FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Tuesday 28th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

MATHEMATICS

TEST PAPER WITH ANSWER

SECTION-A

1. If $\sum_{k=1}^{31}\left({}^{31}C_k\right)\left({}^{31}C_{k-1}\right)-\sum_{k=1}^{30}\left({}^{30}C_k\right)\left({}^{30}C_{k-1}\right)=\frac{\alpha(60!)}{(30!)(31!)}$,

Where $\alpha\in R$, then the value of 16α is equal to

(A) 1411 (B) 1320

(C) 1615 (D) 1855

Official Ans. by NTA (A)

Allen Ans. (A)

2. Let a function $f:\mathbb{N}\to\mathbb{N}$ be defined by

$$f(n)=\begin{cases}2n, & n=2,4,6,8,.....\\ n-1, & n=3,7,11,15,.....\\ \frac{n+1}{2}, & n=1,5,9,13,.....\end{cases}$$

then, f is

(A) one-one but not onto

(B) onto but not one-one

(C) neither one-one nor onto

(D) one-one and onto

Official Ans. by NTA (D)

Allen Ans. (D)

3. If the system of linear equations

$2x + 3y - z = -2$

$x + y + z = 4$

$x - y + |\lambda| z = 4\lambda - 4$

where $\lambda\in\mathbb{R}$, has no solution, then

(A) $\lambda = 7$ (B) $\lambda = -7$

(C) $\lambda = 8$ (D) $\lambda^2 = 1$

Official Ans. by NTA (B)
Allen Ans. (B)

4. Let A be a matrix of order 3 × 3 and det (A) = 2. Then det (det (A) adj (5 adj (A^3))) is equal to ____.

(A) 512×10^6 (B) 256×10^6

(C) 1024×10^6 (D) 256×10^{11}

Official Ans. by NTA (A)

Allen Ans. (A)

5. The total number of 5-digit numbers, formed by using the digits 1, 2, 3, 5, 6, 7 without repetition, which are multiple of 6, is

(A) 36 (B) 48

(C) 60 (D) 72

Official Ans. by NTA (D)
Allen Ans. (D)

6. Let $A_1, A_2, A_3, \ldots\ldots$ be an increasing geometric progression of positive real numbers. If $A_1A_3A_5A_7=\frac{1}{1296}$ and $A_2+A_4=\frac{7}{36}$, then, the value of $A_6+A_8+A_{10}$ is equal to

(A) 33 (B) 37

(C) 43 (D) 47

Official Ans. by NTA (C)
Allen Ans. (C)

7. Let [t] denote the greatest integer less than or equal to t. Then, the value of the integral $\int_0^1[-8x^2+6x-1]dx$ is equal to

(A) -1 (B) $-\frac{5}{4}$

(C) $\frac{\sqrt{17}-13}{8}$ (D) $\frac{\sqrt{17}-16}{8}$

Official Ans. by NTA (C)
Allen Ans. (C)

8. Let $f:\mathbb{R}\to\mathbb{R}$ be defined as

$$f(x)=\begin{cases}[e^x], & x<0\\ ae^x+[x-1], & 0\le x<1\\ b+[\sin(\pi x)], & 1\le x<2\\ [e^{-x}]-c, & x\ge 2\end{cases}$$

where $a,b,c\in\mathbb{R}$ and [t] denotes greatest integer less than or equal to t. Then, which of the following statements is true ?

(A) There exists $a,b,c\in\mathbb{R}$ such that f is continuous of $\mathbb{R}$.

(B) If f is discontinuous at exactly one point, then a + b + c = 1.

(C) If f is discontinuous at exactly one point, then $a+b+c\neq1$.

(D) f is discontinuous at atleast two points, for any values of a, b and c.

Official Ans. by NTA (C)
Allen Ans. (A)

9. The area of the region

$S=\{(x,y): y^2 \le 8x, y \ge \sqrt{2}x, x \ge 1\}$ is

(A) $\frac{13\sqrt{2}}{6}$ (B) $\frac{11\sqrt{2}}{6}$

(C) $\frac{5\sqrt{2}}{6}$ (D) $\frac{19\sqrt{2}}{6}$

Official Ans. by NTA (B)

Allen Ans. (B)

10. Let the solution curve y = y(x) of the differential equation,

$$\left[\frac{x}{\sqrt{x^2-y^2}}+e^{\frac{y}{x}}\right]x\frac{dy}{dx}=x+\left[\frac{x}{\sqrt{x^2-y^2}}+e^{\frac{y}{x}}\right]y$$

pass through the points (1, 0) and $(2\alpha,\alpha), \alpha>0$. Then α is equal to

(A) $\frac{1}{2}\exp\left(\frac{\pi}{6}+\sqrt{e}-1\right)$ (B) $\frac{1}{2}\exp\left(\frac{\pi}{3}+\sqrt{e}-1\right)$

(C) $\exp\left(\frac{\pi}{6}+\sqrt{e}+1\right)$ (D) $2\exp\left(\frac{\pi}{3}+\sqrt{e}-1\right)$

Official Ans. by NTA (A)

Allen Ans. (A)

11. Let y = y(x) be the solution of the differential equation $x\left(1-x^2\right)\frac{dy}{dx}+\left(3x^2y-y-4x^3\right)=0, x>1,$ with y(2) = –2. Then y(3) is equal to

(A) –18 (B) –12

(C) –6 (D) –3

Official Ans. by NTA (A)

Allen Ans. (A)

12. The number of real solutions of $x^7 + 5x^3 + 3x + 1 = 0$ is equal to _____.

(A) 0 (B) 1

(C) 3 (D) 5

Official Ans. by NTA (B)

Allen Ans. (B)

13. Let the eccentricity of the hyperbola $H:\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be $\sqrt{\frac{5}{2}}$ and length of its latus rectum be $6\sqrt{2}$, If y = 2x + c is a tangent to the hyperbola H, then the value of c^2 is equal to

(A) 18 (B) 20

(C) 24 (D) 32

Official Ans. by NTA (B)

Allen Ans. (B)

14. If the tangents drawn at the point O(0, 0) and $P\left(1+\sqrt{5},2\right)$ on the circle $x^2 + y^2 – 2x – 4y = 0$ intersect at the point Q, then the area of the triangle OPQ is equal to

(A) $\frac{3+\sqrt{5}}{2}$ (B) $\frac{4+2\sqrt{5}}{2}$

(C) $\frac{5+3\sqrt{5}}{2}$ (D) $\frac{7+3\sqrt{5}}{2}$

Official Ans. by NTA (C)

Allen Ans. (B)

15. If two distinct point Q, R lie on the line of intersection of the planes –x + 2y – z = 0 and $3x-5y+2z=0$ and $PQ=PR=\sqrt{18}$ where the point P is (1, –2, 3), then the area of the triangle PQR is equal to

(A) $\frac{2}{3}\sqrt{38}$ (B) $\frac{4}{3}\sqrt{38}$

(C) $\frac{8}{3}\sqrt{38}$ (D) $\sqrt{\frac{152}{3}}$

Official Ans. by NTA (B)

Allen Ans. (B)

16. The acute angle between the planes P_1 and P_2, when P_1 and P_2 are the planes passing through the intersection of the planes 5x + 8y + 13z – 29 = 0 and 8x – 7y + z – 20 = 0 and the points (2, 1, 3) and (0, 1, 2), respectively, is

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{12}$

Official Ans. by NTA (A)

Allen Ans. (A)

17. Let the plane $P: \vec{r} \cdot \vec{a} = d$ contain the line of intersection of two planes $\vec{r} \cdot \left(\hat{i} + 3\hat{j} - \hat{k}\right) = 6$ and $\vec{r} \cdot \left(-6\hat{i} + 5\hat{j} - \hat{k}\right) = 7$. If the plane P passes through the point $\left(2, 3, \frac{1}{2}\right)$, then the value of $\frac{|13\vec{a}|^2}{d^2}$ is equal to

(A) 90 (B) 93

(C) 95 (D) 97

Official Ans. by NTA (B)

Allen Ans. (B)

18. The probability, that in a randomly selected 3-digit number at least two digits are odd, is

(A) $\frac{19}{36}$ (B) $\frac{15}{36}$

(C) $\frac{13}{36}$ (D) $\frac{23}{36}$

Official Ans. by NTA (A)

Allen Ans. (A)

19. Let AB and PQ be two vertical poles, 160 m apart from each other. Let C be the middle point of B and Q, which are feet of these two poles. Let $\frac{\pi}{8}$ and θ be the angles of elevation from C to P and A, respectively. If the height of pole PQ is twice the height of pole AB, then $\tan^2\theta$ is equal to

(A) $\frac{3-2\sqrt{2}}{2}$ (B) $\frac{3+\sqrt{2}}{2}$

(C) $\frac{3-2\sqrt{2}}{4}$ (D) $\frac{3-\sqrt{2}}{4}$

Official Ans. by NTA (C)

Allen Ans. (C)

20. Let p, q, r be three logical statements. Consider the compound statements

$S_1 : ((\sim p) \vee q) \vee ((\sim p) \vee r)$ and

$S_2 : p \rightarrow (q \vee r)$

Then, which of the following is **NOT** true ?

(A) If S_2 is True, then S_1 is True

(B) If S_2 is False, then S_1 is False

(C) If S_2 is False, then S_1 is True

(D) If S_1 is False, then S_2 is False

Official Ans. by NTA (C)

Allen Ans. (C)

SECTION-B

1. Let R_1 and R_2 be relations on the set {1, 2, …, 50} such that

$R_1 = \{(p, p^n) : p \text{ is a prime and } n \geq 0 \text{ is an integer}\}$

and $R_2 = \{(p, p^n) : p \text{ is a prime and } n = 0 \text{ or } 1\}$.

Then, the number of elements in $R_1 - R_2$ is _____.

Official Ans. by NTA (8)

Allen Ans. (8)

2. The number of real solutions of the equation $e^{4x} + 4e^{3x} - 58e^{2x} + 4e^x + 1 = 0$ is _____.

Official Ans. by NTA (2)

Allen Ans. (2)

3. The mean and standard deviation of 15 observations are found to be 8 and 3 respectively. On rechecking it was found that, in the observations, 20 was misread as 5. Then, the correct variance is equal to _____.

Official Ans. by NTA (17)

Allen Ans. (17)

4. If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ are coplanar vectors and $\vec{a} \cdot \vec{c} = 5$, $\vec{b} \perp \vec{c}$, then 122 ($c_1 + c_2 + c_3$) is equal to _____.

Official Ans. by NTA (150)

Allen Ans. (150)

5. A ray of light passing through the point P(2, 3) reflects on the x-axis at point A and the reflected ray passes through the point Q(5, 4). Let R be the point that divides the line segment AQ internally into the ratio 2 : 1. Let the co-ordinates of the foot of the perpendicular M from R on the bisector of the angle PAQ be (α, β). Then, the value of $7\alpha + 3\beta$ is equal to _____.

Official Ans. by NTA (31)

Allen Ans. (31)

6. Let ℓ be a line which is normal to the curve $y = 2x^2 + x + 2$ at a point P on the curve. If the point Q(6, 4) lies on the line ℓ and O is origin, then the area of the triangle OPQ is equal to _____.

Official Ans. by NTA (13)

Allen Ans. (13)

7. Let $A = \{1, a_1, a_2 \ldots\ldots a_{18}, 77\}$ be a set of integers with $1 < a_1 < a_2 < \ldots\ldots < a_{18} < 77$. Let the set $A + A = \{x + y : x, y \in A\}$ contain exactly 39 elements. Then, the value of $a_1 + a_2 + \ldots\ldots + a_{18}$ is equal to _____.

Official Ans. by NTA (702)

Allen Ans. (702)

8. The number of positive integers k such that the constant term in the binomial expansion of $\left(2x^3 + \frac{3}{x^k}\right)^{12}$, $x \neq 0$ is $2^8 \cdot \ell$, where ℓ is an odd integer, is _____.

Official Ans. by NTA (2)

Allen Ans. (2)

9. The number of elements in the set $\{z = a + ib \in \mathbb{C} : a, b \in \mathbb{Z} \text{ and } 1 < |z - 3 + 2i| < 4\}$ is _____.

Official Ans. by NTA (40)

Allen Ans. (40)

10. Let the lines $y + 2x = \sqrt{11} + 7\sqrt{7}$ and $2y + x = 2\sqrt{11} + 6\sqrt{7}$ be normal to a circle $C : (x - h)^2 + (y - k)^2 = r^2$. If the line $\sqrt{11}y - 3x = \frac{5\sqrt{77}}{3} + 11$ is tangent to the circle C, then the value of $(5h - 8k)^2 + 5r^2$ is equal to ____.

Official Ans. by NTA (816)

Allen Ans. (816)