FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Tuesday 28th June, 2022) **TIME : 3 : 00 PM to 06 : 00 PM**

MATHEMATICS | TEST PAPER WITH ANSWER

SECTION-A

1. Let $R_1 = \{(a,b) \in N \times N : |a-b| \le 13\}$ and

$R_2 = \{(a,b) \in N \times N : |a-b| \ne 13\}$. Then on N:

(A) Both R_1 and R_2 are equivalence relations

(B) Neither R_1 nor R_2 is an equivalence relation

(C) R_1 is an equivalence relation but R_2 is not

(D) R_2 is an equivalence relation but R_1 is not

Official Ans. by NTA (B)

Allen Ans. (B)

2. Let f(x) be a quadratic polynomial such that f(–2) + f(3) = 0. If one of the roots of f(x) = 0 is –1, then the sum of the roots of f(x) = 0 is equal to :

(A) $\frac{11}{3}$ (B) $\frac{7}{3}$

(C) $\frac{13}{3}$ (D) $\frac{14}{3}$

Official Ans. by NTA (A)

Allen Ans. (A)

3. The number of ways to distribute 30 identical candies among four children C_1, C_2, C_3 and C_4 so that C_2 receives atleast 4 and atmost 7 candies, C_3 receives atleast 2 and atmost 6 candies, is equal to

(A) 205 (B) 615

(C) 510 (D) 430

Official Ans. by NTA (D)

Allen Ans. (D)

4. The term independent of x in the expression of $(1-x^2+3x^3)\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}, x \ne 0$ is

(A) $\frac{7}{40}$ (B) $\frac{33}{200}$

(C) $\frac{39}{200}$ (D) $\frac{11}{50}$

Official Ans. by NTA (B)

Allen Ans. (B)

5. If n arithmetic means are inserted between a and 100 such that the ratio of the first mean to the last mean is 1 : 7 and a + n = 33, then the value of n is

(A) 21 (B) 22

(C) 23 (D) 24

Official Ans. by NTA (C)

Allen Ans. (C)

6. Let $f, g : \mathbf{R} \to \mathbf{R}$ be functions defined by

$$f(x) = \begin{cases} [x] & , \quad x < 0 \\ |1-x| & , \quad x \ge 0 \end{cases} \text{ and}$$

$$g(x) = \begin{cases} e^x - x & , \quad x < 0 \\ (x-1)^2 - 1 & , \quad x \ge 0 \end{cases}$$

where [x] denote the greatest integer less than or equal to x. Then, the function fog is discontinuous at exactly :

(A) one point (B) two points

(C) three points (D) four points

Official Ans. by NTA (B)

Allen Ans. (B)

7. Let $f : \mathbf{R} \to \mathbf{R}$ be a differentiable function such that $f\left(\frac{\pi}{4}\right) = \sqrt{2}, f\left(\frac{\pi}{2}\right) = 0$ and $f'\left(\frac{\pi}{2}\right) = 1$ and let $g(x) = \int_x^{\pi/4} (f'(t)\sec t + \tan t \sec t\, f(t))dt$ for $x \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right)$. Then $\lim_{x \to \left(\frac{\pi}{2}\right)^-} g(x)$ is equal to

(A) 2 (B) 3

(C) 4 (D) –3

Official Ans. by NTA (B)

Allen Ans. (B)

8. Let $f:\mathbf{R}\to\mathbf{R}$ be continuous function satisfying f(x) + f(x + k) = n, for all x ∈ **R** where k > 0 and n is a positive integer. If $I_1 = \int_0^{4nk} f(x)dx$ and $I_2 = \int_{-k}^{3k} f(x)dx$, then

(A) $I_1 + 2I_2 = 4nk$ (B) $I_1 + 2I_2 = 2nk$

(C) $I_1 + nI_2 = 4n^2k$ (D) $I_1 + nI_2 = 6n^2k$

Official Ans. by NTA (C)

Allen Ans. (C)

9. The area of the bounded region enclosed by the curve $y = 3 - \left|x - \frac{1}{2}\right| - |x+1|$ and the x-axis is

(A) $\frac{9}{4}$ (B) $\frac{45}{16}$

(C) $\frac{27}{8}$ (D) $\frac{63}{16}$

Official Ans. by NTA (C)

Allen Ans. (C)

10. Let x = x(y) be the solution of the differential equation $2ye^{x/y^2}dx + \left(y^2 - 4xe^{x/y^2}\right)dy = 0$ such that x(1) = 0. Then, x(e) is equal to

(A) $e\log_e(2)$ (B) $-e\log_e(2)$

(C) $e^2\log_e(2)$ (D) $-e^2\log_e(2)$

Official Ans. by NTA (D)

Allen Ans. (D)

11. Let the slope of the tangent to a curve y = f(x) at (x, y) be given by 2 tanx (cosx – y). if the curve passes through the point $\left(\pi/4, 0\right)$, then the value of $\int_0^{\pi/2} y\,dx$ is equal to

(A) $(2-\sqrt{2}) + \frac{\pi}{\sqrt{2}}$ (B) $2 - \frac{\pi}{\sqrt{2}}$

(C) $(2+\sqrt{2}) + \frac{\pi}{\sqrt{2}}$ (D) $2 + \frac{\pi}{\sqrt{2}}$

Official Ans. by NTA (B)

Allen Ans. (B)

12. Let a triangle be bounded by the lines L_1 : 2x + 5y = 10; L_2 : – 4x + 3y = 12 and the line L_3, which passes through the point P(2, 3), intersect L_2 at A and L_1 at B. If the point P divides the line-segment AB, internally in the ratio 1 : 3, then the area of the triangle is equal to

(A) $\frac{110}{13}$ (B) $\frac{132}{13}$

(C) $\frac{142}{13}$ (D) $\frac{151}{13}$

Official Ans. by NTA (B)

Allen Ans. (B)

13. Let a > 0, b > 0. Let e and ℓ respectively be the eccentricity and length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Let e' and ℓ' respectively the eccentricity and length of the latus rectum of its conjugate hyperbola. If $e^2 = \frac{11}{14}\ell$ and $(e')^2 = \frac{11}{8}\ell'$, then the value of 77a+ 44b is equal to

(A) 100 (B) 110

(C) 120 (D) 130

Official Ans. by NTA (D)

Allen Ans. (D)

14. Let $\vec{a} = \alpha\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = -2\hat{i} + \alpha\hat{j} + \hat{k}$, where $\alpha \in \mathbf{R}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $\sqrt{15(\alpha^2+4)}$, then the value of $2|\vec{a}|^2 + (\vec{a}\cdot\vec{b})|\vec{b}|^2$ is equal to

(A) 10 (B) 7

(C) 9 (D) 14

Official Ans. by NTA (D)

Allen Ans. (D)

15. If vertex of a parabola is (2, – 1) and the equation of its directrix is 4x – 3y = 21, then the length of its latus rectum is

(A) 2 (B) 8

(C) 12 (D) 16

Official Ans. by NTA (B)

Allen Ans. (B)

16. Let the plane ax + by + cz = d pass through (2, 3, –5) and is perpendicular to the planes 2x + y – 5z = 10 and 3x + 5y – 7z = 12.

If a, b, c, d are integers d > 0 and gcd (|a|, |b|, |c|, d) = 1, then the value of a + 7b + c + 20d is equal to

(A) 18 (B) 20

(C) 24 (D) 22

Official Ans. by NTA (D)

Allen Ans. (D)

17. The probability that a randomly chosen one-one function from the set {a, b, c, d} to the set {1, 2, 3, 4, 5} satisfied f(a) + 2f(b) – f(c) = f(d) is :

(A) $\frac{1}{24}$ (B) $\frac{1}{40}$

(C) $\frac{1}{30}$ (D) $\frac{1}{20}$

Official Ans. by NTA (D)

Allen Ans. (D)

18. The value of $\lim_{n\to\infty} 6\tan\left\{\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{r^2+3r+3}\right)\right\}$ is equal to

(A) 1 (B) 2

(C) 3 (D) 6

Official Ans. by NTA (C)

Allen Ans. (C)

19. Let $\vec{a}$ be a vector which is perpendicular to the vector $3\hat{i}+\frac{1}{2}\hat{j}+2\hat{k}$. If $\vec{a}\times(2\hat{i}+\hat{k})=2\hat{i}-13\hat{j}-4\hat{k}$, then the projection of the vector $\vec{a}$ on the vector $2\hat{i}+2\hat{j}+\hat{k}$ is

(A) $\frac{1}{3}$ (B) 1

(C) $\frac{5}{3}$ (D) $\frac{7}{3}$

Official Ans. by NTA (C)

Allen Ans. (C)

20. If $\cot\alpha = 1$ and $\sec\beta = -\frac{5}{3}$, where $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$, then the value of $\tan(\alpha+\beta)$ and the quadrant in which $\alpha+\beta$ lies, respectively are

(A) $-\frac{1}{7}$ and IVth quadrant

(B) 7 and Ist quadrant

(C) – 7 and IVth quadrant

(D) $\frac{1}{7}$ and Ist quadrant

Official Ans. by NTA (A)

Allen Ans. (A)

SECTION-B

1. Let the image of the point P(1, 2, 3) in the line $L: \frac{x-6}{3}=\frac{y-1}{2}=\frac{z-2}{3}$ be Q. let $R(\alpha,\beta,\gamma)$ be a point that divides internally the line segment PQ in the ratio 1 : 3. Then the value of $22(\alpha+\beta+\gamma)$ is equal to

Official Ans. by NTA (125)

Allen Ans. (125)

2. Suppose a class has 7 students. The average marks of these students in the mathematics examination is 62, and their variance is 20. A student fails in the examination if he/she gets less than 50 marks, then in worst case, the number of students can fail is

Official Ans. by NTA (0)

Allen Ans. (0)

3. If one of the diameters of the circle $x^2+y^2-2\sqrt{2}x-6\sqrt{2}y+14=0$ is a chord of the circle $(x-2\sqrt{2})^2+(y-2\sqrt{2})^2=r^2$, then the value of r^2 is equal to

Official Ans. by NTA (10)

Allen Ans. (10)

4. If $\lim_{x\to 1}\frac{\sin(3x^2-4x+1)-x^2+1}{2x^3-7x^2+ax+b}=-2$, then the value of (a – b) is equal to

Official Ans. by NTA (11)

Allen Ans. (11)

5. Let for n = 1, 2, ….., 50, S_n be the sum of the infinite geometric progression whose first term is n^2 and whose common ratio is $\frac{1}{(n+1)^2}$. Then the value of $\frac{1}{26}+\sum_{n=1}^{50}\left(S_n+\frac{2}{n+1}-n-1\right)$ is equal to

Official Ans. by NTA (41651)

Allen Ans. (41651)

6. If the system of linear equations

$2x-3y=\gamma+5,$

$\alpha x+5y=\beta+1,$ where $\alpha,\beta,\gamma\in\mathbf{R}$ has infinitely many solutions, then the value of $|9\alpha+3\beta+5\gamma|$ is equal to

Official Ans. by NTA (58)

Allen Ans. (58)

7. Let $A=\begin{pmatrix}1+i & 1\\ -i & 0\end{pmatrix}$ where $i=\sqrt{-1}$. Then, the number of elements in the set $\{n\in\{1,2,....,100\}:A^n=A\}$ is

Official Ans. by NTA (25)

Allen Ans. (25)

8. Sum of squares of modulus of all the complex numbers z satisfying $\overline{z}=iz^2+z^2-z$ is equal to

Official Ans. by NTA (2)

Allen Ans. (2)

9. Let S = {1, 2, 3, 4}. Then the number of elements in the set {f : S × S → S : f is onto and f(a, b) = f(b, a) $\geq a\,\forall(a,b)\in S\times S\}$ is

Official Ans. by NTA (37)

Allen Ans. (37)

10. The maximum number of compound propositions, out of

$p\vee r\vee s,\ p\vee r\vee\sim s,\ p\vee\sim q\vee s,$
$\sim p\vee\sim r\vee s,\ \sim p\vee\sim r\vee\sim s,\ \sim p\vee q\vee\sim s,$
$q\vee r\vee\sim s,\ q\vee\sim r\vee\sim s,\ \sim p\vee\sim q\vee\sim s$

that can be made simultaneously true by an assignment of the truth values to p, q, r and s, is equal to

Official Ans. by NTA (9)

Allen Ans. (9)