FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Thursday 28th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS | TEST PAPER WITH ANSWER

SECTION-A

1. Let $S=\left\{x\in[-6,3]-\{-2,2\}:\frac{|x+3|-1}{|x|-2}\geq 0\right\}$ and $T=\{x\in Z:x^2-7|x|+9\leq 0\}$. Then the number of elements in $S\cap T$ is

(A) 7 (B) 5

(C) 4 (D) 3

Official Ans. by NTA (D)

Allen Ans. (D)

2. Let α, β be the roots of the equation $x^2-\sqrt{2}x+\sqrt{6}=0$ and $\frac{1}{\alpha^2}+1,\frac{1}{\beta^2}+1$ be the roots of the equation $x^2 + ax + b = 0$. Then the roots of the equation $x^2 - (a + b - 2) x + (a + b + 2) = 0$ are :

(A) non-real complex numbers

(B) real and both negative

(C) real and both positive

(D) real and exactly one of them is positive

Official Ans. by NTA (B)

Allen Ans. (B)

3. Let A and B be any two 3 × 3 symmetric and skew symmetric matrices respectively. Then which of the following is <u>**NOT**</u> true?

(A) $A^4 - B^4$ is a symmetric matrix

(B) AB – BA is a symmetric matrix

(C) $B^5 - A^5$ is a skew-symmetric matrix

(D) AB + BA is a skew-symmetric matrix

Official Ans. by NTA (C)

Allen Ans. (C)

4. Let $f(x) = ax^2 + bx + c$ be such that f(1) = 3, f(– 2) = λ and f(3) = 4. If f(0) + f(1) + f(–2) + f(3) = 14, then λ is equal to

(A) – 4 (B) $\frac{13}{2}$

(C) $\frac{23}{2}$ (D) 4

Official Ans. by NTA (D)

Allen Ans. (D)

5. The function $f : R \to R$ defined by

$$f(x)=\lim_{n\to\infty}\frac{\cos(2\pi x)-x^{2n}\sin(x-1)}{1+x^{2n+1}-x^{2n}}$$ is

continuous for all x in

(A) R – {–1} (B) R – {– 1, 1}

(C) R – {1} (D) R – {0}

Official Ans. by NTA (B)

Allen Ans. (B)

6. The function $f(x)=xe^{x(1-x)}, x\in R$, is

(A) increasing in $\left(-\frac{1}{2},1\right)$

(B) decreasing in $\left(\frac{1}{2},2\right)$

(C) increasing in $\left(-1,-\frac{1}{2}\right)$

(D) decreasing in $\left(-\frac{1}{2},\frac{1}{2}\right)$

Official Ans. by NTA (A)

Allen Ans. (A)

7. The sum of the absolute maximum and absolute minimum values of the function

$f(x) = \tan^{-1}(\sin x - \cos x)$ in the interval $[0, \pi]$ is

(A) 0 (B) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)-\frac{\pi}{4}$

(C) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)-\frac{\pi}{4}$ (D) $\frac{-\pi}{12}$

Official Ans. by NTA (C)

Allen Ans. (C)

8. Let $x(t)=2\sqrt{2}\cos t\sqrt{\sin 2t}$ and $y(t)=2\sqrt{2}\sin t\sqrt{\sin 2t}$, $t\in\left(0,\frac{\pi}{2}\right)$. Then

$$\frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}} \text{ at } t=\frac{\pi}{4} \text{ is equal to}$$

(A) $\frac{-2\sqrt{2}}{3}$ (B) $\frac{2}{3}$

(C) $\frac{1}{3}$ (D) $\frac{-2}{3}$

Official Ans. by NTA (D)

Allen Ans. (D)

9. Let $I_n(x)=\int_0^x \frac{1}{(t^2+5)^n}dt,\ n=1,2,3,\ldots$. Then

(A) $50I_6-9I_5=xI_5'$ (B) $50I_6-11I_5=xI_5'$

(C) $50I_6-9I_5=I_5'$ (D) $50I_6-11I_5=I_5'$

Official Ans. by NTA (A)

Allen Ans. (A)

10. The area enclosed by the curves $y = \log_e (x + e^2)$, $x=\log_e\left(\frac{2}{y}\right)$ and $x = \log_e 2$, above the line $y = 1$ is

(A) $2+e-\log_e 2$

(B) $1+e-\log_e 2$

(C) $e-\log_e 2$

(D) $1+\log_e 2$

Official Ans. by NTA (B)

Allen Ans. (B)

11. Let $y = y(x)$ be the solution curve of the differential equation $\frac{dy}{dx}+\frac{1}{x^2-1}y=\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$, $x > 1$ passing through the point $\left(2,\sqrt{\frac{1}{3}}\right)$. Then $\sqrt{7}y(8)$ is equal to

(A) $11+6\log_e 3$ (B) 19

(C) $12-2\log_e 3$ (D) $19-6\log_e 3$

Official Ans. by NTA (D)

Allen Ans. (D)

12. The differential equation of the family of circles passing through the points (0, 2) and (0, – 2) is

(A) $2xy\frac{dy}{dx}+(x^2-y^2+4)=0$

(B) $2xy\frac{dy}{dx}+(x^2+y^2-4)=0$

(C) $2xy\frac{dy}{dx}+(y^2-x^2+4)=0$

(D) $2xy\frac{dy}{dx}-(x^2-y^2+4)=0$

Official Ans. by NTA (A)

Allen Ans. (A)

13. Let the tangents at two points A and B on the circle $x^2 + y^2 - 4x + 3 = 0$ meet at origin O (0, 0). Then the area of the triangle of OAB is

(A) $\frac{3\sqrt{3}}{2}$

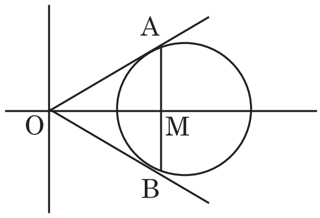
(B) $\frac{3\sqrt{3}}{4}$

(C) $\frac{3}{2\sqrt{3}}$

(D) $\frac{3}{4\sqrt{3}}$

Official Ans. by NTA (B)

Allen Ans. (B)



Sol. C : $(x-2)^2 + y^2 = 1$

Equation of chord AB : $2x = 3$

$OA = OB = \sqrt{3}$

$AM=\frac{\sqrt{3}}{2}$

Area of triangle OAB $=\frac{1}{2}(2AM)(OB)$

$=\frac{3\sqrt{3}}{4}$ sq. units

14. Let the hyperbola H : $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ pass through the point $\left(2\sqrt{2},-2\sqrt{2}\right)$. A parabola is drawn whose focus is same as the focus of H with positive abscissa and the directrix of the parabola passes through the other focus of H. If the length of the latus rectum of the parabola is e times the length of the latus rectum of H, where e is the eccentricity of H, then which of the following points lies on the parabola?

(A) $\left(2\sqrt{3},3\sqrt{2}\right)$ (B) $\left(3\sqrt{3},-6\sqrt{2}\right)$

(C) $\left(\sqrt{3},-\sqrt{6}\right)$ (D) $\left(3\sqrt{6},6\sqrt{2}\right)$

Official Ans. by NTA (B)

Allen Ans. (B)

15. Let the lines $\frac{x-1}{\lambda}=\frac{y-2}{1}=\frac{z-3}{2}$ and $\frac{x+26}{-2}=\frac{y+18}{3}=\frac{z+28}{\lambda}$ be coplanar and P be the plane containing these two lines. Then which of the following points does **NOT** lies on P?

(A) (0, – 2, –2) (B) (– 5, 0, –1)

(C) (3, –1, 0) (D) (0, 4, 5)

Official Ans. by NTA (D)

Allen Ans. (D)

16. A plane P is parallel to two lines whose direction ratios are –2, 1, –3, and –1, 2, –2 and it contains the point (2, 2, –2). Let P intersect the co-ordinate axes at the points A, B, C making the intercepts α, β, γ. If V is the volume of the tetrahedron OABC, where O is the origin and p = $\alpha+\beta+\gamma$, then the ordered pair (V, p) is equal to

(A) (48, –13) (B) (24, –13)

(C) (48, 11) (D) (24, –5)

Official Ans. by NTA (B)

Allen Ans. (B)

17. Let S be the set of all a $\in$ R for which the angle between the vectors $\vec{u}=a\left(\log_e b\right)\hat{i}-6\hat{j}+3\hat{k}$ and $\vec{v}=\left(\log_e b\right)\hat{i}+2\hat{j}+2a\left(\log_e b\right)\hat{k},(b>1)$ is acute. Then S is equal to

(A) $\left(-\infty,-\frac{4}{3}\right)$ (B) Φ

(C) $\left(-\frac{4}{3},0\right)$ (D) $\left(\frac{12}{7},\infty\right)$

Official Ans. by NTA (B)

Allen Ans. (B)

18. A horizontal park is in the shape of a triangle OAB with AB = 16. A vertical lamp post OP is erected at the point O such that ∠PAO = ∠PBO = 15° and ∠PCO = 45°, where C is the midpoint of AB. Then $(OP)^2$ is equal to

(A) $\frac{32}{\sqrt{3}}\left(\sqrt{3}-1\right)$ (B) $\frac{32}{\sqrt{3}}\left(2-\sqrt{3}\right)$

(C) $\frac{16}{\sqrt{3}}\left(\sqrt{3}-1\right)$ (D) $\frac{16}{\sqrt{3}}\left(2-\sqrt{3}\right)$

Official Ans. by NTA (B)

Allen Ans. (B)

19. Let A and B be two events such that $P(B|A)=\frac{2}{5}$, $P(A|B)=\frac{1}{7}$ and $P(A\cap B)=\frac{1}{9}$. Consider

$(S1)\ P(A'\cup B)=\frac{5}{6}$,

$(S2)\ P(A'\cap B')=\frac{1}{18}$. Then

(A) Both (S1) and (S2) are true

(B) Both (S1) and (S2) are false

(C) Only (S1) is true

(D) Only (S2) is true

Official Ans. by NTA (A)

Allen Ans. (A)

20. Let

p : Ramesh listens to music.

q : Ramesh is out of his village

r : It is Sunday

s : It is Saturday

Then the statement "Ramesh listens to music only if he is in his village and it is Sunday or Saturday" can be expressed as

(A) $((\sim q)\wedge(r\vee s))\Rightarrow p$

(B) $(q\wedge(r\vee s))\Rightarrow p$

(C) $p\Rightarrow(q\wedge(r\vee s))$

(D) $p\Rightarrow((\sim q)\wedge(r\vee s))$

Official Ans. by NTA (D)

Allen Ans. (D)

SECTION-B

1. Let the coefficients of the middle terms in the expansion of $\left(\frac{1}{\sqrt{6}}+\beta x\right)^4, (1-3\beta x)^2$ and $\left(1-\frac{\beta}{2}x\right)^6, \beta>0$, respectively form the first three terms of an A.P. If d is the common difference of this A.P., then $50-\frac{2d}{\beta^2}$ is equal to _____

Official Ans. by NTA (57)

Allen Ans. (57)

2. A class contains b boys and g girls. If the number of ways of selecting 3 boys and 2 girls from the class is 168, then b + 3 g is equal to

Official Ans. by NTA (17)

Allen Ans. (17)

3. Let the tangents at the points P and Q on the ellipse $\frac{x^2}{2}+\frac{y^2}{4}=1$ meet at the point $R\left(\sqrt{2}, 2\sqrt{2}-2\right)$. If S is the focus of the ellipse on its negative major axis, then $SP^2 + SQ^2$ is equal to

Official Ans. by NTA (13)

Allen Ans. (13)

4. If $1+(2+{}^{49}C_1+{}^{49}C_2+\ldots..+{}^{49}C_{49})({}^{50}C_2+{}^{50}C_4+\ldots..+{}^{50}C_{50})$ is equal to $2^n.m$, where m is odd, then n + m is equal to _____

Official Ans. by NTA (99)

Allen Ans. (99)

5. Two tangent lines l_1 and l_2 are drawn from the point (2, 0) to the parabola $2y^2 = -x$. If the lines l_1 and l_2 are also tangent to the circle $(x-5)^2 + y^2 = r$, then 17r is equal to

Official Ans. by NTA (9)

Allen Ans. (9)

6. If $\frac{6}{3^{12}}+\frac{10}{3^{11}}+\frac{20}{3^{10}}+\frac{40}{3^9}+\ldots..+\frac{10240}{3}=2^n\cdot m$, where m is odd, then m.n is equal to _____

Official Ans. by NTA (12)

Allen Ans. (12)

7. Let $S=\left[-\pi,\frac{\pi}{2}\right)-\left\{-\frac{\pi}{2},-\frac{\pi}{4},-\frac{3\pi}{4},\frac{\pi}{4}\right\}$. Then the number of elements in the set

$$A=\left\{\theta\in S:\tan\theta\left(1+\sqrt{5}\tan(2\theta)\right)=\sqrt{5}-\tan(2\theta)\right\}$$

is _____

Official Ans. by NTA (5)

Allen Ans. (5)

8. Let z = a + ib, b ≠ 0 be complex numbers satisfying $z^2=\bar{z}\cdot 2^{1-|z|}$. Then the least value of $n \in N$, such that $z^n = (z+1)^n$, is equal to _____

Official Ans. by NTA (6)

Allen Ans. (6)

9. A bag contains 4 white and 6 black balls. Three balls are drawn at random from the bag. Let X be the number of white balls, among the drawn balls. If σ^2 is the variance of X, then $100\,\sigma^2$ is equal to

Official Ans. by NTA (56)

Allen Ans. (56)

10. The value of the integral $\int_0^{\frac{\pi}{2}} 60\frac{\sin(6x)}{\sin x}dx$ is equal to

Official Ans. by NTA (104)

Allen Ans. (104)