FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

(Held On Wednesday 31st January, 2024) TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS — TEST PAPER WITH ANSWER

SECTION-A

1. For $0 < c < b < a$, let $(a + b - 2c)x^2 + (b + c - 2a)x + (c + a - 2b) = 0$ and $\alpha \neq 1$ be one of its root. Then, among the two statements

(I) If $\alpha \in (-1, 0)$, then b cannot be the geometric mean of a and c

(II) If $\alpha \in (0, 1)$, then b may be the geometric mean of a and c

(1) Both (I) and (II) are true

(2) Neither (I) nor (II) is true

(3) Only (II) is true

(4) Only (I) is true

Ans. (1)

2. Let a be the sum of all coefficients in the expansion of $(1 - 2x + 2x^2)^{2023}(3 - 4x^2 + 2x^3)^{2024}$ and $b = \lim\limits_{x \to 0}\left(\dfrac{\int_0^x \frac{\log(1+t)}{t^{2024}+1}dt}{x^2}\right)$. If the equations $cx^2 + dx + e = 0$ and $2bx^2 + ax + 4 = 0$ have a common root, where $c, d, e \in \mathbb{R}$, then $d : c : e$ equals

(1) 2 : 1 : 4 (2) 4 : 1 : 4
(3) 1 : 2 : 4 (4) 1 : 1 : 4

Ans. (4)

3. If the foci of a hyperbola are same as that of the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{25} = 1$ and the eccentricity of the hyperbola is $\dfrac{15}{8}$ times the eccentricity of the ellipse, then the smaller focal distance of the point $\left(\sqrt{2}, \dfrac{14}{3}\sqrt{\dfrac{2}{5}}\right)$ on the hyperbola, is equal to

(1) $7\sqrt{\dfrac{2}{5}} - \dfrac{8}{3}$ (2) $14\sqrt{\dfrac{2}{5}} - \dfrac{4}{3}$

(3) $14\sqrt{\dfrac{2}{5}} - \dfrac{16}{3}$ (4) $7\sqrt{\dfrac{2}{5}} + \dfrac{8}{3}$

Ans. (1)

4. If one of the diameters of the circle $x^2 + y^2 - 10x + 4y + 13 = 0$ is a chord of another circle C, whose center is the point of intersection of the lines $2x + 3y = 12$ and $3x - 2y = 5$, then the radius of the circle C is

(1) $\sqrt{20}$ (2) 4

(3) 6 (4) $3\sqrt{2}$

Ans. (3)

5. The area of the region

$$\left\{(x, y) : y^2 \le 4x, x < 4, \frac{xy(x-1)(x-2)}{(x-3)(x-4)} > 0, x \neq 3\right\}$$

is

(1) $\dfrac{16}{3}$

(2) $\dfrac{64}{3}$

(3) $\dfrac{8}{3}$

(4) $\dfrac{32}{3}$

Ans. (4)

6. If $f(x) = \dfrac{4x+3}{6x-4}, x \neq \dfrac{2}{3}$ and (fof) (x) = g(x), where $g : \mathbb{R} - \left\{\dfrac{2}{3}\right\} \to \mathbb{R} - \left\{\dfrac{2}{3}\right\}$, then (gogog) (4) is equal to

(1) $-\dfrac{19}{20}$ (2) $\dfrac{19}{20}$

(3) – 4 (4) 4

Ans. (4)

7. $\lim\limits_{x \to 0} \dfrac{e^{2|\sin x|} - 2|\sin x| - 1}{x^2}$

(1) is equal to – 1 (2) does not exist

(3) is equal to 1 (4) is equal to 2

Ans. (4)

8. If the system of linear equations

$x-2y+z=-4$

$2x+\alpha y+3z=5$

$3x-y+\beta z=3$

has infinitely many solutions, then $12\alpha + 13\beta$ is equal to

(1) 60 (2) 64
(3) 54 (4) 58

Ans. (4)

9. The solution curve of the differential equation $y\frac{dx}{dy}=x\left(\log_e x-\log_e y+1\right)$, $x > 0$, $y > 0$ passing through the point (e, 1) is

(1) $\left|\log_e \frac{y}{x}\right|=x$ (2) $\left|\log_e \frac{y}{x}\right|=y^2$

(3) $\left|\log_e \frac{x}{y}\right|=y$ (4) $2\left|\log_e \frac{x}{y}\right|=y+1$

Ans. (3)

10. Let $\alpha, \beta, \gamma, \delta \in Z$ and let A (α, β), B (1, 0), C (γ,δ) and D (1, 2) be the vertices of a parallelogram ABCD. If AB = $\sqrt{10}$ and the points A and C lie on the line $3y = 2x + 1$, then $2(\alpha + \beta + \gamma + \delta)$ is equal to

(1) 10 (2) 5
(3) 12 (4) 8

Ans. (4)

11. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx}=\frac{(\tan x)+y}{\sin x(\sec x-\sin x\tan x)}$, $x\in\left(0,\frac{\pi}{2}\right)$ satisfying the condition $y\left(\frac{\pi}{4}\right)=2$. Then, $y\left(\frac{\pi}{3}\right)$ is

(1) $\sqrt{3}\left(2+\log_e\sqrt{3}\right)$

(2) $\frac{\sqrt{3}}{2}(2+\log_e 3)$

(3) $\sqrt{3}(1+2\log_e 3)$

(4) $\sqrt{3}(2+\log_e 3)$

Ans. (1)

12. Let $\vec{a}=3\hat{i}+\hat{j}-2\hat{k}$, $\vec{b}=4\hat{i}+\hat{j}+7\hat{k}$ and $\vec{c}=\hat{i}-3\hat{j}+4\hat{k}$ be three vectors. If a vectors $\vec{p}$ satisfies $\vec{p}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{p}\cdot\vec{a}=0$, then $\vec{p}\cdot\left(\hat{i}-\hat{j}-\hat{k}\right)$ is equal to

(1) 24 (2) 36
(3) 28 (4) 32

Ans. (4)

13. The sum of the series $\frac{1}{1-3\cdot1^2+1^4}+\frac{2}{1-3\cdot2^2+2^4}+\frac{3}{1-3\cdot3^2+3^4}+\ldots$.. up to 10 terms is

(1) $\frac{45}{109}$ (2) $-\frac{45}{109}$

(3) $\frac{55}{109}$ (4) $-\frac{55}{109}$

Ans. (4)

14. The distance of the point Q(0, 2, –2) form the line passing through the point P(5, –4, 3) and perpendicular to the lines $\vec{r}=\left(-3\hat{i}+2\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}+5\hat{k}\right)$, $\lambda\in\mathbb{R}$ and $\vec{r}=\left(\hat{i}-2\hat{j}+\hat{k}\right)+\mu\left(-\hat{i}+3\hat{j}+2\hat{k}\right)$, $\mu\in\mathbb{R}$ is

(1) $\sqrt{86}$

(2) $\sqrt{20}$

(3) $\sqrt{54}$

(4) $\sqrt{74}$

Ans. (4)

15. For $\alpha,\beta,\gamma\neq0$. If $\sin^{-1}\alpha + \sin^{-1}\beta + \sin^{-1}\gamma = \pi$ and $(\alpha + \beta + \gamma)(\alpha - \gamma + \beta) = 3\alpha\beta$, then γ equal to

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{1}{\sqrt{2}}$

(3) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (4) $\sqrt{3}$

Ans. (1)

16. Two marbles are drawn in succession from a box containing 10 red, 30 white, 20 blue and 15 orange marbles, with replacement being made after each drawing. Then the probability, that first drawn marble is red and second drawn marble is white, is

(1) $\frac{2}{25}$ (2) $\frac{4}{25}$

(3) $\frac{2}{3}$ (4) $\frac{4}{75}$

Ans. (4)

17. Let g(x) be a linear function and

$$f(x)=\begin{cases} g(x) & ,x\le 0 \\ \left(\frac{1+x}{2+x}\right)^{\frac{1}{x}} & ,x>0 \end{cases}$$, is continuous at x = 0.

If $f'(1)=f(-1)$, then the value of g(3) is

(1) $\frac{1}{3}\log_e\left(\frac{4}{9e^{1/3}}\right)$ (2) $\frac{1}{3}\log_e\left(\frac{4}{9}\right)+1$

(3) $\log_e\left(\frac{4}{9}\right)-1$ (4) $\log_e\left(\frac{4}{9e^{1/3}}\right)$

Ans. (4)

18. If $f(x)=\begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{vmatrix}$ for all $x\in\mathbb{R}$, then 2f(0) + $f'(0)$ is equal to

(1) 48 (2) 24

(3) 42 (4) 18

Ans. (3)

19. Three rotten apples are accidently mixed with fifteen good apples. Assuming the random variable x to be the number of rotten apples in a draw of two apples, the variance of x is

(1) $\frac{37}{153}$ (2) $\frac{57}{153}$

(3) $\frac{47}{153}$ (4) $\frac{40}{153}$

Ans. (4)

20. Let S be the set of positive integral values of a for which $\frac{ax^2+2(a+1)x+9a+4}{x^2-8x+32}<0,\forall x\in\mathbb{R}$.

Then, the number of elements in S is :

(1) 1

(2) 0

(3) ∞

(4) 3

Ans. (2)

SECTION-B

21. If the integral

$$525\int_0^{\frac{\pi}{2}}\sin 2x\cos^{\frac{11}{2}}x\left(1+\cos^{\frac{5}{2}}x\right)^{\frac{1}{2}}dx$$ is equal to

$\left(n\sqrt{2}-64\right)$, then n is equal to ______

Ans. (176)

22. Let $S=(-1,\infty)$ and $f:S\to\mathbb{R}$ be defined as

$$f(x)=\int_{-1}^{x}\left(e^t-1\right)^{11}(2t-1)^5(t-2)^7(t-3)^{12}(2t-10)^{61}dt$$

Let p = Sum of square of the values of x, where f(x) attains local maxima on S. and q = Sum of the values of x, where f(x) attains local minima on S. Then, the value of p^2+2q is ________

Ans. (27)

23. The total number of words (with or without meaning) that can be formed out of the letters of the word 'DISTRIBUTION' taken four at a time, is equal to _____

Ans. (3734)

24. Let Q and R be the feet of perpendiculars from the point P(a, a, a) on the lines x = y, z = 1 and x = -y, z = –1 respectively. If $\angle QPR$ is a right angle, then $12a^2$ is equal to _____

Ans. (12)

25. In the expansion of

$$(1+x)(1-x^2)\left(1+\frac{3}{x}+\frac{3}{x^2}+\frac{1}{x^3}\right)^5,\ x \neq 0,$$ the

sum of the coefficient of x^3 and x^{-13} is equal to ___

Ans. (118)

26. If α denotes the number of solutions of $|1 - i|^x = 2^x$ and $\beta = \left(\frac{|z|}{\arg(z)}\right)$, where

$$z = \frac{\pi}{4}(1+i)^4\left(\frac{1-\sqrt{\pi}i}{\sqrt{\pi}+i}+\frac{\sqrt{\pi}-i}{1+\sqrt{\pi}i}\right),\ i=\sqrt{-1},$$ then

the distance of the point (α, β) from the line $4x - 3y = 7$ is _____

Ans. (3)

27. Let the foci and length of the latus rectum of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a > b$ be $(\pm 5, 0)$ and $\sqrt{50}$, respectively. Then, the square of the eccentricity of the hyperbola $\frac{x^2}{b^2}-\frac{y^2}{a^2b^2}=1$ equals

Ans. (51)

28. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}|=1, |\vec{b}|=4$ and $\vec{a}\cdot\vec{b}=2$. If $\vec{c}=\left(2\vec{a}\times\vec{b}\right)-3\vec{b}$ and the angle between $\vec{b}$ and $\vec{c}$ is α, then $192\sin^2\alpha$ is equal to______

Ans. (48)

29. Let A = {1, 2, 3, 4} and R = {(1, 2), (2, 3), (1, 4)} be a relation on A. Let S be the equivalence relation on A such that $R \subset S$ and the number of elements in S is n. Then, the minimum value of n is _______

Ans. (16)

30. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by

$$f(x) = \frac{4^x}{4^x+2}$$ and

$$M = \int_{f(a)}^{f(1-a)} x\sin^4\left(x(1-x)\right)dx,$$

$$N = \int_{f(a)}^{f(1-a)} \sin^4\left(x(1-x)\right)dx;\ a \neq \frac{1}{2}.$$ If

$\alpha M = \beta N, \alpha, \beta \in \mathbb{N}$, then the least value of $\alpha^2+\beta^2$ is equal to ______

Ans. (5)

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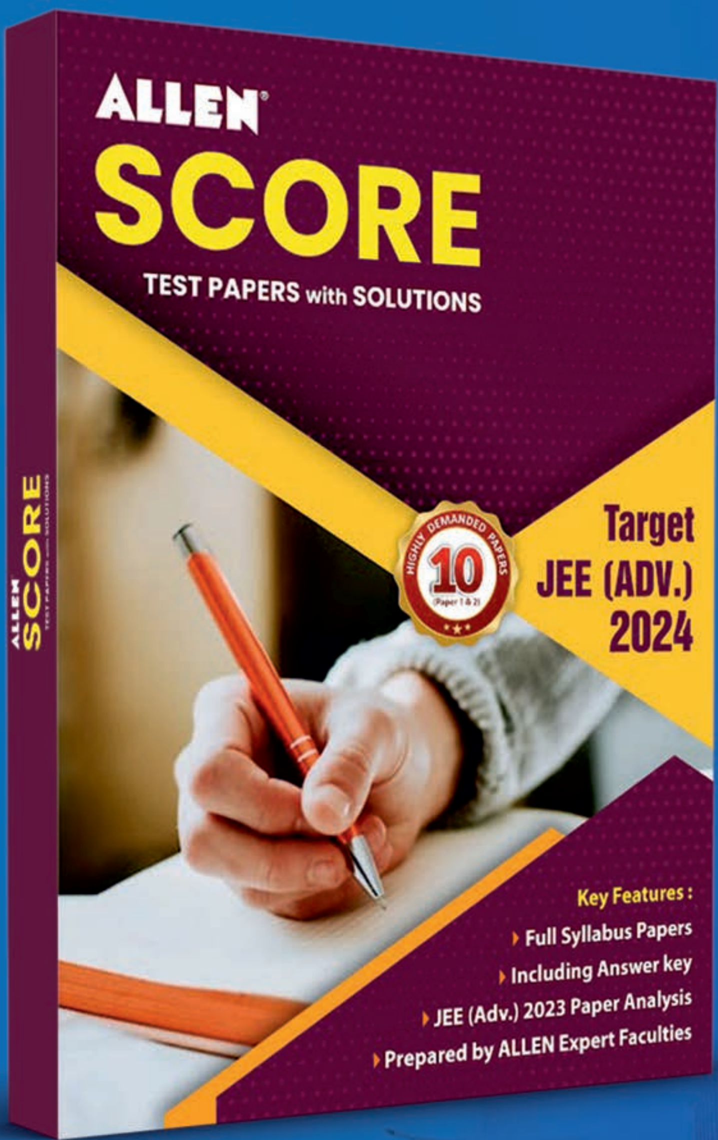

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