FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Monday 27th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS

TEST PAPER WITH ANSWER

SECTION-A

1. The number of points of intersection $|z-(4+3i)|=2|$ and $|z|+|z-4|=6$, $z \in C$ is :

(A) 0 (B) 1

(C) 2 (D) 3

Official Ans. by NTA (C)

Allen Ans. (C)

2. Let $f(x)=\begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, $a \in R$. Then the sum of the squares of all the values of a for $2f'(10)-f'(5)+100=0$ is :

(A) 117 (B) 106

(C) 125 (D) 136

Official Ans. by NTA (C)

Allen Ans. (C)

3. Let for some real numbers α and β, $a = \alpha - i\beta$. If the system of equations $4ix+(1+i)y=0$ and $8\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)x+\bar{a}y=0$ has more than one solution then $\frac{\alpha}{\beta}$ is equal to :

(A) $-2+\sqrt{3}$ (B) $2-\sqrt{3}$

(C) $2+\sqrt{3}$ (D) $-2-\sqrt{3}$

Official Ans. by NTA (B)

Allen Ans. (B)

4. Let A and B be two 3 × 3 matrices such that AB = I and $|A|=\frac{1}{8}$ then $|\text{adj}(B\,\text{adj}(2A))|$ is equal to

(A) 16 (B) 32

(C) 64 (D) 128

Official Ans. by NTA (C)

Allen Ans. (C)

5. Let $S=2+\frac{6}{7}+\frac{12}{7^2}+\frac{20}{7^3}+\frac{30}{7^4}+.....$ then 4S is equal to

(A) $\left(\frac{7}{3}\right)^2$

(B) $\frac{7^3}{3^2}$

(C) $\left(\frac{7}{3}\right)^3$

(D) $\frac{7^2}{3^3}$

Official Ans. by NTA (C)

Allen Ans. (C)

6. If $a_1, a_2, a_3 ....$ and $b_1, b_2, b_3 ....$ are A.P. and $a_1 = 2, a_{10} = 3, a_1b_1 = 1 = a_{10}b_{10}$ then a_4b_4 is equal to

(A) $\frac{35}{27}$

(B) 1

(C) $\frac{27}{28}$

(D) $\frac{28}{27}$

Official Ans. by NTA (D)

Allen Ans. (D)

7. If m and n respectively are the number of local maximum and local minimum points of the function $f(x)=\int_0^{x^2}\frac{t^2-5t+4}{2+e^t}dt$, then the ordered pair (m, n) is equal to

(A) (3, 2) (B) (2, 3)

(C) (2, 2) (D) (3, 4)

Official Ans. by NTA (B)

Allen Ans. (B)

8. Let f be a differentiable function in $\left(0,\frac{\pi}{2}\right)$. If $\int_{\cos x}^{1} t^2 f(t)\,dt=\sin^3 x+\cos x$ then $\frac{1}{\sqrt{3}}f'\left(\frac{1}{\sqrt{3}}\right)$ is equal to :

(A) $6-9\sqrt{2}$ (B) $6-\frac{9}{\sqrt{2}}$

(C) $\frac{9}{2}-6\sqrt{2}$ (D) $\frac{9}{\sqrt{2}}-6$

Official Ans. by NTA (B)

Allen Ans. (B)

9. The integral $\int_0^1 \frac{1}{7^{\left[\frac{1}{x}\right]}}dx$, where [.] denotes the greatest integer function is equal to

(A) $1+6\log_e\left(\frac{6}{7}\right)$ (B) $1-6\log_e\left(\frac{6}{7}\right)$

(C) $\log_e\left(\frac{7}{6}\right)$ (D) $1-7\log_e\left(\frac{6}{7}\right)$

Official Ans. by NTA (A)

Allen Ans. (A)

10. If the solution curve of the differential equation $\left((\tan^{-1}y)-x\right)dy=(1+y^2)dx$ passes through the point (1, 0) then the abscissa of the point on the curve whose ordinate is tan (1) is :

(A) 2e (B) $\frac{2}{e}$

(C) 2 (D) $\frac{1}{e}$

Official Ans. by NTA (B)

Allen Ans. (B)

11. If the equation of the parabola, whose vertex is at (5, 4) and the directrix is $3x+y-29=0$, is $x^2+ay^2+bxy+cx+dy+k=0$ then

a + b + c +d + k is equal to

(A) 575

(B) –575

(C) 576

(D) –576

Official Ans. by NTA (D)

Allen Ans. (D)

12. The set of values of k for which the circle C : $4x^2+4y^2-12x+8y+k=0$ lies inside the fourth quadrant and the point $\left(1,-\frac{1}{3}\right)$ lies on or inside the circle C is :

(A) An empty set

(B) $\left(6,\frac{95}{9}\right]$

(C) $\left[\frac{80}{9},10\right)$

(D) $\left(9,\frac{92}{9}\right]$

Official Ans. by NTA (D)

Allen Ans. (D)

13. Let the foot of the perpendicular from the point (1, 2, 4) on the line $\frac{x+2}{4}=\frac{y-1}{2}=\frac{z+1}{3}$ be P. Then the distance of P from the plane $3x+4y+12z+23=0$

(A) 5 (B) $\frac{50}{13}$

(C) 4 (D) $\frac{63}{13}$

Official Ans. by NTA (A)

Allen Ans. (A)

14. The shortest distance between the lines $\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-1}{-1}$ and $\frac{x+3}{2}=\frac{y-6}{1}=\frac{z-5}{3}$ is :

(A) $\frac{18}{\sqrt{5}}$ (B) $\frac{22}{3\sqrt{5}}$

(C) $\frac{46}{3\sqrt{5}}$ (D) $6\sqrt{3}$

Official Ans. by NTA (A)

Allen Ans. (A)

15. Let $\vec{a}$ and $\vec{b}$ be the vectors along the diagonal of a parallelogram having area $2\sqrt{2}$. Let the angle between $\vec{a}$ and $\vec{b}$ be acute. $|\vec{a}|=1$ and $|\vec{a}.\vec{b}|=|\vec{a}\times\vec{b}|$. If $\vec{c}=2\sqrt{2}(\vec{a}\times\vec{b})-2\vec{b}$, then an angle between $\vec{b}$ and $\vec{c}$ is :

(A) $\frac{\pi}{4}$ (B) $-\frac{\pi}{4}$

(C) $\frac{5\pi}{6}$ (D) $\frac{3\pi}{4}$

Official Ans. by NTA (D)

Allen Ans. (D)

16. The mean and variance of the data4, 5, 6, 6, 7, 8, x, y where $x < y$ are 6, and $\frac{9}{4}$ respectively. Then x^4+y^2 is equal to

(A) 162 (B) 320

(C) 674 (D) 420

Official Ans. by NTA (B)

Allen Ans. (B)

17. If a point A(x, y) lies in the region bounded by the y-axis, straight lines $2y + x = 6$ and $5x - 6y = 30$, then the probability that $y < 1$ is :

(A) $\frac{1}{6}$ (B) $\frac{5}{6}$

(C) $\frac{2}{3}$ (D) $\frac{6}{7}$

Official Ans. by NTA (B)

Allen Ans. (B)

18. The value of $\cot\left(\sum_{n=1}^{50}\tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right)$ is

(A) $\frac{26}{25}$ (B) $\frac{25}{26}$

(C) $\frac{50}{51}$ (D) $\frac{52}{51}$

Official Ans. by NTA (A)

Allen Ans. (A)

19. $\alpha = \sin 36°$ is a root of which of the following equation

(A) $10x^4-10x^2-5=0$ (B) $16x^4+20x^2-5=0$

(C) $16x^4-20x^2+5=0$ (D) $16x^4-10x^2+5=0$

Official Ans. by NTA (C)

Allen Ans. (C)

20. Which of the following statement is a tautology?

(A) $((\sim q)\wedge p)\wedge q$

(B) $((\sim q)\wedge p)\wedge(p\wedge(\sim p))$

(C) $((\sim q)\wedge p)\vee(p\vee(\sim p))$

(D) $(p\wedge q)\wedge(\sim(p\wedge q))$

Official Ans. by NTA (C)

Allen Ans. (C)

SECTION-B

1. Let S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Define

$f: S\to S$ as $f(n)=\begin{cases}2n, & \text{if } n=1,2,3,4,5\\ 2n-11 & \text{if } n=6,7,8,9,10\end{cases}$.

Let $g : S \to S$ be a function such that $fog(n)=\begin{cases}n+1 & ,\text{if } n \text{ is odd}\\ n-1 & ,\text{if } n \text{ is even}\end{cases}$, then

g(10) (g(1) + g(2) + g(3) + g(4) + g(5) is equal to :

Official Ans. by NTA (190)

Allen Ans. (190)

2. Let α, β be the roots of the equation $x^2-4\lambda x+5=0$ and α, γ be the roots of the equation $x^2-\left(3\sqrt{2}+2\sqrt{3}\right)x+7+3\lambda\sqrt{3}=0$. If $\beta+\gamma=3\sqrt{2}$, then $(\alpha+2\beta+\gamma)^2$ is equal to :

Official Ans. by NTA (98)

Allen Ans. (98)

3. Let A be a matrix of order 2×2, whose entries are from the set {0, 1, 2, 3, 4, 5}. If the sum of all the entries of A is a prime number p , $2 < p < 8$, then the number of such matrices A is :

Official Ans. by NTA (180)

Allen Ans. (180)

4. If the sum of the coefficients of all the positive powers of x, in the binomial expansion of $\left(x^n+\frac{2}{x^5}\right)^7$ is 939 , then the sum of all the possible integral values of n is :

Official Ans. by NTA (57)

Allen Ans. (57)

5. Let [t] denote the greatest integer $\le$ t and {t} denote the fractional part of t. Then integral value of α for wich lee left hand limit of the function $f(x)=|1+x|+\frac{\alpha^{2[x]+\{x\}}+[x]-1}{2[x]+\{x\}}$ at x = 0 is equal to $\alpha-\frac{4}{3}$ is ____

Official Ans. by NTA (3)

Allen Ans. (3)

6. If $y(x)=(x^{x^x}), x>0$ then $\frac{d^2x}{dy^2}+20$ at x = 1 is equal to :

Official Ans. by NTA (16)

Allen Ans. (16)

7. If the area of the region $\left\{(x,y): x^{\frac{2}{3}}+y^{\frac{2}{3}} \le 1 x+y \ge 0, y \ge 0\right\}$ is A, then $\frac{256A}{\pi}$ is

Official Ans. by NTA (36)

Allen Ans. (36)

8. Let v be the solution of the differential equation $(1-x^2)dy=\left(xy+(x^3+2)\sqrt{1-x^2}\right)dx, -1<x<1$ and y (0) = 0 if $\int_{-\frac{1}{2}}^{\frac{1}{2}}\sqrt{1-x^2}\,y(x)dx=k$ then k^{-1} is equal to :

Official Ans. by NTA (320)

Allen Ans. (320)

9. Let a circle C of radius 5 lie below the x-axis. The line $L_1=4x+3y-2$ passes through the centre P of the circle C and intersects the line $L_2:3x-4y-11=0$ at Q. The line L_2 touches C at the point Q. Then the distance of P from the line $5x-12y+51=0$ is

Official Ans. by NTA (11)

Allen Ans. (11)

10. Let $S=\{E,E_2....E_8\}$ be a sample space of raddom experiment such that $P(E_n)=\frac{n}{36}$ for every n = 1, 2….8. Then the number of elements in the set $\left\{A \subset S: P(A) \ge \frac{4}{5}\right\}$ is _____

Official Ans. by NTA (19)

Allen Ans. (19)