FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Wednesday 29th June, 2022) **TIME : 3 : 00 PM to 06 : 00 PM**

MATHEMATICS

TEST PAPER WITH ANSWER

SECTION-A

1. Let α be a root of the equation $1 + x^2 + x^4 = 0$. Then the value of $\alpha^{1011} + \alpha^{2022} - \alpha^{3033}$ is equal to:

(A) 1 (B) α

(C) $1 + \alpha$ (D) $1 + 2\alpha$

Official Ans. by NTA (A)

Allen Ans. (A)

2. Let arg (z) represent the principal argument of the complex number z. The, $|z| = 3$ and $\arg(z - 1) - \arg(z + 1) = \frac{\pi}{4}$ intersect:

(A) Exactly at one point

(B) Exactly at two points

(C) Nowhere

(D) At infinitely many points.

Official Ans. by NTA (C)

Allen Ans. (C)

3. Let $A = \begin{pmatrix} 2 & -1 \\ 0 & 2 \end{pmatrix}$. If $B = I - {}^5C_1\,(\text{adj}A) + {}^5C_2\,(\text{adj}A)^2 - \ldots - {}^5C_5\,(\text{adj}A)^5$, then the sum of all elements of the matrix B is:

(A) –5 (B) –6

(C) –7 (D) –8

Official Ans. by NTA (C)

Allen Ans. (C)

4. The sum of the infinite series $1 + \frac{5}{6} + \frac{12}{6^2} + \frac{22}{6^3} + \frac{35}{6^4} + \frac{51}{6^5} + \frac{70}{6^6} + \ldots$ is equal to:

(A) $\frac{425}{216}$ (B) $\frac{429}{216}$

(C) $\frac{288}{125}$ (D) $\frac{280}{125}$

Official Ans. by NTA (C)

Allen Ans. (C)

5. The value of $\lim\limits_{x\to 1} \frac{(x^2-1)\sin^2(\pi x)}{x^4 - 2x^3 + 2x - 1}$ is equal to:

(A) $\frac{\pi^2}{6}$ (B) $\frac{\pi^2}{3}$

(C) $\frac{\pi^2}{2}$ (D) π^2

Official Ans. by NTA (D)

Allen Ans. (D)

6. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = (x-3)^{n_1}(x-5)^{n_2}$, $n_1, n_2 \in N$. The, which of the following is NOT true?

(A) For $n_1 = 3, n_2 = 4$, there exists $\alpha \in (3,5)$ where f attains local maxima.

(B) For $n_1 = 4, n_2 = 3$, there exists $\alpha \in (3,5)$ where f attains local manima.

(C) For $n_1 = 3, n_2 = 5$, there exists $\alpha \in (3,5)$ where f attains local maxima.

(D) For $n_1 = 4, n_2 = 6$, there exists $\alpha \in (3,5)$ where f attains local maxima.

Official Ans. by NTA (C)

Allen Ans. (C)

7. Let f be a real valued continuous function on [0,1] and $f(x) = x + \int_0^1 (x-t)f(t)dt$. Then which of the following points (x,y) lies on the curve $y = f(x)$?

(A) (2, 4) (B) (1, 2)

(C) (4, 17) (D) (6, 8)

Official Ans. by NTA (D)

Allen Ans. (D)

8. If $\int_0^2\left(\sqrt{2x}-\sqrt{2x-x^2}\right)dx =$

$\int_0^1\left(1-\sqrt{1-y^2}-\frac{y^2}{2}\right)dy+\int_1^2\left(2-\frac{y^2}{2}\right)dy+I$

(A) $\int_0^1\left(1+\sqrt{1-y^2}\right)dy$

(B) $\int_0^1\left(\frac{y^2}{2}-\sqrt{1-y^2}+1\right)dy$

(C) $\int_0^1\left(1-\sqrt{1-y^2}\right)dy$

(D) $\int_0^1\left(\frac{y^2}{2}+\sqrt{1-y^2}+1\right)dy$

Official Ans. by NTA (C)

Allen Ans. (C)

9. If y = y (x) is the solution of the differential equation $\left(1+e^{2x}\right)\frac{dy}{dx}+2\left(1+y^2\right)e^x=0$ and y (0) = 0, then $6\left(y'(0)+\left(y\left(\log_e\sqrt{3}\right)\right)^2\right)$ is equal to:

(A) 2 (B) –2

(C) –4 (D) –1

Official Ans. by NTA (C)

Allen Ans. (C)

10. Let P : $y^2 = 4ax$, $a > 0$ be a parabola with focus S.Let the tangents to the parabola P make an angle of $\frac{\pi}{4}$ with the line y = 3x + 5 touch the parabola P at A and B. Then the value of *a* for which A,B and S are collinear is:

(A) 8 only (B) 2 only

(C) $\frac{1}{4}$ only (D) any a > 0

Official Ans. by NTA (D)

Allen Ans. (D)

11. Let a triangle *ABC* be inscribed in the circle $x^2 - \sqrt{2}(x+y)+y^2=0$ such that $\angle BAC = \frac{\pi}{2}$. If the length of side AB is $\sqrt{2}$, then the area of the ΔABC is equal to:

(A) $\left(\sqrt{2}+\sqrt{6}\right)/3$ (B) $\left(\sqrt{6}+\sqrt{3}\right)/2$

(C) $\left(3+\sqrt{3}\right)/4$ (D) $\left(\sqrt{6}+2\sqrt{3}\right)/4$

Official Ans. by NTA (Dropped)

Allen Ans. (Dropped)

12. Let $\frac{x-2}{3}=\frac{y+1}{-2}=\frac{z+3}{-1}$ lie on the plane px – qy + z = 5, for some p, $q\in\mathbb{R}$. The shortest distance of the plane from the origin is:

(A) $\sqrt{\frac{3}{109}}$ (B) $\sqrt{\frac{5}{142}}$

(C) $\sqrt{\frac{5}{71}}$ (D) $\sqrt{\frac{1}{142}}$

Official Ans. by NTA (B)

Allen Ans. (B)

13. The distance of the origin from the centroid of the triangle whose two sides have the equations x – 2y + 1 = 0 and 2x – y – 1 = 0 and whose orthocenter is $\left(\frac{7}{3},\frac{7}{3}\right)$ is:

(A) $\sqrt{2}$ (B) 2

(C) $2\sqrt{2}$ (D) 4

Official Ans. by NTA (C)

Allen Ans. (C)

14. Let Q be the mirror image of the point P(1, 2, 1) with respect to the plane x + 2y + 2z = 16. Let T be a plane passing through the point Q and contains the line $\vec{r}=-\hat{k}+\lambda\left(\hat{i}+\hat{j}+2\hat{k}\right),\lambda\in\mathbb{R}$. Then, which of the following points lies on T?

(A) (2, 1, 0) (B) (1, 2, 1)

(C) (1, 2, 2) (D) (1, 3, 2)

Official Ans. by NTA (B)

Allen Ans. (B)

15. Let A, B, C be three points whose position vectors respectively are:

$\vec{a} = \hat{i} + 4\hat{j} + 3\hat{k}$

$\vec{b} = 2\hat{i} + \alpha\hat{j} + 4\hat{k}, \alpha \in \mathbb{R}$

$\vec{c} = 3\hat{i} - 2\hat{j} + 5\hat{k}$

If α is the smallest positive integer for which $\vec{a}, \vec{b}, \vec{c}$ are non-collinear, then the length of the median, in ΔABC, through A is:

(A) $\frac{\sqrt{82}}{2}$ (B) $\frac{\sqrt{62}}{2}$

(C) $\frac{\sqrt{69}}{2}$ (D) $\frac{\sqrt{66}}{2}$

Official Ans. by NTA (A)

Allen Ans. (A)

16. The probability that a relation R from {x,y} to {x,y} is both symmetric and transitive, is equal to:

(A) $\frac{5}{16}$ (B) $\frac{9}{16}$

(C) $\frac{11}{16}$ (D) $\frac{13}{16}$

Official Ans. by NTA (A)

Allen Ans. (A)

17. The number of values of $a \in \mathbb{N}$ such that the variance of 3, 7, 12 a, 43 – a is a natural number is:

(A) 0 (B) 2

(C) 5 (D) infinite

Official Ans. by NTA (A)

Allen Ans. (A)

18. From the base of a pole of height 20 meter, the angle of elevation of the top of a tower is 60°. The pole subtends an angle 30° at the top of the tower. Then the height of the tower is:

(A) $15\sqrt{3}$ (B) $20\sqrt{3}$

(C) $20+10\sqrt{3}$ (D) 30

Official Ans. by NTA (D)

Allen Ans. (D)

19. Negation of the Boolean statement $(p \vee q) \Rightarrow ((\sim r) \vee p)$ is equivalent to:

(A) $p \wedge (\sim q) \wedge r$ (B) $(\sim p) \wedge (\sim q) \wedge r$

(C) $(\sim p) \wedge q \wedge r$ (D) $p \wedge q \wedge (\sim r)$

Official Ans. by NTA (C)

Allen Ans. (C)

20. Let $n \geq 5$ be an integer. If $9^n - 8n - 1 = 64\,\alpha$ and $6^n - 5n - 1 = 25\,\beta$, then $\alpha - \beta$ is equal to:

(A) $1 + {}^nC_2(8-5) + {}^nC_3(8^2-5^2) + ... + {}^nC_n(8^{n-1} - 5^{n-1})$

(B) $1 + {}^nC_3(8-5) + {}^nC_4(8^2-5^2) + ... + {}^nC_n(8^{n-2} - 5^{n-2})$

(C) ${}^nC_3(8-5) + {}^nC_4(8^2-5^2) + ... + {}^nC_n(8^{n-2} - 5^{n-2})$

(D) ${}^nC_4(8-5) + {}^nC_5(8^2-5^2) + ... + {}^nC_n(8^{n-3} - 5^{n-3})$

Official Ans. by NTA (C)

Allen Ans. (C)

SECTION-B

1. Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} + \left(\vec{b} \times \vec{c}\right) = \vec{0}$ and $\vec{b} \cdot \vec{c} = 5$. Then, the value of $3\left(\vec{c} \cdot \vec{a}\right)$ is equal to____.

Official Ans. by NTA (DROP)

Allen Ans. (Bonus)

2. Let y = y(x), x > 1, be the solution of the differential equation $(x-1)\frac{dy}{dx} + 2xy = \frac{1}{x-1}$, with $y(2) = \frac{1+e^4}{2e^4}$. If $y(3) = \frac{e^\alpha + 1}{\beta e^\alpha}$. then the value of $\alpha + \beta$ is equal to_____.

Official Ans. by NTA (14)

Allen Ans. (14)

3. Let 3, 6, 9, 12,... upto 78 terms and 5, 9, 13, 17,... upto 59 terms be two series. Then, the sum of the terms common to both the series is equal to___.

Official Ans. by NTA (2223)

Allen Ans. (2223)

4. The number of solutions of the equation $\sin x = \cos^2 x$ in the interval (0,10) is__.

Official Ans. by NTA (4)

Allen Ans. (4)

5. For real numbers a,b ($a > b > 0$), let

$$Area\left\{(x,y): x^2+y^2 \le a^2 \text{and } \frac{x^2}{a^2}+\frac{y^2}{b^2} \ge 1\right\} = 30\pi$$

and

$$Area\left\{(x,y): x^2+y^2 \ge b^2 \text{and } \frac{x^2}{a^2}+\frac{y^2}{b^2} \le 1\right\} = 18\pi$$

Then the value of $(a-b)^2$ is equal to__.

Official Ans. by NTA (12)

Allen Ans. (12)

6. Let f and g be twice differentiable even functions on (–2, 2) such that $f\left(\frac{1}{4}\right)=0, f\left(\frac{1}{2}\right)=0, f(1)=1$ and $g\left(\frac{3}{4}\right)=0, g(1)=2$ Then, the minimum number of solutions of $f(x)\, g''(x) + f'(x)g'(x) = 0$ in (–2,2) is equal to__.

Official Ans. by NTA (4)

Allen Ans. (4)

7. Let the coefficients of x^{-1} and x^{-3} in the expansion of $\left(2x^{\frac{1}{5}} - \frac{1}{x^{\frac{1}{5}}}\right)^{15}$, x > 0, be m and n respectively. If r is a positive integer such $mn^2 = {}^{15}C_r \cdot 2^r$, then the value of r is equal to__.

Official Ans. by NTA (5)

Allen Ans. (5)

8. The total number of four digit numbers such that each of the first three digits is divisible by the last digit, is equal to_____.

Official Ans. by NTA (1086)

Allen Ans. (1086)

9. Let $M = \begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix}$, where α is a non-zero real number an $N = \sum_{k=1}^{49} M^{2k}$. If $(I - M^2)N = -2I$, then the positive integral value of α is ____.

Official Ans. by NTA (1)

Allen Ans. (1)

10. Let f(x) and g(x) be two real polynomials of degree 2 and 1 respectively. If $f(g(x)) = 8x^2 - 2x$, and $g(f(x)) = 4x^2 + 6x + 1$, then the value of f(2) + g(2) is_____.

Official Ans. by NTA (18)

Allen Ans. (18)