SOLUTION

THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

57^{th} NMTC - SCREENING TEST - BHASKARA CONTEST

NMTC - JUNIOR LEVEL - IX & X GRADES

1. The greatest 4-digit number such that when divided by 16, 24 and 36 leaves 4 as remainder in each case is

(A) 9994 (B) 9940 (C) 9094 (D) 9904

Ans. (B)

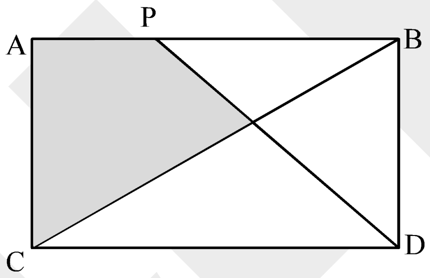
Sol. No which is divisible by 16, 24, 36

$= \text{LCM}(16,24,36) = 144$

Largest 4 digit no divisible by 144 = $144 \times 69 = 9936$

So largest 4 digit no divisible by 16, 24, 36 leave the remainder 4 = $9936 + 4 = 9940$ Ans (B)

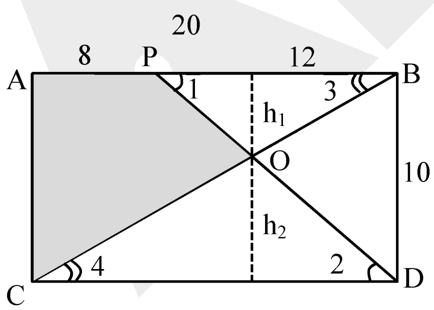
2. ABCD is a rectangle whose length AB is 20 units and breadth is 10 units. Also, given AP = 8 units. The area of the shaded region is $\frac{p}{q}$ sq unit, where p, q are natural numbers with no common factors other than 1. The value of p + q is



(A) 167 (B) 147 (C) 157 (D) 137

Ans. (C)

Sol.



$\angle 1 = \angle 2$ (Alternate interior angle)

$\angle 3 = \angle 4$ (Alternate interior angle)

By AA

$\Delta POB \sim \Delta DOC$

$$\frac{PB}{CD} = \frac{h_1}{h_2}$$

$$\frac{12}{20} = \frac{h_1}{h_2} \Rightarrow h_1 : h_2 = 3:5$$

$$h_1 = \frac{3}{8} \times 10 = 3.75$$

$$\text{ar } \Delta POB = \frac{1}{2} PB \times h_1$$

$$= \frac{1}{2} \times 12 \times 3.75$$

$$= \frac{45}{2}$$

$$\text{ar } \Delta ABC = \frac{1}{2} \text{ Area of rectangle ABCD}$$

$$= \frac{1}{2} \times 10 \times 20 = 100$$

Shaded area = ar ΔABC – arΔPOB

$$= 100 - \frac{45}{2}$$

$$= \frac{200-45}{2} = \frac{155}{2} = \frac{p}{q}$$

$p + q = 155 + 2 = 157$ Option (C)

3. The solution of $\frac{\sqrt[7]{12+x}}{x} + \frac{\sqrt[7]{12+x}}{12} = \frac{64}{3}\left(\sqrt[7]{x}\right)$ is of the form $\frac{a}{b}$ where a, b are natural numbers with GCD (a,b) = 1; then (b – a) is equal to

(A) 115 (B) 114 (C) 113 (D) 125

Ans. (A)

Sol. $\sqrt[7]{12+x}\left(\frac{1}{x} + \frac{1}{12}\right) = \frac{64}{3}\sqrt[7]{x}$

$$\sqrt[7]{12+x}\left(\frac{12+x}{12x}\right) = \frac{64}{3}\sqrt[7]{x}$$

$$(12+x)^{\frac{8}{7}} = \frac{64}{3} \times 12x\sqrt[7]{x}$$

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$(12+x)^{\frac{8}{7}} = 256x^{8/7}$

$\left(\frac{12+x}{x}\right)^{\frac{8}{7}} = 256$

$\frac{12+x}{x} = (256)^{7/8} = 128$

12 + x 128x

12 = 127x

$x = \frac{12}{127} = \frac{a}{b}$

b – a = 127 – 12

= 115 Option (A)

4. The value of $\left(52+6\sqrt{43}\right)^{3/2} - \left(52-6\sqrt{43}\right)^{3/2}$ is

(A) 858 (B) 918 (C) 758 (D) 828

Ans. (D)

Sol. $x = \left(52+6\sqrt{43}\right)^{3/2} - \left(52-6\sqrt{43}\right)^{3/2}$

$= \left(\sqrt{52+6\sqrt{43}}\right)^3 - \left(\sqrt{52-6\sqrt{43}}\right)^3$

$= \left(\sqrt{52+2\sqrt{387}}\right)^3 - \left(\sqrt{52-2\sqrt{387}}\right)^3$

$= \left(\sqrt{43}+\sqrt{9}\right)^3 - \left(\sqrt{43}-\sqrt{9}\right)^3$

$= 2(3)^3 + 6\left(\sqrt{43}\right)^2(3)$

= 54 + 18 × 43

= 54 + 774

= 828 Option (D)

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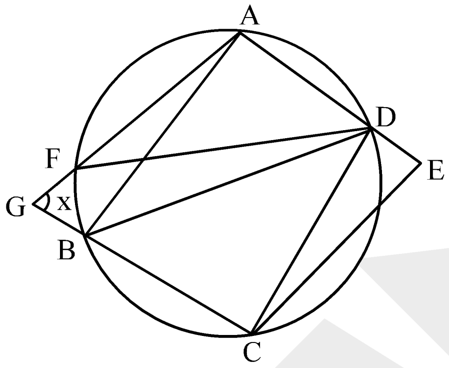
NMTC - JUNIOR LEVEL - IX & X GRADES

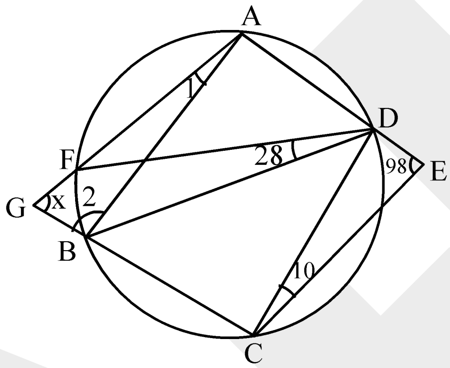
5. In the adjoining figure ∠DCE =10°, ∠CED = 98° , ∠BDF = 28°.
Then the measure of angle x is



(A) 72° (B) 76° (C) 44° (D) 82°

Ans. (C)

Sol.



∠1=∠BDF (Angle in a same segment)

∠1= 28

∠ADC = 10° + 98° (exterior angle equal to sum of interior of angle)

= 108°

∠2 = ∠ADC (exterior angle of cyclic quadrilateral is equal to interior opp. angle)

= 108°

In ΔABG

x = 180 – (∠ 1+∠2)

= 180 – (28 + 108)

= 44°

SOLUTION

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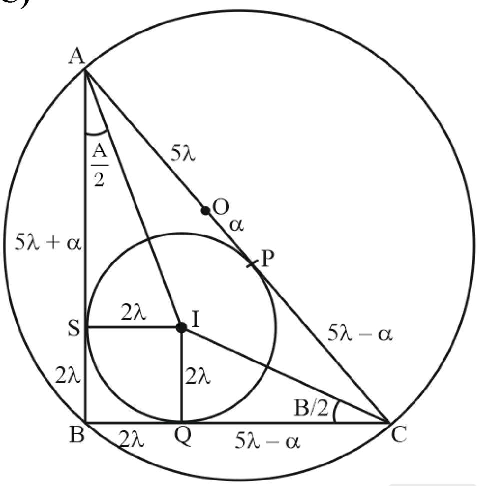
NMTC - JUNIOR LEVEL - IX & X GRADES

6. ABC is a right triangle in which $\angle B = 90°$. The inradius of the triangle is r and the circumradius of the triangle is R. If R:r = 5:2, then the value of $\cot^2\frac{A}{2}+\cot^2\frac{C}{2}$ is

(A) $\frac{25}{4}$ (B) 17 (C) 13 (D) 14

Ans. (C)

Sol.



$\frac{R}{r}=\frac{5}{2}$

$R = 5\lambda$

$r = 2\lambda$

$AP = 5\lambda + \alpha$ $CP = 5\lambda - \alpha$

$AS = AP = 5\lambda + \alpha$ $CQ = CP = 5\lambda - \alpha$

$AB = AS + BS$ $AC = 2R = 10\,\lambda$

$= 5\lambda + \alpha + 2\lambda$

$= 7\lambda + \alpha$

$BC = BQ + QC$

$= 2\lambda + 5\lambda - \alpha$

$= 7\lambda - \alpha$

$AC^2 = AB^2 + BC^2$

$(10\lambda)^2 = (7\lambda + \alpha)^2 + (7\lambda + \alpha)^2$

$\lambda^2 = \alpha^2$

$\lambda = \alpha$

$\cot\frac{A}{2}=\frac{AS}{SI}=\frac{5\lambda+\alpha}{2\lambda}=\frac{6\lambda}{2\lambda}=3$

$\cot\frac{C}{2}=\frac{CQ}{QI}=\frac{5\lambda-\alpha}{2\lambda}=\frac{4\lambda}{2\lambda}=2$

$\cot^2\frac{A}{2}+\cot^2\frac{C}{2}=3^2+2^2=13$. Option (C)

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7. If (α , β) and (γ , β) are the roots of the simultaneous equations:

$$|x-1| + |y-5| = 1;\ y = 5 + |x-1|$$

then the value of α + β + γ is

(A) $\frac{15}{2}$ (B) $\frac{17}{2}$ (C) $\frac{14}{3}$ (D) $\frac{19}{2}$

Ans. (A)

Sol. $y = 5 + |x - 1|$

From this we conclude $y - 5 \geq 0$

Case-I $x - 1 \geq 0$ & $y - 5 \geq 0$

$x - 1 + y - 5 = 1$ & $y = 5 + x - 1$

$x + y = 7$ ...(1) $x - y = -4$ ...(2)

on solving (1) & (2)

$$x = \frac{3}{2},\quad y = \frac{11}{2}$$

Case-II $x - 1 < 0$

$-(x - 1) + y - 5 = 1$ & $y = 5 - (x - 1)$

$-x + 1 + y - 5 = 1$ $y = 5 - x + 1$

$-x + y = 5$ ...(3) $x + y = 6$ ...(4)

on solving (3) & (4)

$$y = \frac{11}{2},\quad x = \frac{1}{2}$$

$$\therefore\ \alpha = \frac{1}{2},\ \beta = \frac{11}{2},\ \gamma = \frac{3}{2}$$

$$\alpha + \beta + \gamma = \frac{1}{2} + \frac{11}{2} + \frac{3}{2} = \frac{15}{2}$$

8. Three persons Ram, Ali and Peter were to be hired to paint a house. Ram and Ali can paint the whole house in 30 days, Ali and Peter in 40 days while Peter and Ram can do it in 60 days. If all of them were hired together, in how many days can they all three complete 50% of the work?

(A) $24\frac{1}{3}$ (B) $25\frac{1}{2}$ (C) $26\frac{1}{3}$ (D) $26\frac{2}{3}$

Ans. (Bonus)

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Sol. No of days required by Ram to paint a house = r

No of days required by Peter to paint a house = p

No of days required by Ali to paint a house = a

ATQ. $\frac{1}{r}+\frac{1}{a}=\frac{1}{30}$ ..(1)

$\frac{1}{a}+\frac{1}{p}=\frac{1}{40}$ ..(2)

$\frac{1}{p}+\frac{1}{r}=\frac{1}{60}$ ..(3)

Add (1), (2), (3)

$$2\left(\frac{1}{r}+\frac{1}{a}+\frac{1}{p}\right)=\frac{1}{30}+\frac{1}{40}+\frac{1}{60}$$

$$=\frac{4+3+2}{120}=\frac{9}{120}$$

$$\frac{1}{r}+\frac{1}{a}+\frac{1}{p}=\frac{9}{240}=\frac{3}{80}$$

so all of them complete the work in $\frac{80}{3}$ days.

so to complete 50% of work they required = $\frac{1}{2}\times\frac{80}{3}=\frac{40}{3}=13\frac{1}{3}$ days.

Bonus as the answer is not in option

9. $\frac{\sqrt{a+3b}+\sqrt{a-3b}}{\sqrt{a+3b}-\sqrt{a-3b}}=x$, then the value of $\frac{3bx^2+3b}{ax}$ is

(A) 1 (B) 2 (C) 3 (D) 4

Ans. (B)

Sol. $\frac{x}{1}=\frac{\sqrt{a+3b}+\sqrt{a-3b}}{\sqrt{a+3b}-\sqrt{a-3b}}$

Apply compodendo & dividendo

$$\frac{x+1}{x-1}=\frac{\sqrt{a+3b}}{\sqrt{a-3b}}$$

Squaring on both side

$$\frac{(x+1)^2}{(x-1)^2}=\frac{a+3b}{a-3b}$$

Apply compodendo & dividendo

$$\frac{(x+1)^2+(x-1)^2}{(x+1)^2-(x-1)^2}=\frac{a}{3b}$$

$$\frac{2(x^2+1)}{4x}=\frac{a}{3b}$$

$3bx^2+3b = 2xa$

$$\frac{3bx^2+3b}{ax}=2 \quad \text{Option (B)}$$

10. The number of integral solutions of the inequation $\left|\frac{2}{x-13}\right|>\frac{8}{9}$ is

(A) 1 (B) 2 (C) 3 (D) 4

Ans. (D)

Sol. $\left|\frac{2}{x-13}\right|>\frac{8}{9}$

$$\frac{2}{x-13}>\frac{8}{9} \text{ or } \frac{2}{x-13}<\frac{-8}{9}$$

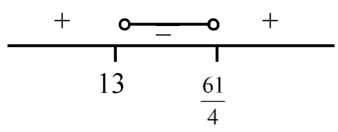
$$\frac{1}{x-13}-\frac{4}{9}>0 \text{ or } \frac{1}{x-13}<\frac{-4}{9}$$

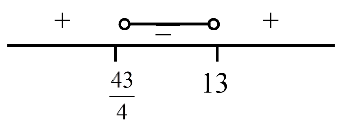
$$\frac{9-4(x-13)}{9(x-13)}>0 \text{ or } \frac{1}{x-13}+\frac{4}{9}<0$$

$$\frac{9-4x+52}{9(x-13)}>0 \text{ or } \frac{9+4(x-13)}{(x-13)9}<0$$

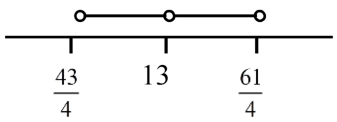
$$\frac{-4x+61}{9(x-13)}>0 \text{ or } \frac{9+4x-52}{9(x-13)}<0$$

$$\frac{4x-61}{9(x-13)}<0 \text{ or } \frac{4x-43}{9(x-13)}<0$$





Final solution



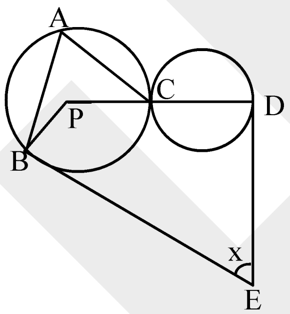
so $x \in \left(\frac{43}{4}, \frac{61}{4}\right) - \{13\}$

$x = 11, 12, 14, 15$

so 4 integral solution

Option (D)

11. In the adjoining figure, P is the centre of the first circle, which touches the other circle in C. PCD is along the diameter of the second circle. $\angle PBA = 20°$ and $\angle PCA = 30°$. The tangents at B and D meet at E. The measure of the angle x is



(A) 75° (B) 80° (C) 70° (D) 85°

Ans. (B)

Sol. $\angle BAC = 20 + 30 = 50°$ $\because (\angle BAP = \angle PBA$ and $\angle CAP = \angle PCA)$

$\angle BPC = 2 \times \angle BAC$

$= 2 \times 50 = 100°$

DEBP is a cyclic quadrilateral.

$\angle BPC + x = 180$

$x = 180 - 100 = 80°$

12. If α, β are the values of x satisfying the equation $3\sqrt{\log_2 x} - \log_2 8x + 1 = 0$, where α < β, then the value of $\left(\frac{\beta}{\alpha}\right)$ is

(A) 2 (B) 4 (C) 6 (D) 8

Ans. (D)

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Sol. $3\sqrt{\log_2 x} - \log_2 8x + 1 = 0$ $\alpha < \beta$

$3\sqrt{\log_2 x} - 3 - \log_2 x + 1 = 0$

$\log_2 x = t^2$

$3t - 3 - t^2 + 1 = 0$

$t = 1,2$

$\Rightarrow \log_2 x = 1$ and $\log_2 x = 4$

$\Rightarrow x = 2$ and $x = 16$

$\Rightarrow \alpha = 2, \beta = 16$

$\Rightarrow \frac{\beta}{\alpha} = \frac{16}{2} = 8$

13. When a natural number is divided by 11, the remainder is 4. When the square of this number is divided by 11, the remainder is

(A) 4 (B) 5 (C) 7 (D) 9

Ans. (B)

Sol. $N = 11m + 4$

$N^2 = 121m^2 + 88m + 16$

$N^2 = 11(11m^2 + 8m + 1) + 5$

$N^2 = 11K+5$

Remainder will be 5.

14. The unit's digit of a 2-digit number is twice the ten's digit. When the number is multiplied by the sum of the digits the result is 144. For another 2-digit number, the ten's digit is twice the unit's digit and the product of the number with the sum of its digits is 567. Then the sum of the two 2-digit numbers is

(A) 68 (B) 86 (C) 98 (D) 87

Ans. (D)

Sol. Let two digit numbers are $10a + b$ and $10c + d$.

A.T.Q

$b = 2a$

$(10a + b)(a + b) = 144$

$(12a)(3a) = 144$

$a = 2, b = 4$

$10a + b = 24$

and c = 2d

(10c + d) (c + d) = 567

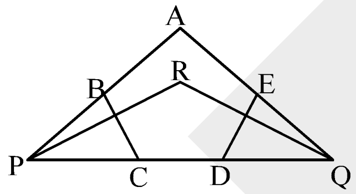
(21d) (3d) = 567

d = 3, c = 6

10c + d = 63

Sum of two, two digit number is 24 + 63 = 87

15. ABCDE is a pentagon. ∠AED = 126°, ∠BAE = ∠CDE and ∠ABC is 4° less than ∠BAE and ∠BCD is 6° less than ∠CDE. PR, QR the bisectors of ∠BPC, ∠EQD respectively, meet at R. Points P, C, D, Q are collinear. Then measure of ∠PRQ is



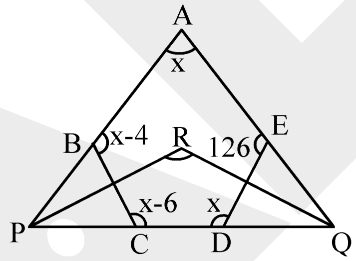
(A) 151° (B) 137° (C) 141° (D) 143°

Ans. **(D)**

Sol. Let ∠BAE = ∠CDE = x

∠ABC = x – 4

∠BCD = x – 6



In pentagone ABCDE:

Angle sum:

x + x – 4 + x – 6 + x + 126 = 540

x = 106°

$$\angle PRQ = 90 + \frac{\angle PAQ}{2}$$

$$= 90 + \frac{x}{2} = 90 + 53 = \boxed{143}$$

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Section B (Fill in the Blanks)

16. a, b, c are real numbers such that $b - c = 8$ and $bc + a^2 + 16 = 0$. The numerical value of $a^{2025} + b^{2025} + c^{2025}$ is ______.

Ans. **0**

Sol. $a, b, c \in R$

$b - c = 8$

$bc + a^2 + 16 = 0$

$a^2 + c(c + 8) + 16 = 0$

$a^2 + c^2 + 8c + 16 = 0$

$a^2 + (c + 4)^2 = 0$

$\Rightarrow\ a = 0, c = -4$

$b = c + 8 = 4$

$a^{2025} + b^{2025} + c^{2025} = 0 + 4^{2025} + (-4)^{2025} = 0$

17. Given $f(x) = \dfrac{2025x}{x+1}$ where $x \neq -1$. Then the value of x for which $f(f(x)) = (2025)^2$ is _______.

Ans. $-\dfrac{1}{2025}$

Sol. $f(x) = \dfrac{2025x}{x+1}\quad x \neq -1$

$f(f(x)) = (2025)^2$

$$\Rightarrow \frac{2025 \times \frac{2025x}{x+1}}{\frac{2025x}{x+1}+1} = (2025)^2$$

$$\Rightarrow \frac{(2025)^2 x}{2025x + x + 1} = (2025)^2$$

$x = 2026x + 1$

$x = -\dfrac{1}{2025}$

18. The sum of all the roots of the equation $\sqrt[3]{16 - x^3} = 4 - x$ is ________.

Ans. **2**

Sol. $\sqrt[3]{16 - x^3} = 4 - x$

$16 - x^3 = (4 - x)^3$

$16 - x^3 = 64 - x^3 - 48x + 12x^2$

$12x^2 - 48x + 48 = 0$

$x^2 - 4x + 4 = 0$

$x = 2$

Sum of roots = 2

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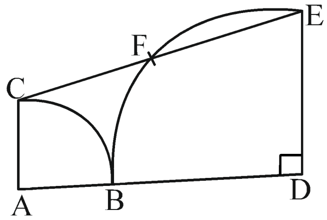
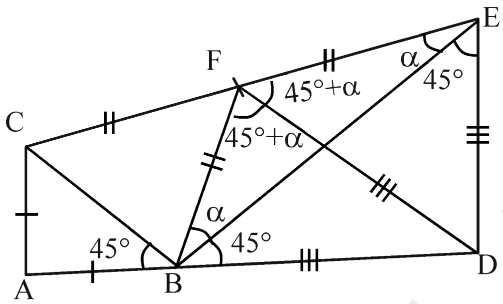
19. In the adjoining figure, two Quadrants are touching at B, CE is joined by a straight line, whose mid-point is F. The measure of ∠CED _______.



Ans. 67.5°

Sol.



$\angle CBE = 180° - (45 + 45) = 90°$
In right ΔCBE: $\angle B = 90°$
So, FC = FE = FB
Let $\angle FEB = \alpha$
$\angle FBD = \angle BFD = 45 + \alpha$ ($\because$ BD = FD)
$\angle DEF = \angle DFE = 45 + \alpha$ ($\because$ DE = DF)
For ΔBFE: $\alpha + 90 + 2\alpha + \alpha = 180$
$4\alpha = 90$
$\alpha = 22.5°$
$\angle CED = \alpha + 45° = 22.5° + 45 = 67.5°$

20. The value of k for which the equation $x^3 - 6x^2 + 11x + (6 - k) = 0$ has exactly three positive integer solutions is _______.

Ans. 12

Sol. $x^3 - 6x^2 + 11x + 6 - k = 0$
Let α, β, γ are roots
$\alpha + \beta + \gamma = 6$,
there are three possibility for $(\alpha, \beta, \gamma) = (1, 2, 3), (2, 2, 2), (1, 1, 4)$
only (1, 2, 3) satisfy the equation
$\alpha\beta + \beta\gamma + \gamma\alpha = 11$
So, $\alpha\beta\gamma = k - 6$
$k - 6 = 1 \times 2 \times 3 \Rightarrow k = 12$

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21. The number of 3-digit numbers of the form ab5 (where a, b are digits) which are divisible by 9 is ______.

Ans. 10

Sol. ab5

It is divisible by 9 then a + b + 5 is a multiple of 9

$a + b + 5 = 9 / 18 / 27$

$a + b = 4 / 13 / 22 \quad \Rightarrow a + b = 22$ (not possible)

So possible pairs are (4, 0), (3, 1), (2, 2) (1, 3), (9, 4), (8, 5) (7, 6), (6, 7), (5, 8), (4, 9)

So total possible pairs are 10.

22. If $a = \sqrt{(2025)^3 - (2023)^3}$, the value of $\sqrt{\frac{a^2 - 2}{6}}$ is _______.

Ans. 2024

Sol. $a = \sqrt{(2025)^3 - (2023)^3}$

$a^2 = (2025)^3 - (2023)^3$

$a^2 = (2024 + 1)^3 - (2024 - 1)^3$

$a^2 = (2024)^3 + (1)^3 + 3\,(2024)^2 + 3\,(2024) - (2024)^3 + (1)^3 + 3\,(2024)^2 - 3(2024)$

$a^2 = 2\,[(1)^3 + 3(2024)^2]$

$a^2 = 2 + 6\,(2024)^2$

$a^2 - 2 = 6\,(2024)^2$

$\sqrt{\frac{a^2 - 2}{6}} = 2024$

23. In a math Olympiad examination, 12% of the students who appeared from a class did not solve any problem; 32% solved with some mistakes. The remaining 14 students solved the paper fully and correctly. The number of students in the class is ________.

Ans. 25

Sol. Let total number of students be 100x

Did not solve any problem = 12x

Solved with some mistake = 32x

Fully correctly solved = 100x – (12 + 32)x = 56x

56x = 14

100x = 25

So total number of students in a class are 25.

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24. When a = 2025, the numerical value of $|2a^3 - 3a^2 - 2a + 1| - |2a^3 - 3a^2 - 3a - 2025|$ is _______.

Ans. 4051

Sol. $|2a^3 - 3a^2 - 2a + 1| - |2a^3 - 3a^2 - 3a - 2025|$

Let $x = 2a^3 - 3a^2 - 2a + 1$

$a(2a^2 - 3a - 2) + 1$

$a(2a + 1)(a - 2) + 1$

Now for a = 2025

$a(2a + 1)(a - 2) + 1 > 0$

So $x > 0$

Let $y = 2a^3 - 3a^2 - 3a - a$

$a(2a^2 - 3a - 4)$

$a(2a^2 - 3a - 5 + 1)$

$a(2a^2 - 3a - 5) + a$

$a(a + 1)(2a - 5) + a$

$a[(a + 1)(2a - 5) + 1]$

Now for a = 2025 it is also positive

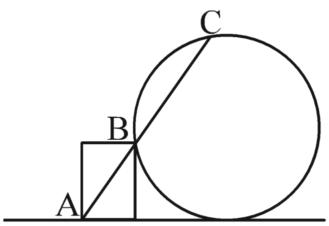
$\therefore$ $|2a^3 - 3a^2 - 2a + 1| - |2a^3 - 3a^2 - 3a - 2025|$

$2a^3 - 3a^2 - 2a + 1 - 2a^3 + 3a^2 + 3a + 2025$

$a + 2026$

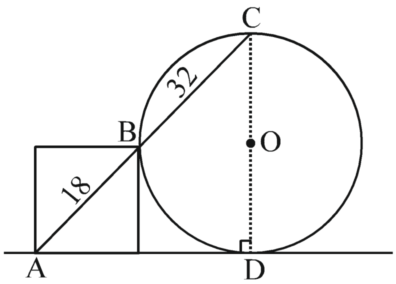
$2025 + 2026 = 4051$

25. A circular hoop and a rectangular frame are standing on the level ground as shown. The diagonal AB is extended to meet the circular hoop at the highest point C. If AB = 18 cm, BC = 32 cm, the radius of the hoop (in cm) is ________.



Ans. 20

Sol.



We know that $AD^2 = AB \times AC$

$AD^2 = 18 \times 50$

$AD = 30$ cm

Now in ΔACD

$AC^2 = AD^2 + CD^2$

$50^2 - 30^2 = CD^2$

$CD = 40$ cm

$\because\ r = 20$ cm

26. 'n' is a natural number. The number of 'n' for which $\frac{16(n^2-n-1)^2}{2n-1}$ is a natural number is ___.

Ans. 3

Sol. $\frac{16(n^2-n-1)^2}{2n-1}$

$\Rightarrow \dfrac{16\left[\left(n-\frac{1}{2}\right)^2-\frac{5}{4}\right]^2}{2n-1}$

$\Rightarrow \dfrac{16\left[(2n-1)^2-5\right]^2}{16\ (2n-1)}$

$\Rightarrow \dfrac{\left[(2n-1)^2-5\right]^2}{(2n-1)}$

$\Rightarrow (2n-1)^3 - 10\,.\,(2n-1) + \frac{25}{2n-1}$ will be natural number If $\frac{25}{2n-1} \in N$

$2n - 1 = 1, 5, 25$

$\therefore\ n = 1, 3, 13$

SOLUTION

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27. The number of solutions (x, y) of the simultaneous equations $\log_4 x - \log_2 y = 0$, $x^2 = 8 + 2y^2$ is ______.

Ans. 1

Sol. $\log_4 x - \log_2 y = 0$

$\log_2 \sqrt{x} - \log_2 y = 0$

$\log_2 \left(\frac{\sqrt{x}}{y}\right) = 0$

$\frac{\sqrt{x}}{y} = 1$

$\boxed{\sqrt{x} - y = 0}$ ...(1)

$x^2 = 8 + 2y^2$ ...(2)

$x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0$

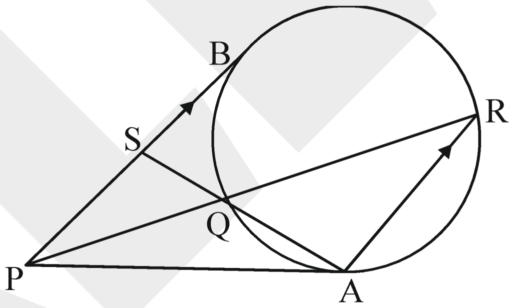
$x = 4$ and -2

$x = -2$ is not possible

If $x = 4$ then $y = 2$

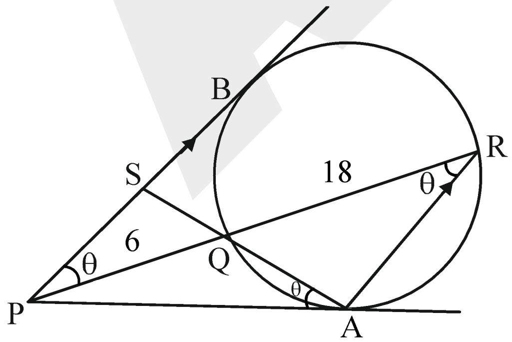
So, only one solution is possible.

28. In the adjoining figure, PA, PB are tangents. AR is parallel to PB. PQ = 6; QR = 18. Length SB = _________.



Ans. 6

Sol.



$PA^2 = PQ \times PR$

$PA^2 = 6 \times 24$

PA = 12 cm

PA = PB = 12 cm

Now let $\angle PAQ = \angle PRA = \theta$ (By alternate segment theorem)

$\angle SPQ = \angle QRA = \theta$ (alternate interior angle)

$\therefore$ $\Delta PQS \sim \Delta RQA$

$$\frac{PQ}{QR} = \frac{QS}{QA} = \frac{PS}{AR} = \frac{6}{18} = \frac{1}{3} \qquad \ldots(1)$$

Similarly $\Delta PAQ \sim \Delta PRA$

$$\frac{PA}{PR} = \frac{AQ}{AR} = \frac{PQ}{AP}$$

$$\frac{AQ}{AR} = \frac{6}{12} = \frac{1}{2} \qquad \ldots(2)$$

Now, let AR = 6x

$\therefore$ AQ = 3x, SQ = x, SP = 2x

$\therefore$ SB = 12 – 2x

Now $(SB)^2 = SQ \times SA$

$(12 - 2x)^2 = x \times 4x$

$144 - 48x + 4x^2 = 4x^2$

48x = 144

x = 3

$\therefore$ SB = 12 – 2 (3)

SB = 6 cm

29. A large watermelon weighs 20 kg with 98% of its weight being water. It is left outside in the sunshine for some time. Some water evaporated and the water content in the watermelon is now 95% of its weight in water. The reduced weight in kg is _______.

Ans. 12

Sol.

20 kg

Water content	Pulp content
98%	2%
19.6 kg	0.4 kg

Let x kg weight reduced.

Remaining weight of water melon = 20 – x

Now pulp content = $\frac{5}{100} \times (20 - x)$

$\frac{5}{100} \times (20 - x) = 0.4$

x = 12 kg

30. In a geometric progression, the fourth term exceeds the third term by 24 and the sum of the second and third term is 6. Then, the sum of the second, third and fourth terms is ______.

Ans. 35.1, 15.9

Sol. According to question

$a_4 - a_3 = 24$

$a_2 + a_3 = 6$

Add both the equation

$a_2 + a_4 = 30$

Let us assume a G.P., according to question

$T_2 = 6 - a_3$,

$T_3 = a_3$,

$T_4 = a_3 + 24$

By property of geometric mean

$(a_3)^2 = (6 - a_3)(a_3 + 24)$

$(a_3)^2 = 144 + 6a_3 - a_3^2 - 24a_3$

$\Rightarrow 2a_3^2 + 18a_3 - 144 = 0$

$a_3^2 + 9a_3 - 72 = 0$

SOLUTION

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$a_3 = \dfrac{-9 \pm \sqrt{81 - 4(1)(-72)}}{2}$

$a_3 = \dfrac{-9 \pm \sqrt{369}}{2} = \left(\dfrac{-9 \pm 19.20}{2}\right)$

So, $a_3 = \dfrac{-9 + 19.20}{2} = 5.10$

or $a_3 = \dfrac{-9 - 19.20}{2} = -14.10$

Then

$a_2 + a_4 + a_3 = 30 + 5.10 = 35.10$

or $a_2 + a_4 + a_3 = 30 - 14.10 = 15.90$