FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

(Held On Wednesday 31st January, 2024) TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS | **TEST PAPER WITH ANSWER**

SECTION-A

1. The number of ways in which 21 identical apples can be distributed among three children such that each child gets at least 2 apples, is

(1) 406 (2) 130 (3) 142 (4) 136

Ans. (4)

2. Let A (a, b), B(3, 4) and (-6, -8) respectively denote the centroid, circumcentre and orthocentre of a triangle. Then, the distance of the point P(2a + 3, 7b + 5) from the line 2x + 3y – 4 = 0 measured parallel to the line x – 2y – 1 = 0 is

(1) $\frac{15\sqrt{5}}{7}$ (2) $\frac{17\sqrt{5}}{6}$

(3) $\frac{17\sqrt{5}}{7}$ (4) $\frac{\sqrt{5}}{17}$

Ans. (3)

3. Let z_1 and z_2 be two complex number such that $z_1 + z_2 = 5$ and $z_1^3 + z_2^3 = 20 + 15i$. Then $\left|z_1^4 + z_2^4\right|$ equals-

(1) $30\sqrt{3}$ (2) 75

(3) $15\sqrt{15}$ (4) $25\sqrt{3}$

Ans. (2)

4. Let a variable line passing through the centre of the circle $x^2 + y^2 - 16x - 4y = 0$, meet the positive co-ordinate axes at the point A and B. Then the minimum value of OA + OB, where O is the origin, is equal to

(1) 12 (2) 18

(3) 20 (4) 24

Ans. (2)

5. Let $f, g : (0, \infty) \to R$ be two functions defined by $f(x) = \int_{-x}^{x}\left(|t| - t^2\right)dt$ and $g(x) = \int_0^{x^2} t^{1/2}e^{-t}dt$.

Then the value of $\left(f\left(\sqrt{\log_e 9}\right) + g\left(\sqrt{\log_e 9}\right)\right)$ is equal to

(1) 6 (2) 9

(3) 8 (4) 10

Ans. (3)

6. Let (α, β, γ) be mirror image of the point (2, 3, 5) in the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Then $2\alpha + 3\beta + 4\gamma$ is equal to

(1) 32 (2) 33

(3) 31 (4) 34

Ans. (2)

7. Let P be a parabola with vertex (2, 3) and directrix 2x + y = 6. Let an ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$ of eccentricity $\frac{1}{\sqrt{2}}$ pass through the focus of the parabola P. Then the square of the length of the latus rectum of E, is

(1) $\frac{385}{8}$ (2) $\frac{347}{8}$

(3) $\frac{512}{25}$ (4) $\frac{656}{25}$

Ans. (4)

8. The temperature T(t) of a body at time t = 0 is 160° F and it decreases continuously as per the differential equation $\frac{dT}{dt} = -K(T - 80)$, where K is positive constant. If T(15) = 120°F, then T(45) is equal to

(1) 85° F (2) 95° F

(3) 90° F (4) 80° F

Ans. (3)

9. Let 2^{nd}, 8^{th}, 44^{th}, terms of a non-constant A.P. be respectively the 1^{st}, 2^{nd} and 3^{rd} terms of G.P. If the first term of A.P. is 1 then the sum of first 20 terms terms is equal to-

(1) 980 (2) 960

(3) 990 (4) 970

Ans. (4)

10. Let $f :\to R \to (0,\infty)$ be strictly increasing function such that $\lim\limits_{x\to\infty} \frac{f(7x)}{f(x)} = 1$. Then, the value of $\lim\limits_{x\to\infty}\left[\frac{f(5x)}{f(x)} - 1\right]$ is equal to

(1) 4 (2) 0 (3) 7/5 (4) 1

Ans. (2)

11. The area of the region enclosed by the parabola $y = 4x - x^2$ and $3y = (x - 4)^2$ is equal to

(1) $\frac{32}{9}$ (2) 4 (3) 6 (4) $\frac{14}{3}$

Ans. (3)

12. Let the mean and the variance of 6 observation a, b, 68, 44, 48, 60 be 55 and 194, respectively if a > b, then a + 3b is

(1) 200 (2) 190 (3) 180 (4) 210

Ans. (3)

13. If the function $f:(-\infty,-1]\to(a,b]$ defined by $f(x) = e^{x^3-3x+1}$ is one-one and onto, then the distance of the point P(2b + 4, a + 2) from the line $x + e^{-3}y = 4$ is :

(1) $2\sqrt{1+e^6}$ (2) $4\sqrt{1+e^6}$

(3) $3\sqrt{1+e^6}$ (4) $\sqrt{1+e^6}$

Ans. (1)

14. Consider the function $f:(0,\infty)\to R$ defined by $f(x) = e^{-|\log_e x|}$. If m and n be respectively the number of points at which f is not continuous and f is not differentiable, then m + n is

(1) 0 (2) 3 (3) 1 (4) 2

Ans. (3)

15. The number of solutions, of the equation $e^{\sin x} - 2e^{-\sin x} = 2$ is

(1) 2 (2) more than 2

(3) 1 (4) 0

Ans. (4)

16. If $a = \sin^{-1}(\sin(5))$ and $b = \cos^{-1}(\cos(5))$, then $a^2 + b^2$ is equal to

(1) $4\pi^2 + 25$

(2) $8\pi^2 - 40\pi + 50$

(3) $4\pi^2 - 20\pi + 50$

(4) 25

Ans. (2)

17. If for some m, n; ${}^6C_m + 2\left({}^6C_{m+1}\right) + {}^6C_{m+2} > {}^8C_3$ and ${}^{n-1}P_3 : {}^nP_4 = 1:8$, then ${}^nP_{m+1} + {}^{n+1}C_m$ is equal to

(1) 380 (2) 376

(3) 384 (4) 372

Ans. (4)

18. A coin is based so that a head is twice as likely to occur as a tail. If the coin is tossed 3 times, then the probability of getting two tails and one head is-

(1) $\frac{2}{9}$ (2) $\frac{1}{9}$ (3) $\frac{2}{27}$ (4) $\frac{1}{27}$

Ans. (1)

19. Let A be a 3×3 real matrix such that

$$A\begin{pmatrix}1\\0\\1\end{pmatrix} = 2\begin{pmatrix}1\\0\\1\end{pmatrix}, A\begin{pmatrix}-1\\0\\1\end{pmatrix} = 4\begin{pmatrix}-1\\0\\1\end{pmatrix}, A\begin{pmatrix}0\\1\\0\end{pmatrix} = 2\begin{pmatrix}0\\1\\0\end{pmatrix}.$$

Then, the system $(A-3I)\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1\\2\\3\end{pmatrix}$ has

(1) unique solution

(2) exactly two solutions

(3) no solution

(4) infinitely many solutions

Ans. (1)

20. The shortest distance between lines L_1 and L_2, where $L_1 \frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+4}{2}$ and L_2 is the line passing through the points $A(-4,4,3).B(-1,6,3)$ and perpendicular to the line $\frac{x-3}{-2}=\frac{y}{3}=\frac{z-1}{1}$, is

(1) $\frac{121}{\sqrt{221}}$

(2) $\frac{24}{\sqrt{117}}$

(3) $\frac{141}{\sqrt{221}}$

(4) $\frac{42}{\sqrt{117}}$

Ans. **(3)**

SECTION-B

21. $\left|\frac{120}{\pi^3}\int_0^{\pi}\frac{x^2\sin x\cos x}{\sin^4 x+\cos^4 x}dx\right|$ is equal to ________.

Ans. **(15)**

22. Let a, b, c be the length of three sides of a triangle satisfying the condition $(a^2 + b^2)x^2 - 2b(a + c).x + (b^2 + c^2) = 0$. If the set of all possible values of x is the interval (α,β), then $12(\alpha^2+\beta^2)$ is equal to ______.

Ans. **(36)**

23. Let A(–2, –1), B(1, 0), $C(\alpha,\beta)$ and $D(\gamma,\delta)$ be the vertices of a parallelogram ABCD. If the point C lies on $2x - y = 5$ and the point D lies on $3x - 2y = 6$, then the value of $|\alpha+\beta+\gamma+\delta|$ is equal to _________.

Ans. **(32)**

24. Let the coefficient of x^r in the expansion of

$$(x+3)^{n-1}+(x+3)^{n-2}(x+2)+(x+3)^{n-3}(x+2)^2+\ldots\ldots+(x+2)^{n-1}$$

be α_r. If $\sum_{r=0}^{n}\alpha_r=\beta^n-\gamma^n, \beta,\gamma\in N$, then the value of $\beta^2+\gamma^2$ equals __________.

Ans. **(25)**

25. Let A be a 3×3 matrix and det (A) = 2. If

$$n=\underbrace{\det(\text{adj}(\text{adj}(\ldots\ldots(\text{adj}A))))}_{2024\text{-times}}$$

Then the remainder when n is divided by 9 is equal to ___________.

Ans. **(7)**

26. Let $\vec{a}=3\hat{i}+2\hat{j}+\hat{k}, \vec{b}=2\hat{i}+\hat{j}+3\hat{k}$ and $\vec{c}$ be a vector such that $(\vec{a}+\vec{b})\times\vec{c}=2(\vec{a}\times\vec{b})+24\hat{j}-6\hat{k}$ and $(\vec{a}-\vec{b}+\hat{i})(\vec{a}-\vec{b}+\hat{i}).\vec{c}=-3$. Then $|\vec{c}|^2$ is equal to ________.

Ans. **(38)**

27. If $\lim\limits_{x\to0}\frac{ax^2e^x-b\log_e(1+x)+cxe^{-x}}{x^2\sin x}$, then $16(a^2 + b^2 + c^2)$ is equal to ______.

Ans. **(81)**

28. A line passes through A(4, –6, –2) and B(16, –2,4). The point P(a, b, c) where a, b, c are non-negative integers, on the line AB lies at a distance of 21 units, from the point A. The distance between the points P(a, b, c) and Q(4, –12, 3) is equal to _________.

Ans. **(22)**

29. Let y = y(x) be the solution of the differential equation

$$\sec^2 x dx + \left(e^{2y}\tan^2 x + \tan x\right)dy = 0,$$

$$0 < x < \frac{\pi}{2}, y\left(\frac{\pi}{4}\right) = 0. \text{ If } y\left(\frac{\pi}{6}\right) = \alpha,$$

Then $e^{8\alpha}$ is equal to ______.

Ans. (9)

30. Let A = {1, 2, 3, ………100}. Let R be a relation on A defined by (x, y) $\in$ R if and only if 2x = 3y. Let R_1 be a symmetric relation on A such that $R \subset R_1$ and the number of elements in R_1 is n. Then, the minimum value of n is ___________.

Ans. (66)

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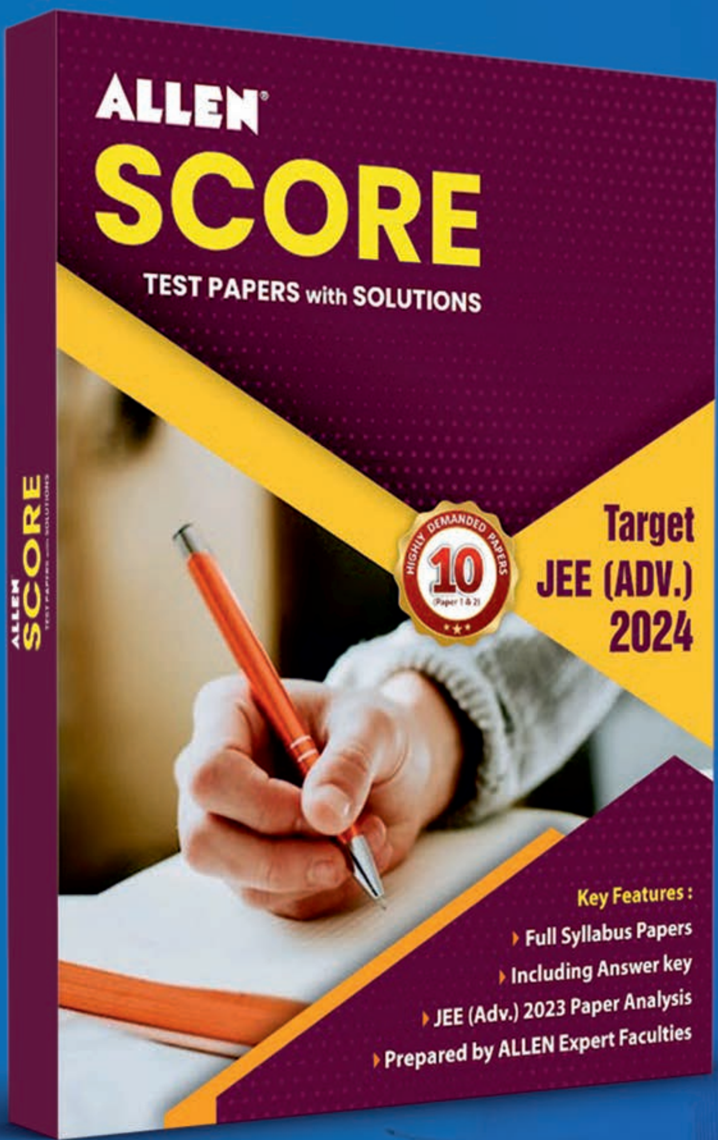

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