

FINAL JEE-MAIN EXAMINATION – JUNE, 2022

 (Held On Friday 24th June, 2022)

TIME : 9 : 00 AM to 12 : 00 PM

MATHEMATICS

SECTION-A

1. Let $A = \{z \in \mathbb{C} : 1 \leq |z - (1 + i)| \leq 2\}$ and $B = \{z \in A : |z - (1 - i)| = 1\}$. Then, B :
 (A) is an empty set
 (B) contains exactly two elements
 (C) contains exactly three elements
 (D) is an infinite set
Official Ans. by NTA (D)
Allen Ans. (D)
2. The remainder when 3^{2022} is divided by 5 is
 (A) 1 (B) 2
 (C) 3 (D) 4
Official Ans. by NTA (D)
Allen Ans. (D)
3. The surface area of a balloon of spherical shape being inflated, increases at a constant rate. If initially, the radius of balloon is 3 units and after 5 seconds,, it becomes 7 units, then its radius after 9 seconds is :
 (A) 9 (B) 10
 (C) 11 (D) 12
Official Ans. by NTA (A)
Allen Ans. (A)
4. Bag A contains 2 white, 1 black and 3 red balls and bag B contains 3 black, 2 red and n white balls. One bag is chosen at random and 2 balls drawn from it at random, are found to be 1 red and 1 black. If the probability that both balls come from Bag A is $\frac{6}{11}$, then n is equal to _____.
 (A) 13 (B) 6
 (C) 4 (D) 3
Official Ans. by NTA (C)
Allen Ans. (C)

TEST PAPER WITH ANSWER

5. Let $x^2 + y^2 + Ax + By + C = 0$ be a circle passing through (0, 6) and touching the parabola $y = x^2$ at (2, 4). Then A + C is equal to _____.
 (A) 16 (B) 88/5
 (C) 72 (D) -8
Official Ans. by NTA (A)
Allen Ans. (A)
6. The number of values of α for which the system of equations :

$$x + y + z = \alpha$$

$$\alpha x + 2\alpha y + 3z = -1$$

$$x + 3\alpha y + 5z = 4$$
 is inconsistent, is
 (A) 0 (B) 1
 (C) 2 (D) 3
Official Ans. by NTA (B)
Allen Ans. (B)
7. If the sum of the squares of the reciprocals of the roots α and β of the equation $3x^2 + \lambda x - 1 = 0$ is 15, then $6(\alpha^3 + \beta^3)^2$ is equal to :
 (A) 18 (B) 24
 (C) 36 (D) 96
Official Ans. by NTA (B)
Allen Ans. (B)
8. The set of all values of k for which $(\tan^{-1} x)^3 + (\cot^{-1} x)^3 = k\pi^3, x \in \mathbb{R}$, is the interval :
 (A) $\left[\frac{1}{32}, \frac{7}{8}\right]$ (B) $\left(\frac{1}{24}, \frac{13}{16}\right)$
 (C) $\left[\frac{1}{48}, \frac{13}{16}\right]$ (D) $\left[\frac{1}{32}, \frac{9}{8}\right]$
Official Ans. by NTA (A)
Allen Ans. (A)

9. Let $S = \{\sqrt{n} : 1 \leq n \leq 50 \text{ and } n \text{ is odd}\}$

Let $a \in S$ and $A = \begin{bmatrix} 1 & 0 & a \\ -1 & 1 & 0 \\ -a & 0 & 1 \end{bmatrix}$

If $\sum_{a \in S} \det(\text{adj} A) = 100\lambda$, then λ is equal to

- (A) 218 (B) 221
(C) 663 (D) 1717

Official Ans. by NTA (B)

Allen Ans. (B)

10. $f(x) = 4 \log_e(x-1) - 2x^2 + 4x + 5$, $x > 1$, which one of the following is NOT correct ?

- (A) f is increasing in $(1, 2)$ and decreasing in $(2, \infty)$
(B) $f(x) = -1$ has exactly two solutions
(C) $f'(e) - f''(2) < 0$
(D) $f(x) = 0$ has a root in the interval $(e, e+1)$

Official Ans. by NTA (C)

Allen Ans. (C)

11. the tangent at the point (x_1, y_1) on the curve $y = x^3 + 3x^2 + 5$ passes through the origin, then (x_1, y_1) does NOT lie on the curve :

- (A) $x^2 + \frac{y^2}{81} = 2$ (B) $\frac{y^2}{9} - x^2 = 8$
(C) $y = 4x^2 + 5$ (D) $\frac{x}{3} - y^2 = 2$

Official Ans. by NTA (D)

Allen Ans. (D)

12. The sum of absolute maximum and absolute minimum values of the function

$f(x) = |2x^2 + 3x - 2| + \sin x \cos x$ in the interval $[0, 1]$ is :

- (A) $3 + \frac{\sin(1) \cos^2(\frac{1}{2})}{2}$ (B) $3 + \frac{1}{2} (1 + 2\cos(1)) \sin(1)$
(C) $5 + \frac{1}{2} (\sin(1) + \sin(2))$ (D) $2 + \sin\left(\frac{1}{2}\right) \cos\left(\frac{1}{2}\right)$

Official Ans. by NTA (B)

Allen Ans. (B)

13. If $\{a_i\}_{i=1}^n$ where n is an even integer, is an arithmetic progression with common difference 1, and $\sum_{i=1}^n a_i = 192$, $\sum_{i=1}^{n/2} a_{2i} = 120$, then n is equal to:

- (A) 48 (B) 96
(C) 92 (D) 104

Official Ans. by NTA (B)

Allen Ans. (B)

14. If $x = x(y)$ is the solution of the differential equation $y \frac{dx}{dy} = 2x + y^3(y+1)e^y$, $x(1) = 0$; then $x(e)$

is equal to :

- (A) $e^3(e^e - 1)$ (B) $e^e(e^3 - 1)$
(C) $e^2(e^e + 1)$ (D) $e^e(e^2 - 1)$

Official Ans. by NTA (A)

Allen Ans. (A)

15. Let $\lambda x - 2y = \mu$ be a tangent to the hyperbola

$a^2x^2 - y^2 = b^2$. Then $\left(\frac{\lambda}{a}\right)^2 - \left(\frac{\mu}{b}\right)^2$ is equal to:

- (A) -2 (B) -4
(C) 2 (D) 4

Official Ans. by NTA (D)

Allen Ans. (D)

16. Let \hat{a}, \hat{b} be unit vectors. If \vec{c} be a vector such that the angle between \hat{a} and \vec{c} is $\frac{\pi}{12}$, and

$\hat{b} = \vec{c} + 2(\vec{c} \times \hat{a})$, then $|\vec{c}|^2$ is equal to

- (A) $6(3 - \sqrt{3})$ (B) $3 + \sqrt{3}$
(C) $6(3 + \sqrt{3})$ (D) $6(\sqrt{3} + 1)$

Official Ans. by NTA (C)

Allen Ans. (C)

17. If a random variable X follows the Binomial distribution $B(33, p)$ such that $3P(X=0) = P(X=1)$, then the value of $\frac{P(X=15)}{P(X=18)} - \frac{P(X=16)}{P(X=17)}$ is equal

to

- (A) 1320 (B) 1088
(C) $\frac{120}{1331}$ (D) $\frac{1088}{1089}$

Official Ans. by NTA (A)

Allen Ans. (A)

18. The domain of the function

$$f(x) = \frac{\cos^{-1}\left(\frac{x^2 - 5x + 6}{x^2 - 9}\right)}{\log_e(x^2 - 3x + 2)} \text{ is}$$

- (A) $(-\infty, 1) \cup (2, \infty)$
(B) $(2, \infty)$
(C) $\left[-\frac{1}{2}, 1\right) \cup (2, \infty)$
(D) $\left[-\frac{1}{2}, 1\right) \cup (2, \infty) - \left\{\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}\right\}$

Official Ans. by NTA (DROP)

Allen Ans. (D)

19. Let

$$S = \left\{ \theta \in [-\pi, \pi] - \left\{ \pm \frac{\pi}{2} \right\} : \sin \theta \tan \theta + \tan \theta = \sin 2\theta \right\}.$$

If $T = \sum_{\theta \in S} \cos 2\theta$, then $T + n(S)$ is equal

- (A) $7 + \sqrt{3}$ (B) 9
(C) $8 + \sqrt{3}$ (D) 10

Official Ans. by NTA (B)

Allen Ans. (B)

20. The number of choices of $\Delta \in \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$, such that $(p\Delta q) \Rightarrow ((p\Delta \sim q) \vee ((\sim p)\Delta q))$ is a tautology, is

- (A) 1 (B) 2
(C) 3 (D) 4

Official Ans. by NTA (B)

Allen Ans. (B)

SECTION-B

1. The number of one-one function $f: \{a, b, c, d\} \rightarrow \{0, 1, 2, \dots, 10\}$ such that $2f(a) - f(b) + 3f(c) + f(d) = 0$ is _____.

Official Ans. by NTA (31)

Allen Ans. (31)

2. In an examination, there are 5 multiple choice questions with 3 choices, out of which exactly one is correct. There are 3 marks for each correct answer, -2 marks for each wrong answer and 0 mark if the question is not attempted. Then, the number of ways a student appearing in the examination gets 5 marks is _____.

Official Ans. by NTA (40)

Allen Ans. (40)

3. Let $A\left(\frac{3}{\sqrt{a}}, \sqrt{a}\right)$, $a > 0$, be a fixed point in the xy -plane. The image of A in y -axis be B and the image of B in x -axis be C . If $D(3 \cos \theta, a \sin \theta)$ is a point in the fourth quadrant such that the maximum area of ΔACD is 12 square units, then a is equal to _____.

Official Ans. by NTA (8)

Allen Ans. (8)

4. Let a line having direction ratios 1, -4, 2 intersect the lines $\frac{x-7}{3} = \frac{y-1}{-1} = \frac{z+2}{1}$ and $\frac{x}{2} = \frac{y-7}{3} = \frac{z}{1}$ at the point A and B . Then $(AB)^2$ is equal to _____.

Official Ans. by NTA (84)

Allen Ans. (84)

5. The number of points where the function

$$f(x) = \begin{cases} |2x^2 - 3x - 7| & \text{if } x \leq -1 \\ [4x^2 - 1] & \text{if } -1 < x < 1 \\ |x+1| + |x-2| & \text{if } x \geq 1 \end{cases}$$

$[t]$ denotes the greatest integer $\leq t$, is

discontinuous is _____.

Official Ans. by NTA (7)

Allen Ans. (7)

6. Let $f(\theta) = \sin \theta + \int_{-\pi/2}^{\pi/2} (\sin \theta + t \cos \theta) f(t) dt$. Then the

value of $\left| \int_0^{\pi/2} f(\theta) d\theta \right|$ is _____.

Official Ans. by NTA (1)

Allen Ans. (1)

7. Let $\text{Max}_{0 \leq x \leq 2} \left\{ \frac{9-x^2}{5-x} \right\} = \alpha$ and $\text{Min}_{0 \leq x \leq 2} \left\{ \frac{9-x^2}{5-x} \right\} = \beta$

$$\text{If } \int_{\beta - \frac{8}{3}}^{2\alpha - 1} \text{Max} \left\{ \frac{9-x^2}{5-x}, x \right\} dx = \alpha_1 + \alpha_2 \log_e \left(\frac{8}{15} \right)$$

then $\alpha_1 + \alpha_2$ is equal to _____

Official Ans. by NTA (34)

Allen Ans. (34)

8. If two tangents drawn from a point (α, β) lying on the ellipse $25x^2 + 4y^2 = 1$ to the parabola $y^2 = 4x$ are such that the slope of one tangent is four times the other, then the value of

$$(10\alpha + 5)^2 + (16\beta^2 + 50)^2 \text{ equals } \underline{\hspace{2cm}}$$

Official Ans. by NTA (2929)

Allen Ans. (2929)

9. Let S be the region bounded by the curves $y = x^3$ and $y^2 = x$. The curve $y = 2|x|$ divides S into two regions of areas R_1 and R_2 .

If $\max \{R_1, R_2\} = R_2$, then $\frac{R_2}{R_1}$ is equal to _____.

Official Ans. by NTA (19)

Allen Ans. (19)

10. If the shortest distance between the line

$$\vec{r} = (-\hat{i} + 3\hat{k}) + \lambda(\hat{i} - a\hat{j}) \text{ and}$$

$$\vec{r} = (-\hat{j} + 2\hat{k}) + \mu(\hat{i} - \hat{j} + \hat{k}) \text{ is } \sqrt{\frac{2}{3}}, \text{ then the integral}$$

value of a is equal to

Official Ans. by NTA (2)

Allen Ans. (2)