _____.



Ans. 360°

Sol.



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SUB-JUNIOR LEVEL - VII & VIII GRADES

According to figure in ΔABC

$\angle B = 50°$

Each interior angle of equilateral Δ is $60°$

Similarity square = $90°$

Regular hexagon = $120°$

$120° + 100° + 90° + X = 360°$

$Y + 60° + 50° + 120° = 360°$

$60° + 30° + 90° + Z = 360°$

$\angle X = 50°, \angle Y = 130°, \angle Z = 180°$

$X + Y + Z = 360°$

19. The mean of 5 numbers is 105. The first number is $\frac{2}{5}$ times the sum of the other 4 numbers. The first number is _____.

Ans. 150

Sol. Let number are x_1, x_2, x_3, x_4, x_5

$x_1 + x_2 + x_3 + x_4 + x_5 = 5 \times 105 = 525$

$$x_1 = \frac{2}{5}[x_2 + x_3 + x_4 + x_5]$$

$5x_1 = 2(x_2 + x_3 + x_4 + x_5)$

$5x_1 = 2[525 - x_1]$

$5x_1 = 1050 - 2x_1$

$x_1 = 150$

20. PQRS is a square. The sides PQ and RS are increased by 30% each and the sides QR and PS are increased by 20% each. The area of the quadrilateral thus obtained exceeds the area of the square by _____%.

Ans. 56

Sol. PQ = QR = RS = PS = a(side)

Area = a^2

New sides are

$$PQ = \frac{13a}{10}, RS = \frac{13a}{10}$$

$$QR = \frac{6a}{5}, PS = \frac{6a}{5}$$

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New area $= \frac{13a}{10} \times \frac{6a}{5} = \frac{39a^2}{25}$

% area increased $= \frac{\frac{39a^2}{25} - a^2}{a^2} \times 100$

$= 56\%$

21. If $x^2 + (2+\sqrt{3})x - 1 = 0$ and $x^2 + \frac{1}{x^2} = a + b\sqrt{c}$, then $(a + b + c)$ is _____.

Ans. 16

Sol. $x^2 + (2+\sqrt{3})x - 1 = 0$

$x^2 - 1 = -(2+\sqrt{3})x$

$\frac{x^2 - 1}{x} = -(2+\sqrt{3})$

$x - \frac{1}{x} = -(2+\sqrt{3})$

$\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$

$(-(2+\sqrt{3}))^2 + 2 = x^2 + \frac{1}{x^2}$

$4 + 3 + 4\sqrt{3} + 2 = x^2 + \frac{1}{x^2}$

$9 + 4\sqrt{3} = x^2 + \frac{1}{x^2}$

$x^2 + \frac{1}{x^2} = a + b\sqrt{c}$

By comparing

$a = 9, b = 4, c = 3$

$a + b + c = 9 + 4 + 3 = 16$

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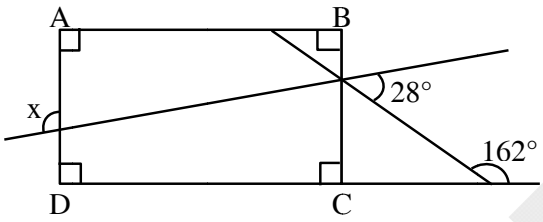
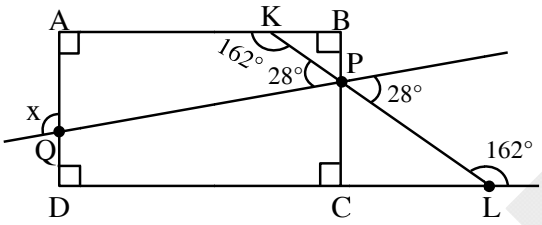
22. In the given figure, ABCD is a rectangle. The measure of angle x is_____ degrees.



Ans. 100°

Sol.



AB||DC

∠AKL = 162 (Alternate interior angles)

∠KPQ = 28 (Vertically opposite angle)

In □AQPK

∠A = 90°

∠AQP = 180° – x (Linear pair)

∠KPQ = 28

∠AKL = 162

So, 90° + 180° – x + 28 + 162 = 360°

–x = 360 – 460

–x = –100 x = 100°

23. The sum of all positive integers m, n which satisfy $m^2 + 2mn + n = 44$ is _____.

Ans. 18

Sol. $2mn + n = 44 - m^2$

$n[2m + 1] = 44 - m^2$

$$n = \frac{44 - m^2}{2m + 1}$$

Put m = 1 then $n = \frac{43}{3}$ (not a integer)

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$m = 2,\ n = \frac{40}{5} = 8$ (m = 2 and n = 8)

$m = 3,\ n = \frac{35}{7} = 5$ (m = 3 and n = 5)

$m = 4,\ n = \frac{28}{9}$ (not a integer)

$m = 5,\ n = \frac{19}{11}$ (not integer)

$m = 6,\ n = \frac{8}{13}$ (not integer)

So, m = 2 and n = 8 and m = 3, n = 5 are the two pairs of solution of m and n

So, sum of all positive integers m, n is 2 + 8 + 3 + 5 = 18

24. Given a = 2025, b = 2024, the numerical value of

$$\left(a+b-\frac{4ab}{a+b}\right)\div\left(\frac{a}{a+b}-\frac{b}{b-a}+\frac{2ab}{b^2-a^2}\right) \text{ is } \_\_\_\_\_.$$

Ans. 1

Sol. $\left(\frac{(a+b)^2-4ab}{(a+b)}\right)\div\left(\frac{a(b-a)-b(a+b)}{(b+a)(b-a)}+\frac{2ab}{b^2-a^2}\right)$

$\left(\frac{a^2+b^2+2ab-4ab}{a+b}\right)\div\left(\frac{ab-a^2-ab-b^2}{b^2-a^2}+\frac{2ab}{b^2-a^2}\right)$

$\left(\frac{a^2+b^2-2ab}{a+b}\right)\div\left(-\frac{(a^2+b^2)}{b^2-a^2}+\frac{2ab}{b^2-a^2}\right)$

$\frac{(a-b)^2}{(a+b)}\div\left(-\frac{(a^2+b^2-2ab)}{b^2-a^2}\right)$

$\frac{(a-b)^2}{(a+b)}\times\frac{(b^2-a^2)}{-(a-b)^2}$

$\frac{(b+a)(b-a)}{-(a+b)} = -(b-a) = (a-b) = 2025 - 2024 = 1$

SOLUTION

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25. In the sequence 0, 7, 26, 63, 124, …. ….. ….. the 6^{th} term is _____.

Ans. 215

Sol. 0, 7, 26, 63, 124, ……

$0 = 1^3 - 1$ (1^{st} term)

$7 = 2^3 - 1$ (2^{nd} term)

$26 = 3^3 - 1$ (3^{rd} term)

$63 = 4^3 - 1$ (4^{th} term)

$124 = 5^3 - 1$ (5^{th} term)

6^{th} term $= 6^3 - 1 = 215$

26. If $A = \sqrt{281+\sqrt{53+\sqrt{112+\sqrt{81}}}}$, $B = \sqrt{92+\sqrt{55+\sqrt{75+\sqrt{36}}}}$, then A – B is _____.

Ans. 7

Sol. $A = \sqrt{281+\sqrt{53+\sqrt{112+\sqrt{81}}}}$

$A = \sqrt{281+\sqrt{53+\sqrt{112+9}}}$

$A = \sqrt{281+\sqrt{53+\sqrt{121}}}$

$A = \sqrt{281+\sqrt{53+11}}$

$A = \sqrt{281+\sqrt{64}}$

$A = \sqrt{281+8}$

$A = \sqrt{289} = 17$

$B = \sqrt{92+\sqrt{55+\sqrt{75+\sqrt{36}}}}$

$B = \sqrt{92+\sqrt{55+\sqrt{75+6}}}$

$B = \sqrt{92+\sqrt{55+\sqrt{81}}}$

$B = \sqrt{92+\sqrt{55+9}}$

$B = \sqrt{92+\sqrt{64}}$

$B = \sqrt{92+8}$

$B = \sqrt{100}$

$B = 10$

$A - B = 17 - 10 = 7$

SOLUTION

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27. The average of the numbers a, b, c, d is (b + 4). The average of pairs (a, b), (b, c) and (c, a) are respectively 16, 26 and 25. Then the average of d and 67 is _____.

Ans. 42

Sol. $\frac{a+b+c+d}{4} = b+4$

$\frac{a+b}{2} = 16$

$a + b = 32$ .....(1)

$\frac{b+c}{2} = 26$

$b + c = 52$ .....(2)

$\frac{a+c}{2} = 25$

$a + c = 50$ .....(3)

Add (1),(2),(3)

$2(a + b + c) = 134$

$a + b + c = 67$

$a + b + c = 67$

$a + c = 50$ (from (3))

$a + c + b = 67$

$50 + b = 67$

$b = 67 - 50$

$b = 17$

Now, $\frac{a+b+c+d}{4} = b+4$ .....(5)

$\left\{\begin{matrix} a + b + c = 67, \\ b = 17 \end{matrix}\right\}$

$\Rightarrow$ from (5)

$\frac{67+d}{4} = b + 4$

$67 + d = 4(b + 4) = 4(17 + 4)$

$67 + d = 84$

Now, average of 67 and d

$\frac{67+d}{2} = \frac{84}{2} = 42$

SOLUTION

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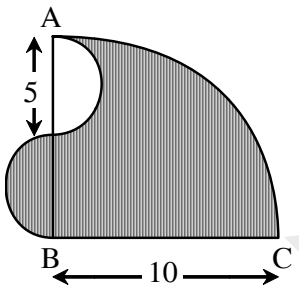
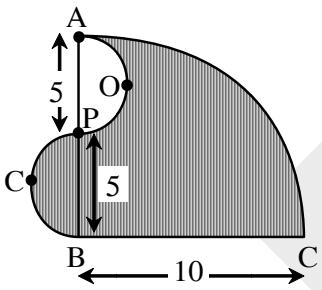
28. ABC is a quadrant of a circle of radius 10 cm. Two semicircles are drawn as in the figure. The area of the shaded portion is kπ , where k is a positive integer. The value of k is _____.



Ans. 25

Sol.



Area semicircle (PCB) = area of semicircle (AOP)

So, area of shaded is equivalent to area of quadrant ABC

So, area of quadrant = $\frac{\pi r^2}{4}$

$= \frac{\pi(10)^2}{4}$

$= \frac{\pi \times 100}{4}$

$= 25\pi$

Comparing with kπ

k = 25

SOLUTION

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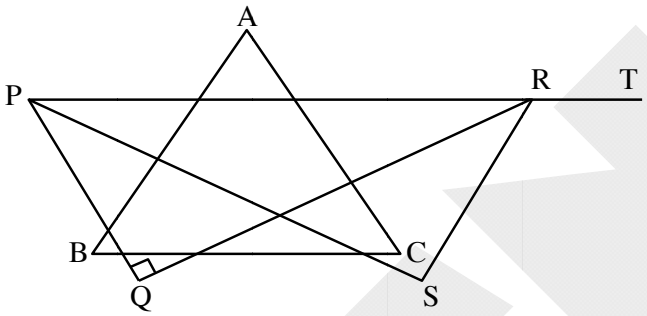
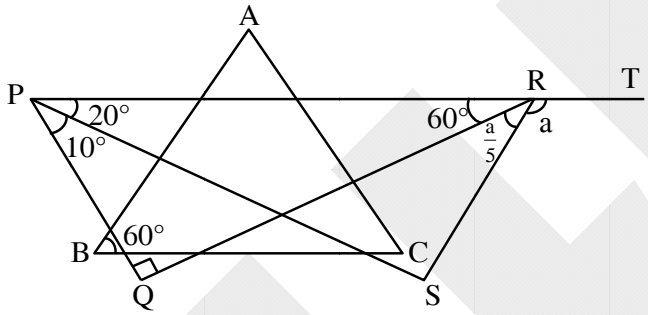
29. In the figure, ABC and PQR are two triangles such that $\angle A : \angle B : \angle C = 5 : 6 : 7$ and $\angle PRQ = \angle B$. PS makes an angle $\frac{\angle P}{3}$ with PQ and RS makes an angle $\frac{\angle SRT}{5}$ with RQ. Then the measure of $\angle S$ is _____.



Ans. **80°**

Sol.



$\angle A = 5x$

$\angle B = 6x$

$\angle C = 7x$

$\angle A + \angle B + \angle C = 180°$

$11x + 7x = 180°$

$18x = 180°$

$x = 10$

$\angle A = 5 \times 10 = 50°$

$\angle B = 6 \times 60°$

$\angle C = 7 \times 10 = 70°$

$\angle PRQ = \angle B$

$\angle PRQ = 60°$ ….(1)

In ΔPRQ

SOLUTION

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$\angle P + \angle Q + \angle R = 180°$

$\angle P + 90 + 60 = 180°$

$\angle P = 180 - 150$

$\angle P = 30$

PS makes angle $\frac{\angle P}{3}$ with PQ

So, $\angle QPS = \frac{\angle P}{3} = \frac{30}{3} = 10$

$\angle SRT = a$

$\angle QRS = \frac{a}{5}$

In ΔPQR

$\angle PRQ = \angle B = 60°$ (given)

So, $\angle PRQ + \angle QRS + \angle SRT = 180°$

$a + \frac{a}{5} + 60 = 180$

$\frac{6a}{5} + 60 = 180$

$\frac{6a}{5} = 120$

$a = \frac{120 \times 5}{6}$

$a = 100$

In ΔPSR

Exterior angle SRT

$Ext(\angle SRT) = 20 + \angle S$

$a = 20 + \angle S$

use $a = 100$

$100 = 20 + \angle S$

$\angle S = 100 - 20$

$\angle S = 80°$

SOLUTION

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30. In a two-digit positive integer, the units digit is one less than the tens digit. The product of one less than the units digit and one more than the tens digit is 40. The number of such two-digit integers is _____.

Ans. 1

Sol. Let suppose two digit number = 10x + y

$y = x - 1$ ....(1)

$(y - 1)(x + 1) = 40$ ....(2)

$(y - 1)(y + 2) = 40$

$y^2 + 2y - y - 2 = 40$

$y^2 + y - 42 = 0$

$y^2 + y - 42 = 0$

$y^2 + 7y - 6y - 42 = 0$

$y(y + 7) - 6(y + 7) = 0$

$y = 6, y = -7$

$x = 7$

Number is = $10 \times 7 + 6 = 76$

Number of such 2 digit integers = 1