FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Wednesday 29th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

MATHEMATICS — TEST PAPER WITH ANSWER

SECTION-A

1. Question ID: 101761

The probability that a randomly chosen 2 × 2 matrix with all the entries from the set of first 10 primes, is singular, is equal to :

(A) $\frac{133}{10^4}$ (B) $\frac{18}{10^3}$

(C) $\frac{19}{10^3}$ (D) $\frac{271}{10^4}$

Official Ans. by NTA (C)

Allen Ans. (C)

2. Question ID: 101762

Let the solution curve of the differential equation

$x\frac{dy}{dx} - y = \sqrt{y^2 + 16x^2}$, $y(1) = 3$ be $y = y(x)$.

Then $y(2)$ is equal to :

(A) 15 (B) 11

(C) 13 (D) 17

Official Ans. by NTA (A)

Allen Ans. (A)

3. Question ID: 101763

If the mirror image of the point (2, 4, 7) in the plane $3x - y + 4z = 2$ is (a, b, c), the $2a + b + 2c$ is equal to :

(A) 54 (B) 50

(C) -6 (D) -42

Official Ans. by NTA (C)

Allen Ans. (C)

4. Question ID: 101764

Let $f : R \to R$ be a function defined by :

$$f(x) = \begin{cases} \max\limits_{t \le x}\{t^3 - 3t\}; x \le 2 \\ x^2 + 2x - 6; 2 < x < 3 \\ [x-3] + 9 \;; 3 \le x \le 5 \\ 2x + 1 \quad ; \quad x > 5 \end{cases}$$

Where [t] is the greatest integer less than or equal to t. Let m be the number of points where f is not differentiable and $I = \int_{-2}^{2} f(x)dx$. Then the ordered pair (m, I) is equal to :

(A) $\left(3, \frac{27}{4}\right)$ (B) $\left(3, \frac{23}{4}\right)$

(C) $\left(4, \frac{27}{4}\right)$ (D) $\left(4, \frac{23}{4}\right)$

Official Ans. by NTA (C)

Allen Ans. (C)

5. Question ID: 101765

Let $\vec{a} = \alpha\hat{i} + 3\hat{j} - \hat{k}, \vec{b} = 3\hat{i} - \beta\hat{j} + 4\hat{k}$ and

$\vec{c} = \hat{i} + 2\hat{j} - 2\hat{k}$ where $\alpha, \beta \in R$, be three vectors. If the projection of $\vec{a}$ on $\vec{c}$ is $\frac{10}{3}$ and

$\vec{b} \times \vec{c} = -6\hat{i} + 10\hat{j} + 7\hat{k}$, then the value of $\alpha + \beta$ equal to :

(A) 3 (B) 4

(C) 5 (D) 6

Official Ans. by NTA (A)

Allen Ans. (A)

6. Question ID : 101766

The area enclosed by $y^2 = 8x$ and $y = \sqrt{2}x$ that lies outside the triangle formed by $y = \sqrt{2}x, x = 1, y = 2\sqrt{2}$, is equal to :

(A) $\frac{16\sqrt{2}}{6}$ (B) $\frac{11\sqrt{2}}{6}$

(C) $\frac{13\sqrt{2}}{6}$ (D) $\frac{5\sqrt{2}}{6}$

Official Ans. by NTA (C)

Allen Ans. (C)

7. Question ID: 101767

If the system of linear equations

$2x + y - z = 7$

$x - 3y + 2z = 1$

$x + 4y + \delta z = k$, where $\delta, k \in R$

has infinitely many solutions, then $\delta + k$ is equal to:

(A) -3 (B) 3

(C) 6 (D) 9

Official Ans. by NTA (B)

Allen Ans. (B)

8. Question ID: 101768

Let α and β be the roots of the equation $x^2 + (2i - 1) = 0$. Then, the value of $|\alpha^8 + \beta^8|$ is equal to :

(A) 50 (B) 250

(C) 1250 (D) 1500

Official Ans. by NTA (A)

Allen Ans. (A)

9. Question ID: 101769

Let $\Delta \in \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$ be such that

$((p \wedge q)\Delta(p \vee q) \Rightarrow q)$ is a tautology. Then Δ is equal to :

(A) $\wedge$ (B) $\vee$

(C) $\Rightarrow$ (D) $\Leftrightarrow$

Official Ans. by NTA (C)

Allen Ans. (C)

10. Question ID: 101770

Let $A = [a_{ij}]$ be a square matrix of order 3 such that $a_{ij} = 2^{j-i}$, for all i, j = 1, 2, 3. Then, the matrix $A^2 + A^3 + \ldots + A^{10}$ is equal to :

(A) $\left(\frac{3^{10}-3}{2}\right)A$ (B) $\left(\frac{3^{10}-1}{2}\right)A$

(C) $\left(\frac{3^{10}+1}{2}\right)A$ (D) $\left(\frac{3^{10}+3}{2}\right)A$

Official Ans. by NTA (A)
Allen Ans. (A)

11. Question ID: 101771

Let a set $A = A_1 \cup A_2 \cup \ldots \cup A_k$, where $A_i \cap A_j = \phi$ for $i \neq j$ $1 \leq i, j \leq k$. Define the relation R from A to A by $R = \{(x, y): y \in A_i$ if and only if $x \in A_i, 1 \leq i \leq k\}$. Then, R is :

(A) reflexive, symmetric but not transitive

(B) reflexive, transitive but not symmetric

(C) reflexive but not symmetric and transitive

(D) an equivalence relation

Official Ans. by NTA (D)

Allen Ans. (D)

12. Question ID: 101772

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 2a_{n+1} - a_n + 1$ for all $n \geq 0$. Then, $\sum_{n=2}^{\infty} \frac{a_n}{7^n}$ is equal to

(A) $\frac{6}{343}$ (B) $\frac{7}{216}$

(C) $\frac{8}{343}$ (D) $\frac{49}{216}$

Official Ans. by NTA (B)

Allen Ans. (B)

13. Question ID: 101773

The distance between the two points A and A' which lie on y = 2 such that both the line segments AB and A' B (where B is the point (2, 3)) subtend angle $\frac{\pi}{4}$ at the origin, is equal to :

(A) 10 (B) $\frac{48}{5}$

(C) $\frac{52}{5}$ (D) 3

Official Ans. by NTA (C)

Allen Ans. (C)

14. Question ID: 101774

A wire of length 22 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into an equilateral triangle. Then, the length of the side of the equilateral triangle, so that the combined area of the square and the equilateral triangle is minimum, is :

(A) $\frac{22}{9+4\sqrt{3}}$ (B) $\frac{66}{9+4\sqrt{3}}$

(C) $\frac{22}{4+9\sqrt{3}}$ (D) $\frac{66}{4+9\sqrt{3}}$

Official Ans. by NTA (B)

Allen Ans. (B)

15. Question ID: 101775

The domain of the function $\cos^{-1}\left(\frac{2sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi}\right)$ is :

(A) $R-\left\{-\frac{1}{2},\frac{1}{2}\right\}$

(B) $(-\infty,-1]\cup[1,\infty)\cup\{0\}$

(C) $\left(-\infty,\frac{-1}{2}\right)\cup\left(\frac{1}{2},\infty\right)\cup\{0\}$

(D) $\left(-\infty,\ \frac{-1}{\sqrt{2}}\right]\cup\left[\frac{1}{\sqrt{2}},\infty\right)\cup\{0\}$

Official Ans. by NTA (D)

Allen Ans. (D)

16. Question ID: 101776

If the constant term in the expansion of $\left(3x^3-2x^2+\frac{5}{x^5}\right)^{10}$ is $2^k.\ l$, where l is an odd integer, then the value of k is equal to :

(A) 6 (B) 7

(C) 8 (D) 9

Official Ans. by NTA (D)

Allen Ans. (D)

17. Question ID: 101777

$\int_0^5 \cos\left(\pi(x-\left[\frac{x}{2}\right])\right)dx,$

Where [t] denotes greatest integer less than or equal to t, is equal to :

(A) -3 (B) -2

(C) 2 (D) 0

Official Ans. by NTA (D)

Allen Ans. (D)

18. Question ID: 101778

Let PQ be a focal chord of the parabola $y^2 = 4x$ such that it subtends an angle of $\frac{\pi}{2}$ at the point (3, 0). Let the line segment PQ be also a focal chord of the ellipse E: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a^2>b^2$. If e is the eccentricity of the ellipse E, then the value of $\frac{1}{e^2}$ is equal to :

(A) $1+\sqrt{2}$ (B) $3+2\sqrt{2}$

(C) $1+2\sqrt{3}$ (D) $4+5\sqrt{3}$

Official Ans. by NTA (B)

Allen Ans. (B)

19. Question ID: 101779

Let the tangent to the circle $C_1: x^2+y^2=2$ at the point $M(-1,1)$ intersect the circle $C_2: (x-3)^2+(y-2)^2=5$, at two distinct points A and B. If the tangents to C_2 at the points A and B intersect at N, then the area of the triangle ANB is equal to :

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{6}$ (D) $\frac{5}{3}$

Official Ans. by NTA (C)

Allen Ans. (C)

20. Question ID: 101780

Let the mean and the variance of 5 observations x_1, x_2, x_3, x_4, x_5 be $\frac{24}{5}$ and $\frac{194}{25}$ respectively. If the mean and variance of the first 4 observation are $\frac{7}{2}$ and a respectively, then $(4a+x_5)$ is equal to:

(A) 13 (B) 15 (C) 17 (D) 18

Official Ans. by NTA (B)

Allen Ans. (B)

SECTION-B

1. Question ID: 101781

Let $S=\{z\in C: |z-2|\le 1, z(1+i)+\bar{z}(1-i)\le 2\}$. Let $|z-4i|$ attains minimum and maximum values, respectively, at $z_1\in S$ and $z_2\in S$. If $5(|z_1|^2+|z_2|^2)=\alpha+\beta\sqrt{5}$, where α and β are integers, then the value of $\alpha+\beta$ is equal to _____.

Official Ans. by NTA (26)

Allen Ans. (26)

2. Question ID: 101782

Let y = y(x) be the solution of the differential equation

$\frac{dy}{dx} + \frac{\sqrt{2}y}{2cos^4x - cos2x} = xe^{\tan^{-1}(\sqrt{2}\cot 2x)}, 0 < x < \pi/2$ with $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{32}$.

If $y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{18} e^{-\tan^{-1}(\alpha)}$, then the value of $3\alpha^2$ is equal to _____.

Official Ans. by NTA (2)

Allen Ans. (2)

3. Question ID: 101783

Let d be the distance between the foot of perpendiculars of the points P(1, 2 – 1) and Q(2, – 1, 3) on the plane – x + y + z = 1. Then d^2 is equal to _____.

Official Ans. by NTA (26)

Allen Ans. (26)

4. Question ID: 101784

The number of elements in the set $S = \{\theta \epsilon [-4\pi, 4\pi] : 3\,cos^2\,2\theta + 6\,cos2\theta - 10cos^2\theta + 5 = 0$ is _____.

Official Ans. by NTA (32)

Allen Ans. (32)

5. Question ID: 101785

The number of solutions of the equation $2\theta - cos^2\theta + \sqrt{2} = 0$ is R is equal to _____.

Official Ans. by NTA (1)

Allen Ans. (1)

6. Question ID: 101786

$50\,tan\left(3tan^{-1}\left(\frac{1}{2}\right) + 2cos^{-1}\left(\frac{1}{\sqrt{5}}\right)\right) + 4\sqrt{2}\tan\left(\frac{1}{2}tan^{-1}(2\sqrt{2})\right)$ is equal to _____.

Official Ans. by NTA (29)

Allen Ans. (29)

7. Question ID: 101787

Let c, k ∈ R. If $f(x) = (c + 1)x^2 + (1 - c^2)x + 2k$ and $f(x + y) = f(x) + f(y) - xy$, for all x, y ∈ R, then the value of $|2(f(1) + f(2) + f(3) + \ldots\ldots + f(20))|$ is equal to _____.

Official Ans. by NTA (3395)

Allen Ans. (3395)

8. Question ID: 101788

Let $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, a > 0, b > 0, be a hyperbola such that the sum of lengths of the transverse and the conjugate axes is $4\left(2\sqrt{2} + \sqrt{14}\right)$. If the eccentricity H is $\frac{\sqrt{11}}{2}$, then value of $a^2 + b^2$ is equal to _____.

Official Ans. by NTA (88)

Allen Ans. (88)

9. Question ID: 101789

Let $P_1 : \vec{r}.(2\hat{i} + \hat{j} - 3\hat{k}) = 4$ be a plane. Let P_2 be another plane which passes through the points (2, -3, 2) (2, – 2, – 3) and (1, –4, 2). If the direction ratios of the line of intersection of P_1 and P_2 be 16, α, β, then the value of α + β is equal to _____.

Official Ans. by NTA (28)

Allen Ans. (28)

10. Question ID: 101790

Let $b_1b_2b_3b_4$ be a 4-element permutation with $b_i \in \{1, 2, 3, \ldots\ldots, 100\}$ for $1 \le i \le 4$ and $b_i \ne b_j$ for $i \ne j$, such that either b_1, b_2, b_3 are consecutive integers or b_2, b_3, b_4 are consecutive integers.

Then the number of such permutations $b_1b_2b_3b_4$ is equal to _____.

Official Ans. by NTA (18915)

Allen Ans. (18915)