FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

(Held On Wednesday 31st January, 2024) **TIME : 9 : 00 AM to 12 : 00 NOON**

PHYSICS — TEST PAPER WITH SOLUTION

SECTION-A

31. The parameter that remains the same for molecules of all gases at a given temperature is :

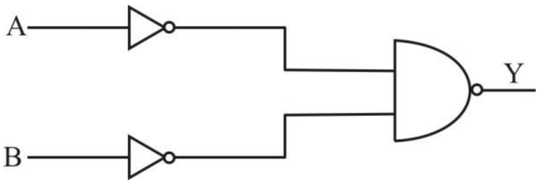
(1) kinetic energy (2) momentum
(3) mass (4) speed

Ans. (1)

Sol. $KE = \frac{f}{2}kT$

Conceptual

32. Identify the logic operation performed by the given circuit.



(1) NAND (2) NOR
(3) OR (4) AND

Ans. (3)

Sol. $Y = \overline{\overline{A}\cdot\overline{B}} = \overline{\overline{A}} + \overline{\overline{B}} = A + B$

(De-Morgan's law)

33. The relation between time 't' and distance 'x' is $t = \alpha x^2 + \beta x$, where α and β are constants. The relation between acceleration (a) and velocity (v) is:

(1) $a = -2\alpha v^3$ (2) $a = -5\alpha v^5$
(3) $a = -3\alpha v^2$ (4) $a = -4\alpha v^4$

Ans. (1)

Sol. $t = \alpha x^2 + \beta x$ (differentiating wrt time)

$$\frac{dt}{dx} = 2\alpha x + \beta$$

$$\frac{1}{v} = 2\alpha x + \beta$$

(differentiating wrt time)

$$-\frac{1}{v^2}\frac{dv}{dt} = 2\alpha\frac{dx}{dt}$$

$$\frac{dv}{dt} = -2\alpha v^3$$

34. The refractive index of a prism with apex angle A is cot A/2. The angle of minimum deviation is :

(1) $\delta_m = 180° - A$
(2) $\delta_m = 180° - 3A$
(3) $\delta_m = 180° - 4A$
(4) $\delta_m = 180° - 2A$

Ans. (4)

Sol. $$\mu = \frac{\sin\left(\frac{A+\delta m}{2}\right)}{\sin\frac{A}{2}}$$

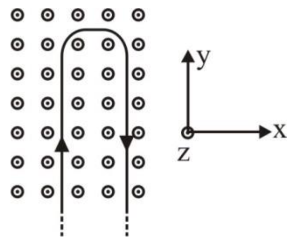
$$\frac{\cos\frac{A}{2}}{\sin\frac{A}{2}} = \frac{\sin\left(\frac{A+\delta m}{2}\right)}{\sin\frac{A}{2}}$$

$$\sin\left(\frac{\pi}{2} - \frac{A}{2}\right) = \sin\left(\frac{A+\delta_m}{2}\right)$$

$$\frac{\pi}{2} - \frac{A}{2} = \frac{A}{2} + \frac{\delta m}{2}$$

$$\delta_m = \pi - 2A$$

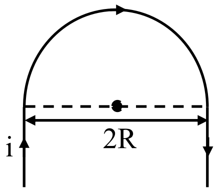
35. A rigid wire consists of a semicircular portion of radius R and two straight sections. The wire is partially immerged in a perpendicular magnetic field $B = B_0\hat{j}$ as shown in figure. The magnetic force on the wire if it has a current i is :



(1) $-iBR\hat{j}$ (2) $2iBR\hat{j}$
(3) $iBR\hat{j}$ (4) $-2iBR\hat{j}$

Ans. (4)

Sol.



Note : Direction of magnetic field is in $+\hat{k}$

$\vec{F} = i\,\vec{\ell} \times \vec{B}$

$\ell = 2R$

$\vec{F} = -2iRB\hat{j}$

36. If the wavelength of the first member of Lyman series of hydrogen is λ. The wavelength of the second member will be

(1) $\frac{27}{32}\lambda$ (2) $\frac{32}{27}\lambda$

(3) $\frac{27}{5}\lambda$ (4) $\frac{5}{27}\lambda$

Ans. (1)

Sol. $\frac{1}{\lambda} = \frac{13.6z^2}{hc}\left[\frac{1}{1^2} - \frac{1}{2^2}\right]$ ….. (i)

$\frac{1}{\lambda'} = \frac{13.6z^2}{hc}\left[\frac{1}{1^2} - \frac{1}{3^2}\right]$ ….. (ii)

On dividing (i) & (ii)

$\lambda' = \frac{27}{32}\lambda$

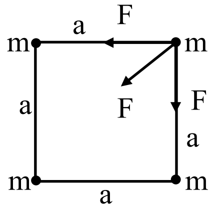
37. Four identical particles of mass m are kept at the four corners of a square. If the gravitational force exerted on one of the masses by the other masses is $\left(\frac{2\sqrt{2}+1}{32}\right)\frac{Gm^2}{L^2}$, the length of the sides of the square is

(1) $\frac{L}{2}$ (2) 4L

(3) 3L (4) 2L

Ans. (2)

Sol.



$F_{net} = \sqrt{2}F + F'$

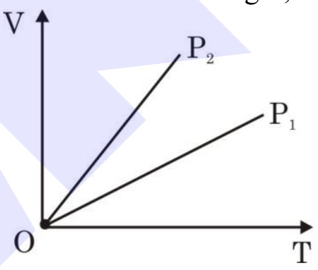
$F = \frac{Gm^2}{a^2}$ and $F' = \frac{Gm^2}{\left(\sqrt{2}a\right)^2}$

$F_{net} = \sqrt{2}\frac{Gm^2}{a^2} + \frac{Gm^2}{2a^2}$

$\left(\frac{2\sqrt{2}+1}{32}\right)\frac{Gm^2}{L^2} = \frac{Gm^2}{a^2}\left(\frac{2\sqrt{2}+1}{2}\right)$

$a = 4L$

38. The given figure represents two isobaric processes for the same mass of an ideal gas, then



(1) $P_2 \geq P_1$ (2) $P_2 > P_1$

(3) $P_1 = P_2$ (4) $P_1 > P_2$

Ans. (4)

Sol. $PV = nRT$

$V = \left(\frac{nR}{P}\right)T$

Slope $= \frac{nR}{P}$

Slope $\propto \frac{1}{P}$

$(\text{Slope})_2 > (\text{Slope})_1$

$P_2 < P_1$

39. If the percentage errors in measuring the length and the diameter of a wire are 0.1% each. The percentage error in measuring its resistance will be:

(1) 0.2% (2) 0.3%

(3) 0.1% (4) 0.144%

Ans. (2)

Sol. $R = \dfrac{\rho L}{\pi \dfrac{d^2}{4}}$

$$\frac{\Delta R}{R} = \frac{\Delta L}{L} + \frac{2\Delta d}{d}$$

$$\frac{\Delta L}{L} = 0.1\% \text{ and } \frac{\Delta d}{d} = 0.1\%$$

$$\frac{\Delta R}{R} = 0.3\%$$

40. In a plane EM wave, the electric field oscillates sinusoidally at a frequency of 5×10^{10} Hz and an amplitude of 50 Vm^{-1}. The total average energy density of the electromagnetic field of the wave is :

[Use $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2 / Nm^2$]

(1) $1.106 \times 10^{-8}\ Jm^{-3}$

(2) $4.425 \times 10^{-8}\ Jm^{-3}$

(3) $2.212 \times 10^{-8}\ Jm^{-3}$

(4) $2.212 \times 10^{-10}\ Jm^{-3}$

Ans. **(1)**

Sol. $U_E = \dfrac{1}{2} \epsilon_0 E^2$

$$U_E = \frac{1}{2} \times 8.85 \times 10^{-12} \times (50)^2$$

$= 1.106 \times 10^{-8}\ J/m^3$

41. A force is represented by $F = ax^2 + bt^{1/2}$

Where x = distance and t = time. The dimensions of b^2/a are :

(1) $[ML^3T^{-3}]$ (2) $[MLT^{-2}]$

(3) $[ML^{-1}T^{-1}]$ (4) $[ML^2T^{-3}]$

Ans. **(1)**

Sol. $F = ax^2 + bt^{1/2}$

$$[a] = \frac{[F]}{[x^2]} = [M^1L^{-1}T^{-2}]$$

$$[b] = \frac{[F]}{[t^{1/2}]} = [M^1L^1T^{-5/2}]$$

$$\left[\frac{b^2}{a}\right] = \frac{[M^2L^2T^{-5}]}{[M^1L^{-1}T^{-2}]} = [M^1L^3T^{-3}]$$

42. Two charges q and 3q are separated by a distance 'r' in air. At a distance x from charge q, the resultant electric field is zero. The value of x is :

(1) $\dfrac{(1+\sqrt{3})}{r}$

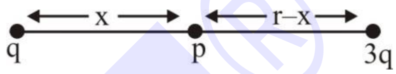
(2) $\dfrac{r}{3(1+\sqrt{3})}$

(3) $\dfrac{r}{(1+\sqrt{3})}$

(4) $r(1+\sqrt{3})$

Ans. **(3)**

Sol.



$$\left(\vec{E}_{net}\right)_P = 0$$

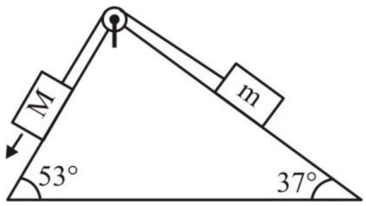
$$\frac{kq}{x^2} = \frac{k \cdot 3q}{(r-x)^2}$$

$$(r-x)^2 = 3x^2$$

$$r - x = \sqrt{3}x$$

$$x = \frac{r}{\sqrt{3}+1}$$

43. In the given arrangement of a doubly inclined plane two blocks of masses M and m are placed. The blocks are connected by a light string passing over an ideal pulley as shown. The coefficient of friction between the surface of the plane and the blocks is 0.25. The value of m, for which M = 10 kg will move down with an acceleration of 2 m/s^2, is : (take g = 10 m/s^2 and tan 37° = 3/4)



(1) 9 kg

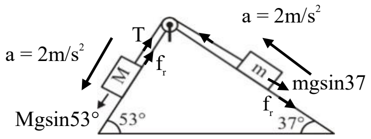
(2) 4.5 kg

(3) 6.5 kg

(4) 2.25 kg

Ans. **(2)**

Sol.



For M block

$10g\sin 53° - \mu\,(10g)\cos 53° - T = 10 \times 2$

$T = 80 - 15 - 20$

$T = 45$ N

For m block

$T - mg\sin 37° - \mu mg\cos 37° = m \times 2$

$45 = 10\,m$

$m = 4.5$ kg

44. A coil is placed perpendicular to a magnetic field of 5000 T. When the field is changed to 3000 T in 2s, an induced emf of 22 V is produced in the coil. If the diameter of the coil is 0.02 m, then the number of turns in the coil is :

(1) 7 (2) 70

(3) 35 (4) 140

Ans. (2)

Sol. $\varepsilon = N\left(\frac{\Delta\phi}{t}\right)$

$\Delta\phi = (\Delta B)A$

$B_i = 5000$ T,

$B_f = 3000$ T

$d = 0.02$ m

$r = 0.01$ m

$\Delta\phi = (\Delta B)A$

$= (2000)\pi(0.01)^2 = 0.2\pi$

$\varepsilon = N\left(\frac{\Delta\phi}{t}\right) \Rightarrow 22 = N\left(\frac{0.2\pi}{2}\right)$

$N = 70$

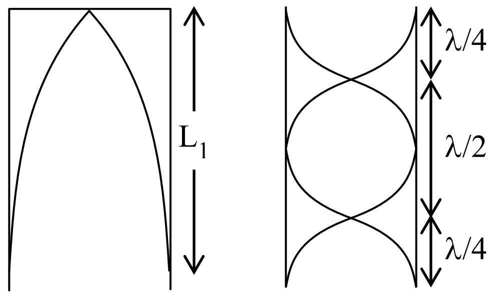
45. The fundamental frequency of a closed organ pipe is equal to the first overtone frequency of an open organ pipe. If length of the open pipe is 60 cm, the length of the closed pipe will be :

(1) 60 cm (2) 45 cm

(3) 30 cm (4) 15 cm

Ans. (4)

Sol.



$\frac{\lambda}{4} = L_1$ $\qquad$ $2\left(\frac{\lambda}{2}\right) = \lambda$

$v = f\lambda$ $\qquad$ $f_2 = \frac{2v}{2L_2}$

$v = f_1(4L_1)$ $\qquad$ $f_2 = \frac{v}{L_2}$

$f_1 = \frac{v}{4L_1}$

$f_1 = f_2$

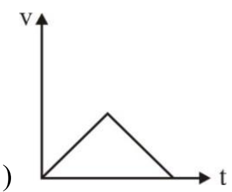
$\frac{v}{4L_1} = \frac{v}{L_2}$

$\Rightarrow L_2 = 4L_1$

$60 = 4 \times L_1$

$L_1 = 15$ cm

46. A small steel ball is dropped into a long cylinder containing glycerine. Which one of the following is the correct representation of the velocity time graph for the transit of the ball?

(1)


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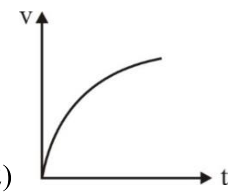

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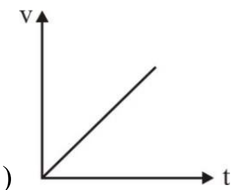

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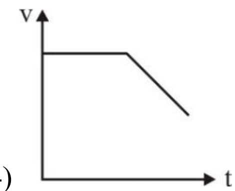
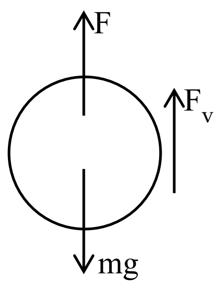


Ans. (2)

Sol.



$$mg - F_B - F_v = ma$$

$$\left(\rho\frac{4}{3}\pi r^3\right)g - \left(\rho_L\frac{4}{3}\pi r^3\right)g - 6\pi\eta rv = m\frac{dv}{dt}$$

$$\text{Let } \frac{4}{3m}\pi R^3 g(\rho-\rho_L) = K_1 \text{ and } \frac{6\pi\eta r}{m} = K_2$$

$$\frac{dv}{dt} = K_1 - K_2 v$$

$$\int_0^v \frac{dv}{K_1 - K_2 v} = \int_0^t dt$$

$$-\frac{1}{K_2}\ell n\left[K_1 - K_2 v\right]_0^v = t$$

$$\ell n\left(\frac{K_1 - K_2 v}{K_1}\right) = -K_2 t$$

$$K_1 - K_2 v = K_1 e^{-K_2 t}$$

$$v = \frac{K_1}{K_2}\left[1 - e^{-K_2 t}\right]$$

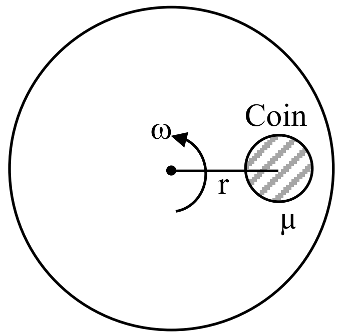
47. A coin is placed on a disc. The coefficient of friction between the coin and the disc is μ. If the distance of the coin from the center of the disc is r, the maximum angular velocity which can be given to the disc, so that the coin does not slip away, is :

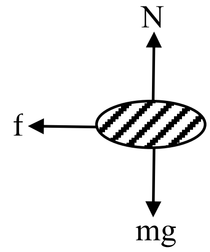
(1) $\frac{\mu g}{r}$ (2) $\sqrt{\frac{r}{\mu g}}$

(3) $\sqrt{\frac{\mu g}{r}}$ (4) $\frac{\mu}{\sqrt{rg}}$

Ans. **(3)**

Sol.





$$N = mg$$
$$f = m\omega^2 r$$
$$f = \mu N$$
$$\mu mg = mr\omega^2$$
$$\omega = \sqrt{\frac{\mu g}{r}}$$

48. Two conductors have the same resistances at 0°C but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coefficients for their series and parallel combinations are :

(1) $\alpha_1 + \alpha_2, \frac{\alpha_1 + \alpha_2}{2}$

(2) $\frac{\alpha_1 + \alpha_2}{2}, \frac{\alpha_1 + \alpha_2}{2}$

(3) $\alpha_1 + \alpha_2, \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2}$

(4) $\frac{\alpha_1 + \alpha_2}{2}, \alpha_1 + \alpha_2$

Ans. **(2)**

Sol. **Series :**

$$R_{eq} = R_1 + R_2$$

$$2R(1+\alpha_{eq}\Delta\theta) = R(1+\alpha_1\Delta\theta) + R(1+\alpha_2\Delta\theta)$$

$$2R(1+\alpha_{eq}\Delta\theta) = 2R + (\alpha_1 + \alpha_2)R\Delta\theta$$

$$\alpha_{eq} = \frac{\alpha_1 + \alpha_2}{2}$$

Parallel :

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{\frac{R}{2}(1+\alpha_{eq}\Delta\theta)} = \frac{1}{R(1+\alpha_1\Delta\theta)} + \frac{1}{R(1+\alpha_2\Delta\theta)}$$

$$\frac{2}{1+\alpha_{eq}\Delta\theta}=\frac{1}{1+\alpha_1\Delta\theta}+\frac{1}{1+\alpha_2\Delta\theta}$$

$$\frac{2}{1+\alpha_{eq}\Delta\theta}=\frac{1+\alpha_2\Delta\theta+1+\alpha_1\Delta\theta}{(1+\alpha_1\Delta\theta)(1+\alpha_2\Delta\theta)}$$

$$2\left[(1+\alpha_1\Delta\theta)(1+\alpha_2\Delta\theta)\right]$$

$$=\left[2+(\alpha_1+\alpha_2)\Delta\theta\right]\left[1+\alpha_{eq}\Delta\theta\right]$$

$$2[1+\alpha_1\Delta\theta+\alpha_2\Delta\theta+\alpha_1\alpha_2\Delta\theta]$$

$=$

$$2+2\alpha_{eq}\Delta\theta+(\alpha_1+\alpha_2)\Delta\theta+\alpha_{eq}(\alpha_1+\alpha_2)\Delta\theta^2$$

Neglecting small terms

$$2+2(\alpha_1+\alpha_2)\Delta\theta=2+2\alpha_{eq}\Delta\theta+(\alpha_1+\alpha_2)\Delta\theta$$

$$(\alpha_1+\alpha_2)\Delta\theta=2\alpha_{eq}\Delta\theta$$

$$\alpha_{eq}=\frac{\alpha_1+\alpha_2}{2}$$

49. An artillery piece of mass M_1 fires a shell of mass M_2 horizontally. Instantaneously after the firing, the ratio of kinetic energy of the artillery and that of the shell is :

(1) $M_1/(M_1+M_2)$ (2) $\frac{M_2}{M_1}$

(3) $M_2/(M_1+M_2)$ (4) $\frac{M_1}{M_2}$

Ans. (2)

Sol. $|\vec{p}_1|=|\vec{p}_2|$

$$KE=\frac{p^2}{2M}\ ;\ p\text{ same}$$

$$KE\propto\frac{1}{m}$$

$$\frac{KE_1}{KE_2}=\frac{p^2/2M_1}{p^2/2M_2}=\frac{M_2}{M_1}$$

50. When a metal surface is illuminated by light of wavelength λ, the stopping potential is 8V. When the same surface is illuminated by light of wavelength 3λ, stopping potential is 2V. The threshold wavelength for this surface is :

(1) 5λ

(2) 3λ

(3) 9λ

(4) 4.5λ

Ans. (3)

Sol. $E=\phi+K_{max}$

$$\phi=\frac{hc}{\lambda_0}$$

$$K_{max}=eV_0$$

$$8e=\frac{hc}{\lambda}-\frac{hc}{\lambda_0}\ ......(i)$$

$$2e=\frac{hc}{3\lambda}-\frac{hc}{\lambda_0}\ ....(ii)$$

on solving (i) & (ii)

$$\lambda_0=9\lambda$$

SECTION-B

51. An electron moves through a uniform magnetic field $\vec{B}=B_0\hat{i}+2B_0\hat{j}$ T. At a particular instant of time, the velocity of electron is $\vec{u}=3\hat{i}+5\hat{j}$ m/s. If the magnetic force acting on electron is $\vec{F}=5e\hat{k}$ N, where e is the charge of electron, then the value of B_0 is ____ T.

Ans. (5)

Sol. $\vec{F}=q(\vec{v}\times\vec{B})$

$$5e\hat{k}=e(3\hat{i}+5\hat{j})\times(B_0\hat{i}+2B_0\hat{j})$$

$$5e\hat{k}=e(6B_0\hat{k}-5B_0\hat{k})$$

$$\Rightarrow B_0=5T$$

52. A parallel plate capacitor with plate separation 5 mm is charged up by a battery. It is found that on introducing a dielectric sheet of thickness 2 mm, while keeping the battery connections intact, the capacitor draws 25% more charge from the battery than before. The dielectric constant of the sheet is ____.

Ans. **(2)**

Sol. Without dielectric

$$Q = \frac{A \in_0}{d} V$$

with dielectric

$$Q = \frac{A \in_0 V}{d - t + \frac{t}{K}}$$

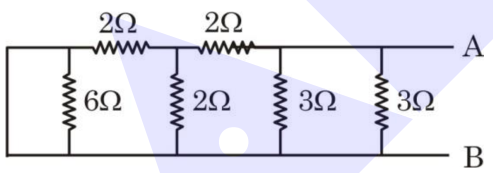
given

$$\frac{A \in_0 V}{d - t + \frac{t}{K}} = (1.25)\frac{A \in_0 V}{d}$$

$$\Rightarrow 1.25\left(3 + \frac{2}{K}\right) = 5$$

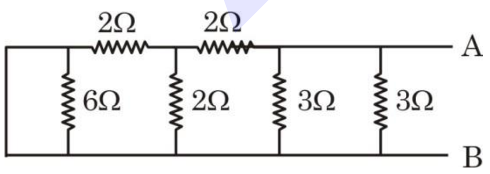
$$\Rightarrow K = 2$$

53. Equivalent resistance of the following network is _____ Ω.

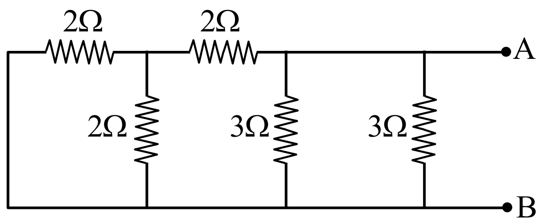


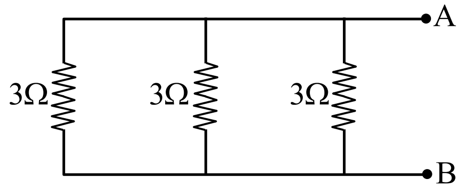
Ans. **(1)**

Sol.



6Ω is short circuit





$$R_{eq} = 3 \times \frac{1}{3} = 1\Omega$$

54. A solid circular disc of mass 50 kg rolls along a horizontal floor so that its center of mass has a speed of 0.4 m/s. The absolute value of work done on the disc to stop it is _______ J.

Ans. **(6)**

Sol. Using work energy theorem

$$W = \Delta KE = 0 - \left(\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2\right)$$

$$W = 0 - \frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right)$$

$$= -\frac{1}{2} \times 50 \times 0.4^2\left(1 + \frac{1}{2}\right) = -6J$$

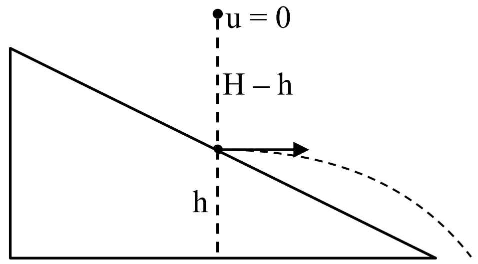
Absolute work = +6J

$W = -6J \quad |W| = 6J$

55. A body starts falling freely from height H hits an inclined plane in its path at height h. As a result of this perfectly elastic impact, the direction of the velocity of the body becomes horizontal. The value of $\frac{H}{h}$ for which the body will take the maximum time to reach the ground is _____.

Ans. **(2)**

Sol.



Total time of flight = T

$$T = \sqrt{\frac{2h}{g}} + \sqrt{\frac{2(H-h)}{g}}$$

For max. time = $\frac{dT}{dh} = 0$

$$\sqrt{\frac{2}{g}}\left(\frac{-1}{2\sqrt{H-h}} + \frac{1}{2\sqrt{h}}\right) = 0$$

$$\sqrt{H-h} = \sqrt{h}$$

$$h = \frac{H}{2} \Rightarrow \frac{H}{h} = 2$$

56. Two waves of intensity ratio 1 : 9 cross each other at a point. The resultant intensities at the point, when (a) Waves are incoherent is I_1(b) Waves are coherent is I_2 and differ in phase by 60°. If $\frac{I_1}{I_2} = \frac{10}{x}$ then x = _____.

Ans. **(13)**

Sol. For incoherent wave $I_1 = I_A + I_B \Rightarrow I_1 = I_0 + 9I_0$

$I_1 = 10I_0$

For coherent wave $I_2 = I_A + I_B + 2\sqrt{I_A I_B}\cos 60°$

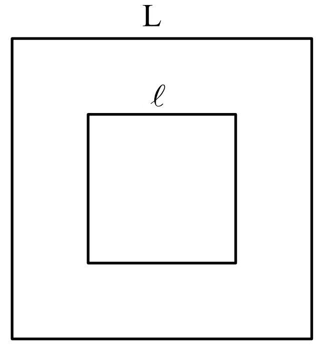
$$I_2 = I_0 + 9I_0 + 2\sqrt{9I_0^2}\cdot\frac{1}{2} = 13\,I_0$$

$$\frac{I_1}{I_2} = \frac{10}{13}$$

57. A small square loop of wire of side ℓ is placed inside a large square loop of wire of side L ($L = \ell^2$). The loops are coplanar and their centers coinside. The value of the mutual inductance of the system is $\sqrt{x} \times 10^{-7}$ H, where x = _____.

Ans. **(128)**

Sol.



Flux linkage for inner loop.

$$\phi = B_{center}\,.\,\ell^2$$

$$= 4 \times \frac{\mu_0 i}{4\pi\frac{L}{2}}(\sin 45 + \sin 45)\ell^2$$

$$\phi = 2\sqrt{2}\frac{\mu_0 i}{\pi L}\ell^2$$

$$M = \frac{\phi}{i} = \frac{2\sqrt{2}\mu_0\ell^2}{\pi L} = 2\sqrt{2}\frac{\mu_0}{\pi}$$

$$= 2\sqrt{2}\frac{4\pi}{\pi}\times 10^{-7}$$

$$= 8\sqrt{2}\times 10^{-7}\,H$$

$$= \sqrt{128}\times 10^{-7}\,H$$

x = 128

58. The depth below the surface of sea to which a rubber ball be taken so as to decrease its volume by 0.02% is _____ m.

(Take density of sea water = 10^3 kgm^{-3}, Bulk modulus of rubber = 9×10^8 Nm^{-2}, and g = 10 ms^{-2})

Ans. **(18)**

Sol. $\beta = \frac{-\Delta P}{\frac{\Delta V}{V}}$

$$\Delta P = -\beta\frac{\Delta V}{V}$$

$$\rho g h = -\beta\frac{\Delta V}{V}$$

$$10^3 \times 10 \times h = -9\times 10^8 \times \left(-\frac{0.02}{100}\right)$$

$\Rightarrow$ h = 18 m

59. A particle performs simple harmonic motion with amplitude A. Its speed is increased to three times at an instant when its displacement is $\frac{2A}{3}$. The new amplitude of motion is $\frac{nA}{3}$. The value of n is ___.

Ans. **(7)**

Sol. $v = \omega\sqrt{A^2 - x^2}$

at $x = \frac{2A}{3}$

$$v = \omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2} = \frac{\sqrt{5}A\omega}{3}$$

New amplitude = A'

$$v' = 3v = \sqrt{5}A\omega = \omega\sqrt{(A')^2 - \left(\frac{2A}{3}\right)^2}$$

$$A' = \frac{7A}{3}$$

60. The mass defect in a particular reaction is 0.4g. The amount of energy liberated is $n \times 10^7$ kWh, where n = _____.

(speed of light = 3×10^8 m/s)

Ans. **(1)**

Sol. $E = \Delta mc^2$

$= 0.4 \times 10^{-3} \times (3 \times 10^8)^2$

$= 3600 \times 10^7$ kWs

$$= \frac{3600 \times 10^7}{3600}\,\text{kWh} = 1 \times 10^7\,\text{kWh}$$