FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

(Held On Tuesday 30th January, 2024) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS

TEST PAPER WITH ANSWER

SECTION-A

1. A line passing through the point A(9, 0) makes an angle of 30º with the positive direction of x-axis. If this line is rotated about A through an angle of 15º in the clockwise direction, then its equation in the new position is

(1) $\frac{y}{\sqrt{3}-2}+x=9$ (2) $\frac{x}{\sqrt{3}-2}+y=9$

(3) $\frac{x}{\sqrt{3}+2}+y=9$ (4) $\frac{y}{\sqrt{3}+2}+x=9$

Ans. (1)

2. Let S_a denote the sum of first n terms an arithmetic progression. If $S_{20} = 790$ and $S_{10} = 145$, then $S_{15} - S_5$ is :

(1) 395 (2) 390

(3) 405 (4) 410

Ans. (1)

3. If z = x + iy, xy $\neq 0$, satisfies the equation $z^2 + i\bar{z} = 0$, then $|z^2|$ is equal to :

(1) 9 (2) 1

(3) 4 (4) $\frac{1}{4}$

Ans. (2)

4. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ be two vectors such that $|\vec{a}| = 1$; $\vec{a}.\vec{b} = 2$ and $|\vec{b}| = 4$. If $\vec{c} = 2(\vec{a}\times\vec{b}) - 3\vec{b}$, then the angle between $\vec{b}$ and $\vec{c}$ is equal to :

(1) $\cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$

(2) $\cos^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

(3) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

(4) $\cos^{-1}\left(\frac{2}{3}\right)$

Ans. (3)

5. The maximum area of a triangle whose one vertex is at (0, 0) and the other two vertices lie on the curve $y = -2x^2 + 54$ at points (x, y) and (-x, y) where y > 0 is :

(1) 88

(2) 122

(3) 92

(4) 108

Ans. (4)

6. The value of $\lim\limits_{n\to\infty}\sum\limits_{k=1}^{n}\frac{n^3}{(n^2+k^2)(n^2+3k^2)}$ is :

(1) $\frac{(2\sqrt{3}+3)\pi}{24}$

(2) $\frac{13\pi}{8(4\sqrt{3}+3)}$

(3) $\frac{13(2\sqrt{3}-3)\pi}{8}$

(4) $\frac{\pi}{8(2\sqrt{3}+3)}$

Ans. (2)

7. Let g : R→R be a non constant twice differentiable such that $g'\left(\frac{1}{2}\right) = g'\left(\frac{3}{2}\right)$. If a real valued function f is defined as $f(x) = \frac{1}{2}[g(x) + g(2-x)]$, then

(1) f''(x) = 0 for atleast two x in (0, 2)

(2) f''(x) = 0 for exactly one x in (0, 1)

(3) f''(x) = 0 for no x in (0, 1)

(4) $f'\left(\frac{3}{2}\right) + f'\left(\frac{1}{2}\right) = 1$

Ans. (1)

8. The area (in square units) of the region bounded by the parabola $y^2 = 4(x-2)$ and the line $y = 2x - 8$

(1) 8

(2) 9

(3) 6

(4) 7

Ans. (2)

9. Let $y = y(x)$ be the solution of the differential equation $\sec x\, dy + \{2(1-x)\tan x + x(2-x)\} dx = 0$ such that $y(0) = 2$. Then $y(2)$ is equal to :

(1) 2

(2) $2\{1-\sin(2)\}$

(3) $2\{\sin(2)+1\}$

(4) 1

Ans. (1)

10. Let (α,β,γ) be the foot of perpendicular from the point (1, 2, 3) on the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$. then $19(\alpha+\beta+\gamma)$ is equal to :

(1) 102

(2) 101

(3) 99

(4) 100

Ans. (2)

11. Two integers x and y are chosen with replacement from the set {0, 1, 2, 3, ….., 10}. Then the probability that $|x-y|>5$ is :

(1) $\frac{30}{121}$

(2) $\frac{62}{121}$

(3) $\frac{60}{121}$

(4) $\frac{31}{121}$

Ans. (1)

12. If the domain of the function $f(x)=\cos^{-1}\left(\frac{2-|x|}{4}\right)+\left(\log_e(3-x)\right)^{-1}$ is $[-\alpha,\beta)-\{\gamma\}$, then $\alpha+\beta+\gamma$ is equal to :

(1) 12

(2) 9

(3) 11

(4) 8

Ans. (3)

13. Consider the system of linear equation $x + y + z = 4\mu$, $x + 2y + 2\lambda z = 10\mu$, $x + 3y + 4\lambda^2 z = \mu^2 + 15$, where $\lambda, \mu \in R$. Which one of the following statements is NOT correct ?

(1) The system has unique solution if $\lambda \neq \frac{1}{2}$ and $\mu \neq 1, 15$

(2) The system is inconsistent if $\lambda = \frac{1}{2}$ and $\mu \neq 1$

(3) The system has infinite number of solutions if $\lambda = \frac{1}{2}$ and $\mu = 15$

(4) The system is consistent if $\lambda \neq \frac{1}{2}$

Ans. (2)

14. If the circles $(x+1)^2+(y+2)^2=r^2$ and $x^2+y^2-4x-4y+4=0$ intersect at exactly two distinct points, then

(1) $5 < r < 9$

(2) $0 < r < 7$

(3) $3 < r < 7$

(4) $\frac{1}{2} < r < 7$

Ans. (3)

15. If the length of the minor axis of ellipse is equal to half of the distance between the foci, then the eccentricity of the ellipse is :

(1) $\frac{\sqrt{5}}{3}$

(2) $\frac{\sqrt{3}}{2}$

(3) $\frac{1}{\sqrt{3}}$

(4) $\frac{2}{\sqrt{5}}$

Ans. (4)

16. Let M denote the median of the following frequency distribution.

Class	0-4	4-8	8-12	12-16	16-20
Frequency	3	9	10	8	6

Then 20 M is equal to :

(1) 416 (2) 104

(3) 52 (4) 208

Ans. (4)

17. If $f(x)=\begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3+\sin^2 2x \\ 3+2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3+2\sin^4 x & \sin^2 2x \end{vmatrix}$ then $\frac{1}{5}f'(0)$ is equal to ______

(1) 0 (2) 1

(3) 2 (4) 6

Ans. (1)

18. Let A (2, 3, 5) and C(-3, 4, -2) be opposite vertices of a parallelogram ABCD if the diagonal $\overrightarrow{BD}=\hat{i}+2\hat{j}+3\hat{k}$ then the area of the parallelogram is equal to

(1) $\frac{1}{2}\sqrt{410}$ (2) $\frac{1}{2}\sqrt{474}$

(3) $\frac{1}{2}\sqrt{586}$ (4) $\frac{1}{2}\sqrt{306}$

Ans. (2)

19. If $2\sin^3x + \sin 2x \cos x + 4\sin x - 4 = 0$ has exactly 3 solutions in the interval $\left[0,\frac{n\pi}{2}\right]$, $n\in N$, then the roots of the equation $x^2+nx+(n-3)=0$ belong to :

(1) $(0,\infty)$

(2) $(-\infty,0)$

(3) $\left(-\frac{\sqrt{17}}{2},\frac{\sqrt{17}}{2}\right)$

(4) Z

Ans. (2)

20. Let $f:\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\to R$ be a differentiable function such that $f(0)=\frac{1}{2}$, If the $\lim\limits_{x\to 0}\frac{x\int_0^x f(t)dt}{e^{x^2}-1}=\alpha$, then $8\alpha^2$ is equal to :

(1) 16

(2) 2

(3) 1

(4) 4

Ans. (2)

SECTION-B

21. A group of 40 students appeared in an examination of 3 subjects – Mathematics, Physics & Chemistry. It was found that all students passed in at least one of the subjects, 20 students passed in Mathematics, 25 students passed in Physics, 16 students passed in Chemistry, at most 11 students passed in both Mathematics and Physics, at most 15 students passed in both Physics and Chemistry, at most 15 students passed in both Mathematics and Chemistry. The maximum number of students passed in all the three subjects is ________.

Ans. (10)

22. If d_1 is the shortest distance between the lines $x + 1 = 2y = -12z$, $x = y + 2 = 6z - 6$ and d_2 is the shortest distance between the lines $\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}$, $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$, then the value of $\frac{32\sqrt{3}d_1}{d_2}$ is :

Ans. (16)

23. Let the latus rectum of the hyperbola $\frac{x^2}{9}-\frac{y^2}{b^2}=1$ subtend an angle of $\frac{\pi}{3}$ at the centre of the hyperbola. If b^2 is equal to $\frac{l}{m}(1+\sqrt{n})$, where l and m are co-prime numbers, then $l^2+m^2+n^2$ is equal to ___________

Ans. (182)

24. Let A = {1, 2, 3,….7} and let P(1) denote the power set of A. If the number of functions $f:A\to P(A)$ such that $a\in f(a),\forall a\in A$ is m^n, m and $n\in N$ and m is least, then m + n is equal to _______.

Ans. (44)

25. The value $9\int_0^9\left[\sqrt{\frac{10x}{x+1}}\right]dx$, where [t] denotes the greatest integer less than or equal to t, is _____.

Ans. (155)

26. Number of integral terms in the expansion of $\left\{7^{\left(\frac{1}{2}\right)}+11^{\left(\frac{1}{6}\right)}\right\}^{824}$ is equal to ________.

Ans. (138)

27. Let y = y(x) be the solution of the differential equation $(1 - x^2)\,dy = \left[xy+\left(x^3+2\right)\sqrt{3\left(1-x^2\right)}\right]dx$, $-1<x<1, y(0)=0$. If $y\left(\frac{1}{2}\right)=\frac{m}{n}$, m and n are co-prime numbers, then m + n is equal to ________.,

Ans. (97)

28. Let $\alpha,\beta \in N$ be roots of equation $x^2 - 70x + \lambda = 0$, where $\frac{\lambda}{2},\frac{\lambda}{3} \notin N$. If λ assumes the minimum possible value, then $\frac{\left(\sqrt{\alpha-1}+\sqrt{\beta-1}\right)(\lambda+35)}{|\alpha-\beta|}$ is equal to :

Ans. (60)

29. If the function $f(x)=\begin{cases}\frac{1}{|x|} & ,|x|\ge 2\\ ax^2+2b, & |x|<2\end{cases}$ is differentiable on R, then 48 (a + b) is equal to _____.

Ans. (15)

30. Let $\alpha=1^2+4^2+8^2+13^2+19^2+26^2+......$ upto 10 terms and $\beta=\sum_{n=1}^{10}n^4$. If $4\alpha-\beta=55k+40$, then k is equal to ___________.

Ans. (353)