FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Sunday 26th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

MATHEMATICS — TEST PAPER WITH ANSWER

SECTION-A

1. Let $f(x)=\frac{x-1}{x+1}$, $x \in R - \{0, -1, 1\}$. If $f^{n+1}(x) = f(f^{n}(x))$ for all $n \in N$, then $f^6(6) + f^7(7)$ is equal to:

(A) $\frac{7}{6}$ (B) $-\frac{3}{2}$ (C) $\frac{7}{12}$ (D) $-\frac{11}{12}$

Official Ans. by NTA (B)

Allen Ans. (B)

2. Let $A=\left\{z \in C: \left|\frac{z+1}{z-1}\right|<1\right\}$

and $B=\left\{z \in C: \arg\left(\frac{z-1}{z+1}\right)=\frac{2\pi}{3}\right\}$.

Then $A \cap B$ is :

(A) a portion of a circle centred at $\left(0,-\frac{1}{\sqrt{3}}\right)$ that lies in the second and third quadrants only

(B) a portion of a circle centred at $\left(0,-\frac{1}{\sqrt{3}}\right)$ that lies in the second quadrant only

(C) an empty set

(D) a portion of a circle of radius $\frac{2}{\sqrt{3}}$ that lies in the third quadrant only

Official Ans. by NTA (B)

Allen Ans. (B)

3. Let A be a 3×3 invertible matrix. If $|\text{adj } (24A)| = |\text{adj}(3\text{adj}(2A))|$, then $|A|^2$ is equal to :

(A) 6^6 (B) 2^{12} (C) 2^6 (D) 1

Official Ans. by NTA (C)

Allen Ans. (C)

4. The ordered pair (a, b), for which the system of linear equations

$3x - 2y + z = b$

$5x - 8y + 9z = 3$

$2x + y + az = -1$

has no solution, is :

(A) $\left(3,\frac{1}{3}\right)$ (B) $\left(-3,\frac{1}{3}\right)$

(C) $\left(-3,-\frac{1}{3}\right)$ (D) $\left(3,-\frac{1}{3}\right)$

Official Ans. by NTA (C)

Allen Ans. (C)

5. The remainder when $(2021)^{2023}$ is divided by 7 is :

(A) 1 (B) 2 (C) 5 (D) 6

Official Ans. by NTA (C)

Allen Ans. (C)

6. $\lim\limits_{x \to \frac{1}{\sqrt{2}}} \frac{\sin(\cos^{-1} x)-x}{1-\tan(\cos^{-1} x)}$ is equal to :

(A) $\sqrt{2}$ (B) $-\sqrt{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $-\frac{1}{\sqrt{2}}$

Official Ans. by NTA (D)

Allen Ans. (D)

7. Let f, g : R $\to$ R be two real valued functions defined as $f(x)=\begin{cases}-|x+3| & , \quad x<0 \\ e^x & , \quad x \geq 0\end{cases}$ and

$g(x)=\begin{cases}x^2+k_1x & , \quad x<0 \\ 4x+k_2 & , \quad x \geq 0\end{cases}$, where k_1 and k_2 are real constants. If (gof) is differentiable at x = 0, then (gof) (–4) + (gof) (4) is equal to :

(A) $4(e^4 + 1)$ (B) $2(2e^4 + 1)$

(C) $4e^4$ (D) $2(2e^4 - 1)$

Official Ans. by NTA (D)

Allen Ans. (D)

8. The sum of the absolute minimum and the absolute maximum values of the function $f(x) = |3x - x^2 + 2| - x$ in the interval $[-1, 2]$ is :

(A) $\frac{\sqrt{17}+3}{2}$ (B) $\frac{\sqrt{17}+5}{2}$

(C) 5 (D) $\frac{9-\sqrt{17}}{2}$

Official Ans. by NTA (A)

Allen Ans. (A)

9. Let S be the set of all the natural numbers, for which the line $\frac{x}{a}+\frac{y}{b}=2$ is a tangent to the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$ at the point (a, b), $ab \neq 0$. Then:

(A) $S = \phi$ (B) $n(S) = 1$

(C) $S = \{2k : k \in N\}$ (D) $S = N$

Official Ans. by NTA (D)

Allen Ans. (D)

10. The area bounded by the curve $y = |x^2 - 9|$ and the line $y = 3$ is :

(A) $4\left(2\sqrt{3}+\sqrt{6}-4\right)$ (B) $4\left(4\sqrt{3}+\sqrt{6}-4\right)$

(C) $8\left(4\sqrt{3}+3\sqrt{6}-9\right)$ (D) $8\left(4\sqrt{3}+\sqrt{6}-9\right)$

Official Ans. by NTA (DROP)

Allen Ans. (Bonus)

11. Let R be the point (3, 7) and let P and Q be two points on the line $x + y = 5$ such that PQR is an equilateral triangle. Then the area of ΔPQR is :

(A) $\frac{25}{4\sqrt{3}}$ (B) $\frac{25\sqrt{3}}{2}$ (C) $\frac{25}{\sqrt{3}}$ (D) $\frac{25}{2\sqrt{3}}$

Official Ans. by NTA (D)

Allen Ans. (D)

12. Let C be a circle passing through the points A(2, –1) and B(3, 4). The line segment AB is not a diameter of C. If r is the radius of C and its centre lies on the circle $(x - 5)^2 + (y - 1)^2 = \frac{13}{2}$, then r^2 is equal to :

(A) 32 (B) $\frac{65}{2}$ (C) $\frac{61}{2}$ (D) 30

Official Ans. by NTA (B)

Allen Ans. (B)

13. Let the normal at the point P on the parabola $y^2 = 6x$ pass through the point (5, –8). If the tangent at P to the parabola intersects its directrix at the point Q, then the ordinate of the point Q is :

(A) –3 (B) $-\frac{9}{4}$ (C) $-\frac{5}{2}$ (D) –2

Official Ans. by NTA (B)

Allen Ans. (B)

14. If the two lines $l_1 : \frac{x-2}{3} = \frac{y+1}{-2}$, $z = 2$ and $l_2 : \frac{x-1}{1} = \frac{2y+3}{\alpha} = \frac{z+5}{2}$ perpendicular, then an angle between the lines l_2 and $l_3 : \frac{1-x}{3} = \frac{2y-1}{-4} = \frac{z}{4}$ is :

(A) $\cos^{-1}\left(\frac{29}{4}\right)$ (B) $\sec^{-1}\left(\frac{29}{4}\right)$

(C) $\cos^{-1}\left(\frac{2}{29}\right)$ (D) $\cos^{-1}\left(\frac{2}{\sqrt{29}}\right)$

Official Ans. by NTA (B)

Allen Ans. (B)

15. Let the plane $2x + 3y + z + 20 = 0$ be rotated through a right angle about its line of intersection with the plane $x - 3y + 5z = 8$. If the mirror image of the point $\left(2,-\frac{1}{2},2\right)$ in the rotated plane is B(a, b, c), then :

(A) $\frac{a}{8}=\frac{b}{5}=\frac{c}{-4}$ (B) $\frac{a}{4}=\frac{b}{5}=\frac{c}{-2}$

(C) $\frac{a}{8}=\frac{b}{-5}=\frac{c}{4}$ (D) $\frac{a}{4}=\frac{b}{5}=\frac{c}{2}$

Official Ans. by NTA (A)

Allen Ans. (A)

16. If $\vec{a}\cdot\vec{b}=1$, $\vec{b}\cdot\vec{c}=2$ and $\vec{c}\cdot\vec{a}=3$, then the value of $\left[\vec{a}\times\left(\vec{b}\times\vec{c}\right),\vec{b}\times\left(\vec{c}\times\vec{a}\right),\vec{c}\times\left(\vec{b}\times\vec{a}\right)\right]$ is :

(A) 0 (B) $-6\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)$

(C) $12\vec{c}\cdot\left(\vec{a}\times\vec{b}\right)$ (D) $-12\vec{b}\cdot\left(\vec{c}\times\vec{a}\right)$

Official Ans. by NTA (A)

Allen Ans. (A)

17. Let a biased coin be tossed 5 times. If the probability of getting 4 heads is equal to the probability of getting 5 heads, then the probability of getting atmost two heads is:

(A) $\frac{275}{6^5}$ (B) $\frac{36}{5^4}$ (C) $\frac{181}{5^5}$ (D) $\frac{46}{6^4}$

Official Ans. by NTA (D)

Allen Ans. (D)

18. The mean of the numbers a, b, 8, 5, 10 is 6 and their variance is 6.8. If M is the mean deviation of the numbers about the mean, then 25 M is equal to:

(A) 60 (B) 55 (C) 50 (D) 45

Official Ans. by NTA (A)

Allen Ans. (A)

19. Let $f(x) = 2\cos^{-1}x + 4\cot^{-1}x - 3x^2 - 2x + 10$, $x \in [-1, 1]$. If [a, b] is the range of the function then 4a – b is equal to:

(A) 11 (B) $11 - \pi$ (C) $11 + \pi$ (D) $15 - \pi$

Official Ans. by NTA (B)

Allen Ans. (B)

20. Let $\Delta, \nabla \in \{\wedge, \vee\}$ be such that $p \nabla q \Rightarrow ((p \Delta q) \nabla r)$ is a tautology. Then $(p \nabla q) \Delta r$ is logically equivalent to :

(A) $(p \Delta r) \vee q$ (B) $(p \Delta r) \wedge q$

(C) $(p \wedge r) \Delta q$ (D) $(p \nabla r) \wedge q$

Official Ans. by NTA (A)

Allen Ans. (A)

SECTION-B

1. The sum of the cubes of all the roots of the equation $x^4 - 3x^3 - 2x^2 + 3x + 1 = 10$ is _____.

Official Ans. by NTA (36)

Allen Ans. (36)

2. There are ten boys B_1, B_2, ..., B_{10} and five girls G_1, G_2, ..., G_5 in a class. Then the number of ways of forming a group consisting of three boys and three girls, if both B_1 and B_2 together should not be the members of a group, is_____.

Official Ans. by NTA (1120)

Allen Ans. (1120)

3. Let the common tangents to the curves $4(x^2 + y^2) = 9$ and $y^2 = 4x$ intersect at the point Q. Let an ellipse, centered at the origin O, has lengths of semi-minor and semi-major axes equal to OQ and 6, respectively. If e and l respectively denote the eccentricity and the length of the latus rectum of this ellipse, then $\frac{l}{e^2}$ is equal to_____.

Official Ans. by NTA (4)

Allen Ans. (4)

4. Let $f(x) = \max\{|x + 1|, |x + 2|, ..., |x + 5|\}$. Then $\int_{-6}^{0} f(x)dx$ is equal to _____.

Official Ans. by NTA (21)

Allen Ans. (21)

5. Let the solution curve $y = y(x)$ of the differential equation $(4 + x^2)dy - 2x(x^2 + 3y + 4)dx = 0$ pass through the origin. Then y(2) is equal to_____.

Official Ans. by NTA (12)

Allen Ans. (12)

6. If $\sin^2(10°)\sin(20°)\sin(40°)\sin(50°)\sin(70°) = \alpha - \frac{1}{16}\sin(10°)$, then $16 + \alpha^{-1}$ is equal to _____.

Official Ans. by NTA (80)

Allen Ans. (80)

7. Let A = {n ∈ N : H.C.F. (n, 45) = 1} and

Let B = {2k : k ∈ {1, 2, ..., 100}}. Then the sum of all the elements of A ∩ B is _____.

Official Ans. by NTA (5264)

Allen Ans. (5264)

8. The value of the integral $\frac{48}{\pi^4}\int_0^{\pi}\left(\frac{3\pi x^2}{2}-x^3\right)\frac{\sin x}{1+\cos^2 x}dx$ is equal to _____.

Official Ans. by NTA (6)

Allen Ans. (6)

9. Let $A=\sum_{i=1}^{10}\sum_{j=1}^{10}\min\{i,j\}$ and

$B=\sum_{i=1}^{10}\sum_{j=1}^{10}\max\{i,j\}$. Then A + B is equal to _____.

Official Ans. by NTA (1100)

Allen Ans. (1100)

10. Let $S=(0,2\pi)-\left\{\frac{\pi}{2},\frac{3\pi}{4},\frac{3\pi}{2},\frac{7\pi}{4}\right\}$. Let y = y(x), x ∈ S, be the solution curve of the differential equation $\frac{dy}{dx}=\frac{1}{1+\sin 2x}$, $y\left(\frac{\pi}{4}\right)=\frac{1}{2}$. if the sum of abscissas of all the points of intersection of the curve y = y(x) with the curve $y=\sqrt{2}\sin x$ is $\frac{k\pi}{12}$, then k is equal to _____.

Official Ans. by NTA (42)

Allen Ans. (42)