FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Friday 29th July, 2022) **TIME : 3 : 00 PM to 06 : 00 PM**

MATHEMATICS

TEST PAPER WITH ANSWER

SECTION-A

1. If $z \neq 0$ be a complex number such that $\left|z-\frac{1}{z}\right|=2$, then the maximum value of $|z|$ is:

(A) $\sqrt{2}$ (B) 1

(C) $\sqrt{2}-1$ (D) $\sqrt{2}+1$

Official Ans. by NTA (D)

Allen Ans. (D)

2. Which of the following matrices can NOT be obtained from the matrix $\begin{bmatrix}-1 & 2\\ 1 & -1\end{bmatrix}$ by a single elementary row operation?

(A) $\begin{bmatrix}0 & 1\\ 1 & -1\end{bmatrix}$ (B) $\begin{bmatrix}1 & -1\\ -1 & 2\end{bmatrix}$

(C) $\begin{bmatrix}-1 & 2\\ -2 & 7\end{bmatrix}$ (D) $\begin{bmatrix}-1 & 2\\ -1 & 3\end{bmatrix}$

Official Ans. by NTA (C)

Allen Ans. (C)

3. If the system of equations

$x + y + z = 6$

$2x + 5y + \alpha z = \beta$

$x + 2y + 3z = 14$

has infinitely many solutions, then $\alpha + \beta$ is equal to :

(A) 8 (B) 36

(C) 44 (D) 48

Official Ans. by NTA (C)

Allen Ans. (C)

4. Let the function

$$f(x)=\begin{cases}\dfrac{\log_e(1+5x)-\log_e(1+\alpha x)}{x} & ;\text{if } x\neq 0\\ 10 & ;\text{if } x=0\end{cases}$$

be continuous at $x = 0$.

The α is equal to :

(A) 10 (B) –10

(C) 5 (D) –5

Official Ans. by NTA (D)

Allen Ans. (D)

5. If $[t]$ denotes the greatest integer $\leq t$, then the value of $\int_0^1\left[2x-|3x^2-5x+2|+1\right]dx$ is:

(A) $\frac{\sqrt{37}+\sqrt{13}-4}{6}$ (B) $\frac{\sqrt{37}-\sqrt{13}-4}{6}$

(C) $\frac{-\sqrt{37}-\sqrt{13}+4}{6}$ (D) $\frac{-\sqrt{37}+\sqrt{13}+4}{6}$

Official Ans. by NTA (A)

Allen Ans. (A)

6. Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 3a_{n+1} - 2a_n + 1,\ \forall\ n \geq 0$.

Then $a_{25}\,a_{23} - 2\,a_{25}\,a_{22} - 2\,a_{23}\,a_{24} + 4\,a_{22}\,a_{24}$ is equal to:

(A) 483 (B) 528

(C) 575 (D) 624

Official Ans. by NTA (B)

Allen Ans. (B)

7. $\sum_{r=1}^{20}(r^2+1)(r!)$ is equal to:

(A) 22! – 21! (B) 22! – 2 (21!)

(C) 21! – 2 (20!) (D) 21! – 20!

Official Ans. by NTA (B)

Allen Ans. (B)

8. For $I(x)=\int\frac{\sec^2 x-2022}{\sin^{2022}x}dx$, if $I\left(\frac{\pi}{4}\right)=2^{1011}$, then

(A) $3^{1010}I\left(\frac{\pi}{3}\right)-I\left(\frac{\pi}{6}\right)=0$

(B) $3^{1010}I\left(\frac{\pi}{6}\right)-I\left(\frac{\pi}{3}\right)=0$

(C) $3^{1011}I\left(\frac{\pi}{3}\right)-I\left(\frac{\pi}{6}\right)=0$

(D) $3^{1011}I\left(\frac{\pi}{6}\right)-I\left(\frac{\pi}{3}\right)=0$

Official Ans. by NTA (A)

Allen Ans. (A)

9. If the solution curve of the differential equation $\frac{dy}{dx}=\frac{x+y-2}{x-y}$ passes through the point (2,1) and (k + 1,2), k > 0, then

(A) $2\tan^{-1}\left(\frac{1}{k}\right)=\log_e\left(k^2+1\right)$

(B) $\tan^{-1}\left(\frac{1}{k}\right)=\log_e\left(k^2+1\right)$

(C) $2\tan^{-1}\left(\frac{1}{k+1}\right)=\log_e\left(k^2+2k+2\right)$

(D) $2\tan^{-1}\left(\frac{1}{k}\right)=\log_e\left(\frac{k^2+1}{k^2}\right)$

Official Ans. by NTA (A)

Allen Ans. (A)

10. Let y = y (x) be the solution curve of the differential equation $\frac{dy}{dx}+\left(\frac{2x^2+11x+13}{x^3+6x^2+11x+6}\right)y=\frac{(x+3)}{x+1}$, x > –1, which passes through the point (0,1). Then y (1) is equal to:

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$

(C) $\frac{5}{2}$ (D) $\frac{7}{2}$

Official Ans. by NTA (B)

Allen Ans. (B)

11. Let m_1, m_2 be the slopes of two adjacent sides of a square of side a such that $a^2+11a+3\left(m_2^2+m_2^2\right)=220$. If one vertex of the square is (10(cosα – sinα), 10 (sinα + cosα)), where $\alpha\in\left(0,\frac{\pi}{2}\right)$ and the equation of one diagonal is (cosα – sinα) x + (sinα + cosα) y = 10, then 72 $(\sin^4\alpha+\cos^4\alpha)+a^2-3a+13$ is equal to:

(A) 119 (B) 128

(C) 145 (D) 155

Official Ans. by NTA (B)

Allen Ans. (B)

12. The number of elements in the set $S=\left\{x\in\mathbb{R}:2\cos\left(\frac{x^2+x}{6}\right)=4^x+4^{-x}\right\}$ is:

(A) 1 (B) 3

(C) 0 (D) infinite

Official Ans. by NTA (A)

Allen Ans. (A)

13. Let A (α, –2), B (α, 6) and $C\left(\frac{\alpha}{4},-2\right)$ be vertices of a ΔABC. If $\left(5,\frac{\alpha}{4}\right)$ is the circumcentre of ΔABC, then which of the following is NOT correct about ΔABC:

(A) ares is 24 (B) perimeter is 25

(C) circumradius is 5 (D) inradius is 2

Official Ans. by NTA (B)

Allen Ans. (B)

14. Let Q be the food of perpendicular drawn from the point P (1, 2, 3) to the plane x + 2y + z = 14. If R is a point on the plane such that ∠PRQ = 60°, then the area of ΔPQR is equal to:

(A) $\frac{\sqrt{3}}{2}$ (B) $\sqrt{3}$

(C) $2\sqrt{3}$ (D) 3

Official Ans. by NTA (B)

Allen Ans. (B)

15. If (2, 3, 9), (5, 2, 1), (1, λ, 8) and (λ, 2, 3) are coplanar, then the product of all possible values of λ is:

(A) $\frac{21}{2}$ (B) $\frac{59}{8}$

(C) $\frac{57}{8}$ (D) $\frac{95}{8}$

Official Ans. by NTA (D)

Allen Ans. (D)

16. Bag I contains 3 red, 4 black and 3 white balls and Bag II contains 2 red, 5 black and 2 white balls. One ball is transferred from Bag I to Bag II and then a ball is draw from Bag II. The ball so drawn is found to be black in colour. Then the probability, that the transferred ball is red, is:

(A) $\frac{4}{9}$ (B) $\frac{5}{18}$

(C) $\frac{1}{6}$ (D) $\frac{3}{10}$

Official Ans. by NTA (B)

Allen Ans. (B)

17. Let S = $\{z = x + iy : |z - 1 + i| \ge |z|, |z| < 2, |z + i| = |z - 1|\}$. Then the set of all values of x, for which $w = 2x + iy \in S$ for some $y \in \mathbb{R}$, is

(A) $\left(-\sqrt{2}, \frac{1}{2\sqrt{2}}\right]$ (B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{4}\right]$

(C) $\left(-\sqrt{2}, \frac{1}{2}\right]$ (D) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}\right]$

Official Ans. by NTA (B)

Allen Ans. (B)

18. Let $\vec{a}, \vec{b}, \vec{c}$ be three coplanar concurrent vectors such that angles between any two of them is same. If the product of their magnitudes is 14 and $(\vec{a}\times\vec{b}).(\vec{b}\times\vec{c}) + (\vec{b}\times\vec{c}).(\vec{c}\times\vec{a}) + (\vec{c}\times\vec{a}).(\vec{a}\times\vec{b}) = 168$ then $|\vec{a}| + |\vec{b}| + |\vec{c}|$ is equal to:

(A) 10 (B) 14

(C) 16 (D) 18

Official Ans. by NTA (C)

Allen Ans. (C)

19. The domain of the function $f(x) = \sin^{-1}\left(\frac{x^2 - 3x + 2}{x^2 + 2x + 7}\right)$ is :

(A) $[1, \infty)$ (B) $(-1, 2]$

(C) $[-1, \infty)$ (D) $(-\infty, 2]$

Official Ans. by NTA (C)

Allen Ans. (C)

20. The statement $(p \Rightarrow q) \vee (p \Rightarrow r)$ is NOT equivalent to:

(A) $(p \wedge (\sim r)) \Rightarrow q$ (B) $(\sim q) \Rightarrow ((\sim r) \vee p)$

(C) $p \Rightarrow (q \vee r)$ (D) $(p \wedge (\sim q)) \Rightarrow r$

Official Ans. by NTA (B)

Allen Ans. (B)

SECTION-B

1. The sum and product of the mean and variance of a binomial distribution are 82.5 and 1350 respectively. They the number of trials in the binomial distribution is:

Official Ans. by NTA (96)

Allen Ans. (96)

2. Let α, β (α > β) be the roots of the quadratic equation $x^2 - x - 4 = 0$. If $P_n = \alpha^n - \beta^n$, $n \in \mathbb{N}$, then $\frac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^2 + P_{14}P_{15}}{P_{13}P_{14}}$ is equal to _____.

Official Ans. by NTA (16)

Allen Ans. (16)

3. Let $x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}$. For $k \in \mathbb{N}$, if $X' A^k X = 33$, then k is equal to:

Official Ans. by NTA (10)

Allen Ans. (Dropped or 10)

4. The number of natural numbers lying between 1012 and 23421 that can be formed using the digits 2, 3, 4, 5, 6 (repetition of digits is not allowed) and divisible by 55 is____,

Official Ans. by NTA (6)

Allen Ans. (6)

5. If $\sum_{k=1}^{10} K^2\left(10_{C_K}\right)^2 = 22000L$, then L is equal to ____.

Official Ans. by NTA (221)

Allen Ans. (221)

6. If [t] denotes the greatest integer ≤ t, then number of points, at which the function f(x) = 4 | 2x + 3| + $9\left[x+\frac{1}{2}\right]-12\,[x+20]$ is not differentiable in the open interval (–20, 20), is____.

Official Ans. by NTA (79)

Allen Ans. (79)

7. If the tangent to the curve $y = x^3 - x^2 + x$ at the point (a , b) is also tangent to the curve $y = 5x^2 + 2x - 25$ at the point (2, –1), then |2a + 9b| is equal to ______.

Official Ans. by NTA (195)

Allen Ans. (195)

8. Let AB be a chord of length 12 of the circle $(x-2)^2 + (y+1)^2 = \frac{169}{4}$.

If tangents drawn to the circle at points A and B intersect at the point P, then five times the distance of point P from chord AB is equal to ____.

Official Ans. by NTA (72)

Allen Ans. (72)

9. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + 2|\vec{b}|^2, \vec{a}.\vec{b} = 3$ and $|\vec{a}\times\vec{b}|^2 = 75$.

Then $|\vec{a}|^2$ is equal to_____.

Official Ans. by NTA (14)

Allen Ans. (14)

10. Let

$$S = \left\{(x,y)\in\mathbb{N}\times\mathbb{N} : 9(x-3)^2 + 16(y-4)^2 \le 144\right\}$$

and $T = \left\{(x,y)\in\mathbb{R}\times\mathbb{R} : (x-7)^2 + (y-4)^2 \le 36\right\}$.

The $n(S\cap T)$ is equal to______.

Official Ans. by NTA (27)

Allen Ans. (27)