FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Tuesday 26th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS

TEST PAPER WITH ANSWER

SECTION-A

1. The minimum value of the sum of the squares of the roots of $x^2+(3-a)x+1=2a$ is:

(A) 4 (B) 5
(C) 6 (D) 8

Official Ans. by NTA (C)
Allen Ans. (C)

2. If $z = x + iy$ satisfies $|z| - 2 = 0$ and $|z-i|-|z+5i|=0$, then

(A) $x + 2y - 4 = 0$ (B) $x^2 + y - 4 = 0$
(C) $x + 2y + 4 = 0$ (D) $x^2 - y + 3 = 0$

Official Ans. by NTA (C)
Allen Ans. (C)

3. Let $A=\begin{bmatrix}1\\1\\1\end{bmatrix}$ and $B=\begin{bmatrix}9^2 & -10^2 & 11^2\\ 12^2 & 13^2 & -14^2\\ -15^2 & 16^2 & 17^2\end{bmatrix}$, then the value of $A'BA$ is:

(A) 1224 (B) 1042 (C) 540 (D) 539

Official Ans. by NTA (D)
Allen Ans. (D)

4. $\sum\limits_{\substack{i,j=0\\ i\neq j}}^{n} {}^nC_i\ {}^nC_j$ is equal to

(A) $2^{2n} - {}^{2n}C_n$ (B) $2^{2n-1} - {}^{2n-1}C_{n-1}$
(C) $2^{2n} - \frac{1}{2}\,{}^{2n}C_n$ (D) $2^{n-1} + {}^{2n-1}C_n$

Official Ans. by NTA (A)
Allen Ans. (A)

5. Let P and Q be any points on the curves $(x-1)^2+(y+1)^2=1$ and $y = x^2$, respectively. The distance between P and Q is minimum for some value of the abscissa of P in the interval

(A) $\left(0,\frac{1}{4}\right)$ (B) $\left(\frac{1}{2},\frac{3}{4}\right)$
(C) $\left(\frac{1}{4},\frac{1}{2}\right)$ (D) $\left(\frac{3}{4},1\right)$

Official Ans. by NTA (C)
Allen Ans. (C)

6. If the maximum value of a, for which the function $f_a(x)=\tan^{-1}2x-3ax+7$ is non-decreasing in $\left(-\frac{\pi}{6},\frac{\pi}{6}\right)$, is $\bar{a}$, then $f_{\bar{a}}\left(\frac{\pi}{8}\right)$ is equal to

(A) $8-\frac{9\pi}{4\left(9+\pi^2\right)}$ (B) $8-\frac{4\pi}{9\left(4+\pi^2\right)}$
(C) $8\left(\frac{1+\pi^2}{9+\pi^2}\right)$ (D) $8-\frac{\pi}{4}$

Official Ans. by NTA (Drop)
Allen Ans. (Bonus)

7. Let $\beta=\lim\limits_{x\to 0}\frac{\alpha x-\left(e^{3x}-1\right)}{\alpha x\left(e^{3x}-1\right)}$ for some $\alpha\in\mathbb{R}$. Then the value of $\alpha+\beta$ is :

(A) $\frac{14}{5}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{7}{2}$

Official Ans. by NTA (C)
Allen Ans. (C)

8. The value of $\log_e 2\,\frac{d}{dx}(\log_{\cos x}\operatorname{cosec}x)$ at $x=\frac{\pi}{4}$ is

(A) $-2\sqrt{2}$ (B) $2\sqrt{2}$ (C) -4 (D) 4

Official Ans. by NTA (D)
Allen Ans. (D)

9. $\int\limits_0^{20\pi}\left(|\sin x|+|\cos x|\right)^2 dx$ is equal to :-

(A) $10(\pi+4)$ (B) $10(\pi+2)$
(C) $20(\pi-2)$ (D) $20(\pi+2)$

Official Ans. by NTA (D)
Allen Ans. (D)

10. Let the solution curve y = f(x) of the differential equation $\frac{dy}{dx}+\frac{xy}{x^2-1}=\frac{x^4+2x}{\sqrt{1-x^2}}, x\in(-1,1)$ pass through the origin. Then $\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)dx$ is equal to

(A) $\frac{\pi}{3}-\frac{1}{4}$ (B) $\frac{\pi}{3}-\frac{\sqrt{3}}{4}$

(C) $\frac{\pi}{6}-\frac{\sqrt{3}}{4}$ (D) $\frac{\pi}{6}-\frac{\sqrt{3}}{2}$

Official Ans. by NTA (B)

Allen Ans. (B)

11. The acute angle between the pair of tangents drawn to the ellipse $2x^2 + 3y^2 = 5$ from the point (1,3) is

(A) $\tan^{-1}\left(\frac{16}{7\sqrt{5}}\right)$ (B) $\tan^{-1}\left(\frac{24}{7\sqrt{5}}\right)$

(C) $\tan^{-1}\left(\frac{32}{7\sqrt{5}}\right)$ (D) $\tan^{-1}\left(\frac{3+8\sqrt{5}}{35}\right)$

Official Ans. by NTA (B)

Allen Ans. (B)

12. The equation of a common tangent to the parabolas $y = x^2$ and $y = -(x-2)^2$ is

(A) y = 4(x-2) (B) y = 4 (x-1)

(C) y = 4 (x+1) (D) y = 4 (x+2)

Official Ans. by NTA (B)

Allen Ans. (B)

13. Let the abscissae of the two points P and Q on a circle be the roots of $x^2 - 4x - 6 = 0$ and the ordinates of P and Q be the roots of $y^2 + 2y - 7 = 0$.If PQ is a diameter of the circle $x^2 + y^2 + 2ax + 2by + c= 0$, then the value of (a+b-c) is

(A) 12 (B) 13 (C) 14 (D) 16

Official Ans. by NTA (A)

Allen Ans. (A)

14. If the line x-1 = 0, is a directrix of the hyperbola $kx^2 – y^2=6$, then the hyperbola passes through the point

(A) $\left(-2\sqrt{5},6\right)$ (B) $\left(-\sqrt{5},3\right)$

(C) $\left(\sqrt{5},-2\right)$ (D) $\left(2\sqrt{5},3\sqrt{6}\right)$

Official Ans. by NTA (C)

Allen Ans. (C)

15. A vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i},\hat{i}+\hat{j}$ and the plane determined by the vectors $\hat{i}-\hat{j},\hat{i}+\hat{k}$. The obtuse angle between $\vec{a}$ and the vector $\vec{b}=\hat{i}-2\hat{j}+2\hat{k}$ is

(A) $\frac{3\pi}{4}$ (B) $\frac{2\pi}{3}$

(C) $\frac{4\pi}{5}$ (D) $\frac{5\pi}{6}$

Official Ans. by NTA (A)

Allen Ans. (A)

16. If $0<x<\frac{1}{\sqrt{2}}$ and $\frac{\sin^{-1}x}{\alpha}=\frac{\cos^{-1}x}{\beta}$, then a value of $\sin\left(\frac{2\pi\alpha}{\alpha+\beta}\right)$ is

(A) $4\sqrt{(1-x^2)}(1-2x^2)$

(B) $4x\sqrt{(1-x^2)}(1-2x^2)$

(C) $2x\sqrt{(1-x^2)}(1-4x^2)$

(D) $4\sqrt{(1-x^2)}(1-4x^2)$

Official Ans. by NTA (B)

Allen Ans. (B)

17. Negation of the Boolean expression $p \Leftrightarrow (q\Rightarrow p)$ is

(A) $(\sim p)\wedge q$ (B) $p\wedge(\sim q)$

(C) $(\sim p)\vee(\sim q)$ (D) $(\sim p)\wedge(\sim q)$

Official Ans. by NTA (D)

Allen Ans. (D)

18. Let X be a binomially distributed random variable with mean 4 and variance $\frac{4}{3}$. Then 54 P(X≤ 2) is equal to

(A) $\frac{73}{27}$ (B) $\frac{146}{27}$

(C) $\frac{146}{81}$ (D) $\frac{126}{81}$

Official Ans. by NTA (B)

Allen Ans. (B)

19. The integral $\int \frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x-\sin x)}{\left(1+\frac{2}{\sqrt{3}}\sin 2x\right)}dx$ is equal to

(A) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}{\left(\frac{x}{2}+\frac{\pi}{6}\right)}\right|+C$

(B) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\left(\frac{x}{2}+\frac{\pi}{3}\right)}\right|+C$

(C) $\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}\right|+C$

(D) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}-\frac{\pi}{6}\right)}\right|+C$

Official Ans. by NTA (A)

Allen Ans. (A)

20. The area bounded by the curves $y = |x^2-1|$ and $y = 1$ is

(A) $\frac{2}{3}\left(\sqrt{2}+1\right)$

(B) $\frac{4}{3}\left(\sqrt{2}-1\right)$

(C) $2\left(\sqrt{2}-1\right)$

(D) $\frac{8}{3}\left(\sqrt{2}-1\right)$

Official Ans. by NTA (D)

Allen Ans. (D)

SECTION-B

1. Let A = {1,2,3,4,5,6,7} and B = {3,6,7,9}. Then the number of elements in the set $\{C \subseteq A : C \cap B \neq \phi\}$ is_____

Official Ans. by NTA (112)

Allen Ans. (112)

2. The largest value of a, for which the perpendicular distance of the plane containing the lines $\vec{r}=\left(\hat{i}+\hat{j}\right)+\lambda\left(\hat{i}+a\hat{j}-\hat{k}\right)$ and $\vec{r}=\left(\hat{i}+\hat{j}\right)+\mu\left(-\hat{i}+\hat{j}-a\hat{k}\right)$ from the point (2,1,4) is $\sqrt{3}$, is_____.

Official Ans. by NTA (2)

Allen Ans. (2)

3. Numbers are to be formed between 1000 and 3000, which are divisible by 4, using the digits 1,2,3,4,5 and 6 without repetition of digits. Then the total number of such numbers is _____.

Official Ans. by NTA (30)

Allen Ans. (30)

4. If $\sum_{k=1}^{10}\frac{k}{k^4+k^2+1}=\frac{m}{n}$, where m and n are co-prime, then m + n is equal to

Official Ans. by NTA (166)

Allen Ans. (166)

5. If the sum of solutions of the system of equations $2\sin^2\theta-\cos 2\theta=0$ and $2\cos^2\theta+3\sin\theta=0$ in the interval $[0,2\pi]$ is $k\pi$, then k is equal to _____.

Official Ans. by NTA (3)

Allen Ans. (3)

6. The mean and standard deviation of 40 observations are 30 and 5 respectively. It was noticed that two of these observations 12 and 10 were wrongly recorded. If σ is the standard deviation of the data after omitting the two wrong observations from the data, then $38\sigma^2$ is equal to_____.

Official Ans. by NTA (238)

Allen Ans. (238)

7. The plane passing through the line L: ℓ x-y+3(1- ℓ) z = 1, x+2y - z = 2 and perpendicular to the plane 3x+2y+z = 6 is 3x-8y+7z=4. If θ is the acute angle between the line L and the y-axis, then 415 $\cos^2\theta$ is equal to_____.

Official Ans. by NTA (125)

Allen Ans. (125)

8. Suppose y = y(x) be the solution curve to the differential equation $\frac{dy}{dx} - y = 2 - e^{-x}$ such that $\lim_{x\to\infty} y(x)$ is finite. If a and b are respectively the x- and y- intercepts of the tangent to the curve at x=0, then the value of a-4b is equal to _____.

Official Ans. by NTA (3)

Allen Ans. (3)

9. Different A.P.'s are constructed with the first term 100, the last term 199, And integral common differences. The sum of the common differences of all such, A.P's having at least 3 terms and at most 33 terms is.

Official Ans. by NTA (53)

Allen Ans. (53)

10. The number of matrices $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where a,b,c,d $\in$ {-1,0,1,2,3,……,10}, such that A = A^{-1}, is_____.

Official Ans. by NTA (50)

Allen Ans. (50)