FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Monday 27th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

MATHEMATICS

TEST PAPER WITH ANSWER

SECTION-A

1. The area of the polygon, whose vertices are the non-real roots of the equation $\bar{z} = iz^2$ is :

(A) $\frac{3\sqrt{3}}{4}$ (B) $\frac{3\sqrt{3}}{2}$

(C) $\frac{3}{2}$ (D) $\frac{3}{4}$

Official Ans. by NTA (A)

Allen Ans. (A)

2. Let the system of linear equations x + 2y + z = 2, αx + 3y – z = α, -αx + y + 2z = -α be inconsistent. Then α is equal to :

(A) $\frac{5}{2}$ (B) $-\frac{5}{2}$

(C) $\frac{7}{2}$ (D) $-\frac{7}{2}$

Official Ans. by NTA (D)

Allen Ans. (D)

3. If $x=\sum_{n=0}^{\infty}a^n, y=\sum_{n=0}^{\infty}b^n, z=\sum_{n=0}^{\infty}c^n$, where a, b, c are in A.P. and |a| < 1, |b| < 1, |c| < 1, abc ≠ 0, then

(A) x, y, z are in A.P.

(B) x, y, z are in G.P.

(C) $\frac{1}{x},\frac{1}{y},\frac{1}{z}$ are in A.P.

(D) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1-(a+b+c)$

Official Ans. by NTA (C)

Allen Ans. (C)

4. Let $\frac{dy}{dx}=\frac{ax-by+a}{bx+cy+a}$, where a, b, c are constants, represent a circle passing through the point (2, 5). Then the shortest distance of the point (11, 6) from this circle is :

(A) 10 (B) 8

(C) 7 (D) 5

Official Ans. by NTA (B)

Allen Ans. (B)

5. Let a be an integer such that $\lim_{x\to 7}\frac{18-[1-x]}{[x-3a]}$ exists, where [t] is greatest integer ≤ t. Then a is equal to :

(A) -6 (B) -2

(C) 2 (D) 6

Official Ans. by NTA (A)

Allen Ans. (A)

6. The number of distinct real roots of $x^4 - 4x + 1 = 0$ is :

(A) 4 (B) 2

(C) 1 (D) 0

Official Ans. by NTA (B)

Allen Ans. (B)

7. The lengths of the sides of a triangle are $10 + x^2$, $10 + x^2$ and $20 - 2x^2$. If for x = k, the area of the triangle is maximum, then $3k^2$ is equal to :

(A) 5 (B) 8

(C) 10 (D) 12

Official Ans. by NTA (C)

Allen Ans. (C)

8. If $\cos^{-1}\left(\frac{y}{2}\right)=\log_e\left(\frac{x}{5}\right)^5, |y|<2$, then :

(A) $x^2y''+xy'-25y=0$

(B) $x^2y''-xy'-25y=0$

(C) $x^2y''-xy'+25y=0$

(D) $x^2y''+xy'+25y=0$

Official Ans. by NTA (D)

Allen Ans. (D)

9. $\int\frac{\left(x^2+1\right)e^x}{\left(x+1\right)^2}dx=f\left(x\right)e^x+C$, Where C is a constant, then $\frac{d^3f}{dx^3}$ at x = 1 is equal to :

(A) $-\frac{3}{4}$ (B) $\frac{3}{4}$

(C) $-\frac{3}{2}$ (D) $\frac{3}{2}$

Official Ans. by NTA (B)

Allen Ans. (B)

10. The value of the integral $\int_{-2}^{2}\frac{\left|x^3+x\right|}{\left(e^{x|x|}+1\right)}dx$ is equal to:

(A) $5e^2$ (B) $3e^{-2}$

(C) 4 (D) 6

Official Ans. by NTA (D)

Allen Ans. (D)

11. If $\frac{dy}{dx}+\frac{2^{x-y}\left(2^y-1\right)}{2^x-1}=0, x, y>0, y\left(1\right)=1$, then y(2) is equal to :

(A) $2+\log_2 3$ (B) $2+\log_2 2$

(C) $2-\log_2 3$ (D) $2-\log_2 3$

Official Ans. by NTA (D)

Allen Ans. (D)

12. In an isosceles triangle ABC, the vertex A is (6, 1) and the equation of the base BC is 2x + y = 4. Let the point B lie on the line x + 3y = 7. If (α, β) is the centroid ΔABC, then $15\left(\alpha+\beta\right)$ is equal to :

(A) 39 (B) 41

(C) 51 (D) 63

Official Ans. by NTA (C)

Allen Ans. (C)

13. Let the eccentricity of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b,$ be $\frac{1}{4}$. If this ellipse passes through the point $\left(-4\sqrt{\frac{2}{5}},3\right)$, then a^2+b^2 is equal to :

(A) 29 (B) 31

(C) 32 (D) 34

Official Ans. by NTA (B)

Allen Ans. (B)

14. If two straight lines whose direction cosines are given by the relations l + m – n = 0, $3l^2 + m^2 + cnl = 0$ are parallel, then the positive value of c is :

(A) 6 (B) 4

(C) 3 (D) 2

Official Ans. by NTA (A)

Allen Ans. (A)

15. Let $\vec{a}=\hat{i}+\hat{j}-\hat{k}$ and $\vec{c}=2\hat{i}-3\hat{j}+2\hat{k}$. Then the number of vectors $\vec{b}$ such that $\vec{b}\times\vec{c}=\vec{a}$ and $\left|\vec{b}\right|\in\{1, 2,....., 10\}$ is :

(A) 0 (B) 1

(C) 2 (D) 3

Official Ans. by NTA (A)

Allen Ans. (A)

16. Five numbers x_1, x_2, x_3, x_4, x_5 are randomly selected from the numbers 1, 2, 3,……, 18 and are arranged in the increasing order ($x_1 < x_2 < x_3 < x_4 < x_5$). The probability that $x_2 = 7$ and $x_4 = 11$ is :

(A) $\frac{1}{136}$ (B) $\frac{1}{72}$

(C) $\frac{1}{68}$ (D) $\frac{1}{34}$

Official Ans. by NTA (C)

Allen Ans. (C)

17. Let X be a random variable having binomial distribution B(7, p). If P(X = 3) = 5P(X = 4), then the sum of the mean and the variance of X is :

(A) $\frac{105}{16}$ (B) $\frac{7}{16}$

(C) $\frac{77}{36}$ (D) $\frac{49}{16}$

Official Ans. by NTA (C)

Allen Ans. (C)

18. The value of $\cos\left(\frac{2\pi}{7}\right)+\cos\left(\frac{4\pi}{7}\right)+\cos\left(\frac{6\pi}{7}\right)$ is equal to :

(A) -1 (B) $-\frac{1}{2}$

(C) $-\frac{1}{3}$ (D) $-\frac{1}{4}$

Official Ans. by NTA (B)

Allen Ans. (B)

19. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)+\cos^{-1}\left(\cos\frac{7\pi}{6}\right)+\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ is equal to :

(A) $\frac{11\pi}{12}$ (B) $\frac{17\pi}{12}$

(C) $\frac{31\pi}{12}$ (D) $-\frac{3\pi}{4}$

Official Ans. by NTA (A)

Allen Ans. (A)

20. The Boolean expression $(\sim(p\wedge q))\vee q$ is equivalent to :

(A) $q\rightarrow(p\wedge q)$ (B) $p\rightarrow q$

(C) $p\rightarrow(p\rightarrow q)$ (D) $p\rightarrow(p\vee q)$

Official Ans. by NTA (D)

Allen Ans. (D)

SECTION-B

1. Let $f:R\rightarrow R$ be a function defined $f(x)=\frac{2e^{2x}}{e^{2x}+e}$. Then $f\left(\frac{1}{100}\right)+f\left(\frac{2}{100}\right)+f\left(\frac{3}{100}\right)+.....+f\left(\frac{99}{100}\right)$ is equal to_____.

Official Ans. by NTA (99)

Allen Ans. (99)

2. If the sum of all the roots of the equation $e^{2x}-11e^{x}-45e^{-x}+\frac{81}{2}=0$ is $\log_e$ P, then p is equal to _____.

Official Ans. by NTA (45)

Allen Ans. (45)

3. The positive value of the determinant of the matrix A, whose $Adj\left(Adj\left(A\right)\right)=\begin{pmatrix}14 & 28 & -14\\ -14 & 14 & 28\\ 28 & -14 & 14\end{pmatrix}$, is _____.

Official Ans. by NTA (14)

Allen Ans. (14)

4. The number of ways, 16 identical cubes, of which 11 are blue and rest are red, can be placed in a row so that between any two red cubes there should be at least 2 blue cubes, is _____.

Official Ans. by NTA (56)

Allen Ans. (56)

5. If the coefficient of x^{10} in the binomial expansion of $\left(\frac{\sqrt{x}}{5^{\frac{1}{4}}}+\frac{\sqrt{5}}{x^{\frac{1}{3}}}\right)^{60}$ is $5^k l$, where l, k $\in$ N and l is co-prime to 5, then k is equal to _____.

Official Ans. by NTA (5)

Allen Ans. (5)

6. Let

$A_1=\{(x,y):|x|\le y^2,|x|+2y\le 8\}$ and

$A_2=\{(x,y):|x|+|y|\le k\}$. If 27 (Area A_1) = 5 (Area A_2), then k is equal to :

Official Ans. by NTA (6)

Allen Ans. (6)

7. If the sum of the first ten terms of the series

$\frac{1}{5}+\frac{2}{65}+\frac{3}{325}+\frac{4}{1025}+\frac{5}{2501}+....$ is $\frac{m}{n}$, where m and n are co-prime numbers, then m + n is equal to _____.

Official Ans. by NTA (276)

Allen Ans. (276)

8. A rectangle R with end points of the one of its dies as (1, 2) and (3, 6) is inscribed in a circle. If the equation of a diameter of the circle is 2x – y + 4 = 0, then the area of R is _____.

Official Ans. by NTA (16)

Allen Ans. (16)

9. A rectangle R with end points of one of its sides as (1, 2) and (3, 6) is inscribed in a circle. If the equation of a diameter of the circle is 2x – y + 4 =0, then the area of R is _____.

Official Ans. by NTA (63)

Allen Ans. (63)

10. A circle of radius 2 unit passes through the vertex and the focus of the parabola $y^2 = 2x$ and touches the parabola $y=\left(x-\frac{1}{4}\right)^2+\alpha$, where $\alpha>0$. Then $(4\alpha-8)^2$ is equal to _____.

Official Ans. by NTA (137)

Allen Ans. (137)