FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Saturday 25th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

MATHEMATICS

TEST PAPER WITH ANSWER

SECTION-A

1. Let a circle C touch the lines $L_1 : 4x - 3y + K_1 = 0$ and $L_2 : 4x - 3y + K_2 = 0$, $K_1, K_2 \in R$. If a line passing through the centre of the circle C intersects L_1 at (–1, 2) and L_2 at (3, – 6), then the equation of the circle C is

(A) $(x - 1)^2 + (y - 2)^2 = 4$

(B) $(x + 1)^2 + (y - 2)^2 = 4$

(C) $(x - 1)^2 + (y + 2)^2 = 16$

(D) $(x - 1)^2 + (y - 2)^2 = 16$

Official Ans. by NTA (C)
Allen Ans. (C)

2. The value of $\int_0^{\pi} \frac{e^{\cos x} \sin x}{(1+\cos^2 x)(e^{\cos x}+e^{-\cos x})}dx$ is equal to

(A) $\frac{\pi^2}{4}$ (B) $\frac{\pi^2}{2}$

(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

Official Ans. by NTA (C)
Allen Ans. (C)

3. Let a, b and c be the length of sides of a triangle ABC such that $\frac{a+b}{7}=\frac{b+c}{8}=\frac{c+a}{9}$. If r and R are the radius of incircle and radius of circumcircle of the triangle ABC, respectively, then the value of $\frac{R}{r}$ is equal to

(A) $\frac{5}{2}$ (B) 2

(C) $\frac{3}{2}$ (D) 1

Official Ans. by NTA (A)
Allen Ans. (A)

4. Let $f : N \to R$ be a function such that $f(x+y)=2f(x)f(y)$ for natural numbers x and y. If f(1) = 2, then the value of α for which

$$\sum_{k=1}^{10} f(\alpha+k)=\frac{512}{3}(2^{20}-1)$$

holds, is

(A) 2 (B) 3

(C) 4 (D) 6

Official Ans. by NTA (C)
Allen Ans. (C)

5. Let A be a 3 × 3 real matrix such that

$$A\begin{pmatrix}1\\1\\0\end{pmatrix}=\begin{pmatrix}1\\1\\0\end{pmatrix}; A\begin{pmatrix}1\\0\\1\end{pmatrix}=\begin{pmatrix}-1\\0\\1\end{pmatrix} \text{ and } A\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}1\\1\\2\end{pmatrix}.$$

If $X = (x_1, x_2, x_3)^T$ and I is an identity matrix of order 3, then the system $(A - 2I)X = \begin{pmatrix}4\\1\\1\end{pmatrix}$ has

(A) no solution
(B) infinitely many solutions
(C) unique solution
(D) exactly two solutions

Official Ans. by NTA (B)
Allen Ans. (B)

6. Let $f : R \to R$ be defined as $f(x) = x^3 + x - 5$. If g (x) is a function such that f (g(x)) = x, $\forall x \in R$, then g ' (63) is equal to ______.

(A) $\frac{1}{49}$ (B) $\frac{3}{49}$

(C) $\frac{43}{49}$ (D) $\frac{91}{49}$

Official Ans. by NTA (A)
Allen Ans. (A)

7. Consider the following two propositions:

P1 : $\sim(p \to \sim q)$

P2 : $(p \wedge \sim q) \wedge ((\sim p) \vee q)$

If the proposition $p \to ((\sim p) \vee q)$ is evaluated as FALSE, then:

(A) P1 is TRUE and P2 is FALSE

(B) P1 is FALSE and P2 is TRUE

(C) Both P1 and P2 are FALSE

(D) Both P1 and P2 are TRUE

Official Ans. by NTA (C)

Allen Ans. (C)

8. If $\frac{1}{2\cdot 3^{10}}+\frac{1}{2^2\cdot 3^9}+\ldots\frac{1}{2^{10}\cdot 3}=\frac{K}{2^{10}\cdot 3^{10}}$, then the remainder when K is divided by 6 is

(A) 1 (B) 2

(C) 3 (D) 5

Official Ans. by NTA (D)

Allen Ans. (D)

9. Let f (x) be a polynomial function such that $f(x) + f'(x) + f''(x) = x^5 + 64$. Then, the value of $\lim_{x\to 1}\frac{f(x)}{x-1}$

(A) – 15 (B) – 60

(C) 60 (D) 15

Official Ans. by NTA (A)

Allen Ans. (A)

10. Let E_1 and E_2 be two events such that the conditional probabilities $P(E_1|E_2)=\frac{1}{2}$, $P(E_2|E_1)=\frac{3}{4}$ and $P(E_1\cap E_2)=\frac{1}{8}$. Then:

(A) $P(E_1\cap E_2)=P(E_1)\cdot P(E_2)$

(B) $P(E'_1\cap E'_2)=P(E'_1)\cdot P(E_2)$

(C) $P(E_1\cap E'_2)=P(E_1)\cdot P(E_2)$

(D) $P(E'_1\cap E_2)=P(E_1)\cdot P(E_2)$

Official Ans. by NTA (C)

Allen Ans. (C)

11. Let $A=\begin{bmatrix}0 & -2\\ 2 & 0\end{bmatrix}$. If M and N are two matrices given by $M=\sum_{k=1}^{10}A^{2k}$ and $N=\sum_{k=1}^{10}A^{2k-1}$ then MN^2 is

(A) a non-identity symmetric matrix

(B) a skew-symmetric matrix

(C) neither symmetric nor skew-symmetric matrix

(D) an identify matrix

Official Ans. by NTA (A)

Allen Ans. (A)

12. Let g : $(0, \infty) \to R$ be a differentiable function such that

$$\int\left(\frac{x(\cos x-\sin x)}{e^x+1}+\frac{g(x)(e^x+1-xe^x)}{(e^x+1)^2}\right)dx=\frac{xg(x)}{e^x+1}+c,$$

for all x > 0, where c is an arbitrary constant. Then.

(A) g is decreasing in $\left(0,\frac{\pi}{4}\right)$

(B) g' is increasing in $\left(0,\frac{\pi}{4}\right)$

(C) g + g' is increasing in $\left(0,\frac{\pi}{2}\right)$

(D) g – g' is increasing in $\left(0,\frac{\pi}{2}\right)$

Official Ans. by NTA (D)

Allen Ans. (D)

13. Let $f : R \to R$ and $g : R \to R$ be two functions defined by $f(x) = \log_e(x^2 + 1) - e^{-x} + 1$ and $g(x)=\frac{1-2e^{2x}}{e^x}$. Then, for which of the following range of α, the inequality

$$f\left(g\left(\frac{(\alpha-1)^2}{3}\right)\right)>f\left(g\left(\alpha-\frac{5}{3}\right)\right)$$ holds?

(A) (2, 3) (B) (–2, – 1)

(C) (1, 2) (D) (–1, 1)

Official Ans. by NTA (A)

Allen Ans. (A)

14. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ $a_i > 0$, $i = 1, 2, 3$ be a vector which makes equal angles with the coordinates axes OX, OY and OZ. Also, let the projection of $\vec{a}$ on the vector $3\hat{i} + 4\hat{j}$ be 7. Let $\vec{b}$ be a vector obtained by rotating $\vec{a}$ with 90°. If $\vec{a}, \vec{b}$ and x-axis are coplanar, then projection of a vector $\vec{b}$ on $3\hat{i} + 4\hat{j}$ is equal to

(A) $\sqrt{7}$ (B) $\sqrt{2}$

(C) 2 (D) 7

Official Ans. by NTA (B)

Allen Ans. (B)

15. Let $y = y(x)$ be the solution of the differential equation $(x + 1)y' - y = e^{3x}(x + 1)^2$, with $y(0) = \frac{1}{3}$. Then, the point $x = -\frac{4}{3}$ for the curve $y = y(x)$ is:

(A) not a critical point

(B) a point of local minima

(C) a point of local maxima

(D) a point of inflection

Official Ans. by NTA (B)

Allen Ans. (B)

16. If $y = m_1x + c_1$ and $y = m_2x + c_2$, $m_1 \neq m_2$ are two common tangents of circle $x^2 + y^2 = 2$ and parabola $y^2 = x$, then the value of $8|m_1m_2|$ is equal to

(A) $3 + 4\sqrt{2}$ (B) $-5 + 6\sqrt{2}$

(C) $-4 + 3\sqrt{2}$ (D) $7 + 6\sqrt{2}$

Official Ans. by NTA (C)

Allen Ans. (C)

17. Let Q be the mirror image of the point P(1, 0, 1) with respect to the plane S : x + y + z = 5. If a line L passing through (1, –1, –1), parallel to the line PQ meets the plane S at R, then QR^2 is equal to:

(A) 2 (B) 5

(C) 7 (D) 11

Official Ans. by NTA (B)

Allen Ans. (B)

18. If the solution curve $y = y(x)$ of the differential equation $y^2dx + (x^2 - xy + y^2)dy = 0$, which passes through the point (1, 1) and intersects the line $y = \sqrt{3}\,x$ at the point $(\alpha, \sqrt{3}\,\alpha)$, then value of $\log_e(\sqrt{3}\,\alpha)$ is equal to

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$

(C) $\frac{\pi}{12}$ (D) $\frac{\pi}{6}$

Official Ans. by NTA (C)

Allen Ans. (C)

19. Let $x = 2t$, $y = \frac{t^2}{3}$ be a conic. Let S be the focus and B be the point on the axis of the conic such that $SA \perp BA$, where A is any point on the conic. If k is the ordinate of the centroid of ΔSAB, then $\lim\limits_{t \to 1} k$ is equal to

(A) $\frac{17}{18}$ (B) $\frac{19}{18}$ (C) $\frac{11}{18}$ (D) $\frac{13}{18}$

Official Ans. by NTA (D)

Allen Ans. (D)

20. Let a circle C in complex plane pass tltrough the points $z_1 = 3 + 4i$, $z_2 = 4 + 3i$ and $z_3 = 5i$. If $z(\neq z_1)$ is a point on C such that the line through z and z_1 is perpendicular to the line through z_2 and z_3, then arg(z) is equal to :

(A) $\tan^{-1}\left(\frac{2}{\sqrt{5}}\right) - \pi$ (B) $\tan^{-1}\left(\frac{24}{7}\right) - \pi$

(C) $\tan^{-1}(3) - \pi$ (D) $\tan^{-1}\left(\frac{3}{4}\right) - \pi$

Official Ans. by NTA (B)

Allen Ans. (B)

SECTION-B

1. Let C_r denote the binomial coefficient of x^r in the expansion of $(1 + x)^{10}$. If $\alpha, \beta \in R$. $C_1 + 3 \cdot 2C_2 + 5 \cdot 3C_3 + \ldots$ upto 10 terms

$$= \frac{\alpha \times 2^{11}}{2^\beta - 1}\left(C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \ldots\text{upto 10 terms}\right)$$

then the value of $\alpha + \beta$ is equal to

Official Ans. by NTA (286)

Allen Ans. (BONUS)

2. The number of 3-digit odd numbers, whose sum of digits is a multiple of 7, is ______.

Official Ans. by NTA (63)

Allen Ans. (63)

3. Let θ be the angle between the vectors $\vec{a}$ and $\vec{b}$, where $|\vec{a}|=4, \; |\vec{b}|=3 \quad \theta \in\left(\frac{\pi}{4}, \frac{\pi}{3}\right)$. Then $\left|(\vec{a}-\vec{b}) \times(\vec{a}+\vec{b})\right|^{2}+4(\vec{a} \cdot \vec{b})^{2}$ is equal to ____

Official Ans. by NTA (576)

Allen Ans. (576)

4. Let the abscissae of the two points P and Q be the roots of $2x^2 - rx + p = 0$ and the ordinates of P and Q be the roots of $x^2 - sx - q = 0$. If the equation of the circle described on PQ as diameter is $2(x^2 + y^2) - 11x - 14y - 22 = 0$, then $2r + s - 2q + p$ is equal to

Official Ans. by NTA (7)

Allen Ans. (7)

5. The number of values of x in the interval $\left(\frac{\pi}{4}, \frac{7\pi}{4}\right)$ for which $14\operatorname{cosec}^2 x - 2\sin^2 x = 21 - 4\cos^2 x$ holds, is ________

Official Ans. by NTA (4)

Allen Ans. (4)

6. For a natural number n, let $a_n = 19^n - 12^n$. Then, the value of $\frac{31\alpha_9 - \alpha_{10}}{57\alpha_8}$ is

Official Ans. by NTA (4)

Allen Ans. (4)

7. Let $f : R \to R$ be a function defined by $f(x)=\left(2\left(1-\frac{x^{25}}{2}\right)\left(2+x^{25}\right)\right)^{\frac{1}{50}}$. If the function $g(x)=f(f(f(x)))+f(f(x))$, the the greatest integer less than or equal to g (1) is ______

Official Ans. by NTA (2)

Allen Ans. (2)

8. Let the lines

$L_1 : \vec{r}=\lambda(\hat{i}+2\hat{j}+3\hat{k}), \lambda \in R$

$L_2 : \vec{r}=(\hat{i}+3\hat{j}+\hat{k})+\mu(\hat{i}+\hat{j}+5\hat{k}); \mu \in R$

intersect at the point S. If a plane $ax + by - z + d = 0$ passes through S and is parallel to both the lines L_1 and L_2, then the value of $a + b + d$ is equal to _______

Official Ans. by NTA (5)

Allen Ans. (5)

9. Let A be a 3×3 matrix having entries from. the set $\{-1, 0, 1\}$. The number of all such matrices A having sum of all the entries equal to 5, is _______

Official Ans. by NTA (414)

Allen Ans. (414)

10. The greatest integer less than or equal to the sum of first 100 terms of the sequence $\frac{1}{3}, \frac{5}{9}, \frac{19}{27}, \frac{65}{81}, \ldots\ldots$ is equal to

Official Ans. by NTA (98)

Allen Ans. (98)