FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

(Held On Tuesday 30th January, 2024) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS — TEST PAPER WITH ANSWER

SECTION-A

1. Consider the system of linear equations

$x + y + z = 5,\ x + 2y + \lambda^2 z = 9,$

$x + 3y + \lambda z = \mu$, where $\lambda, \mu \in R$. Then, which of the following statement is NOT correct?

(1) System has infinite number of solution if $\lambda = 1$ and $\mu = 13$

(2) System is inconsistent if $\lambda = 1$ and $\mu \neq 13$

(3) System is consistent if $\lambda \neq 1$ and $\mu = 13$

(4) System has unique solution if $\lambda \neq 1$ and $\mu \neq 13$

Ans. (4)

2. For $\alpha, \beta \in \left(0, \frac{\pi}{2}\right)$, let $3\sin(\alpha+\beta) = 2\sin(\alpha-\beta)$ and a real number k be such that $\tan\alpha = k\tan\beta$. Then the value of k is equal to :

(1) $-\frac{2}{3}$ (2) -5

(3) $\frac{2}{3}$ (4) 5

Ans. (Bonus)

3. Let A(α, 0) and B(0, β) be the points on the line 5x + 7y = 50. Let the point P divide the line segment AB internally in the ratio 7 : 3. Let 3x – 25 = 0 be a directrix of the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the corresponding focus be S. If from S, the perpendicular on the x-axis passes through P, then the length of the latus rectum of E is equal to

(1) $\frac{25}{3}$ (2) $\frac{32}{9}$

(3) $\frac{25}{9}$ (4) $\frac{32}{5}$

Ans. (4)

4. Let $\vec{a} = \hat{i} + \alpha\hat{j} + \beta\hat{k},\ \alpha, \beta \in R$. Let a vector $\vec{b}$ be such that the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$ and $|\vec{b}|^2 = 6$, If $\vec{a}.\vec{b} = 3\sqrt{2}$, then the value of $(\alpha^2 + \beta^2)|\vec{a} \times \vec{b}|^2$ is equal to

(1) 90 (2) 75

(3) 95 (4) 85

Ans. (1)

5. Let $f(x) = (x+3)^2(x-2)^3, x \in [-4, 4]$. If M and m are the maximum and minimum values of f, respectively in [–4, 4], then the value of M – m is :

(1) 600 (2) 392

(3) 608 (4) 108

Ans. (3)

6. Let a and b be be two distinct positive real numbers. Let 11th term of a GP, whose first term is a and third term is b, is equal to pth term of another GP, whose first term is a and fifth term is b. Then p is equal to

(1) 20 (2) 25

(3) 21 (4) 24

Ans. (3)

7. If $x^2 - y^2 + 2hxy + 2gx + 2fy + c = 0$ is the locus of a point, which moves such that it is always equidistant from the lines x + 2y + 7 = 0 and 2x – y + 8 = 0, then the value of g + c + h – f equals

(1) 14 (2) 6

(3) 8 (4) 29

Ans. (1)

8. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{b}| = 1$ and $|\vec{b} \times \vec{a}| = 2$. Then $\left|(\vec{b} \times \vec{a}) - \vec{b}\right|^2$ is equal to

(1) 3 (2) 5

(3) 1 (4) 4

Ans. (2)

9. Let $y=f(x)$ be a thrice differentiable function in $(-5, 5)$. Let the tangents to the curve $y=f(x)$ at $(1, f(1))$ and $(3, f(3))$ make angles $\frac{\pi}{6}$ and $\frac{\pi}{4}$, respectively with positive x-axis. If $27\int_1^3\left((f'(t))^2+1\right)f''(t)dt=\alpha+\beta\sqrt{3}$ where α, β are integers, then the value of $\alpha+\beta$ equals

(1) –14 (2) 26

(3) –16 (4) 36

Ans. (2)

10. Let P be a point on the hyperbola $H:\frac{x^2}{9}-\frac{y^2}{4}=1$, in the first quadrant such that the area of triangle formed by P and the two foci of H is $2\sqrt{13}$. Then, the square of the distance of P from the origin is

(1) 18 (2) 26

(3) 22 (4) 20

Ans. (3)

11. Bag A contains 3 white, 7 red balls and bag B contains 3 white, 2 red balls. One bag is selected at random and a ball is drawn from it. The probability of drawing the ball from the bag A, if the ball drawn in white, is :

(1) $\frac{1}{4}$ (2) $\frac{1}{9}$

(3) $\frac{1}{3}$ (4) $\frac{3}{10}$

Ans. (3)

12. Let f : R→ R be defined $f(x)=ae^{2x}+be^x+cx$. If $f(0)=-1$, $f'(\log_e 2)=21$ and $\int_0^{\log_e 4}(f(x)-cx)dx=\frac{39}{2}$, then the value of $|a+b+c|$ equals :

(1) 16 (2) 10

(3) 12 (4) 8

Ans. (4)

13. Let $L_1:\vec{r}=(\hat{i}-\hat{j}+2\hat{k})+\lambda(\hat{i}-\hat{j}+2\hat{k}),\ \lambda\in R$

$L_2:\vec{r}=(\hat{j}-\hat{k})+\mu(3\hat{i}+\hat{j}+p\hat{k}),\ \mu\in R$ and

$L_3:\vec{r}=\delta(\ell\hat{i}+m\hat{j}+n\hat{k})\ \delta\in R$

Be three lines such that L_1 is perpendicular to L_2 and L_3 is perpendicular to both L_1 and L_2. Then the point which lies on L_3 is

(1)) (–1, 7, 4) (2) (–1, –7, 4)

(3) (1, 7, –4) (4) (1, –7, 4)

Ans. (1)

14. Let a and b be real constants such that the function f defined by $f(x)=\begin{cases}x^2+3x+a, & x\le 1\\ bx+2, & x>1\end{cases}$ be differentiable on R. Then, the value of $\int_{-2}^{2}f(x)dx$ equals

(1) $\frac{15}{6}$ (2) $\frac{19}{6}$

(3) 21 (4) 17

Ans. (4)

15. Let $f:\mathbb{R}-\{0\}\to\mathbb{R}$ be a function satisfying $f\left(\frac{x}{y}\right)=\frac{f(x)}{f(y)}$ for all x, y, $f(y)\neq 0$. If $f'(1)=2024$, then

(1) $xf'(x)-2024f(x)=0$

(2) $xf'(x)+2024f(x)=0$

(3) $xf'(x)+f(x)=2024$

(4) $xf'(x)-2023f(x)=0$

Ans. (1)

16. If z is a complex number, then the number of common roots of the equation $z^{1985}+z^{100}+1=0$ and $z^3+2z^2+2z+1=0$, is equal to :

(1) 1 (2) 2

(3) 0 (4) 3

Ans. (2)

17. Suppose 2 – p, p, 2 – α, α are the coefficient of four consecutive terms in the expansion of $(1+x)^n$. Then the value of $p^2-\alpha^2+6\alpha+2p$ equals

(1) 4 (2) 10

(3) 8 (4) 6

Ans. (Bonus)

18. If the domain of the function $f(x) = \log_e\left(\frac{2x+3}{4x^2+x-3}\right)+\cos^{-1}\left(\frac{2x-1}{x+2}\right)$ is $(\alpha,\beta]$, then the value of $5\beta - 4\alpha$ is equal to

(1) 10 (2) 12

(3) 11 (4) 9

Ans. (2)

19. Let $f : R \to R$ be a function defined $f(x)=\frac{x}{(1+x^4)^{1/4}}$ and $g(x)=f(f(f(f(x))))$ then $18\int_0^{\sqrt{2\sqrt{5}}} x^2 g(x)dx$

(1) 33 (2) 36

(3) 42 (4) 39

Ans. (4)

20. Let $R = \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix}$ be a non-zero 3 × 3 matrix, where $x\sin\theta = y\sin\left(\theta+\frac{2\pi}{3}\right)=z\sin\left(\theta+\frac{4\pi}{3}\right)$ $\neq 0, \theta\in(0,2\pi)$. For a square matrix M, let trace (M) denote the sum of all the diagonal entries of M. Then, among the statements:

(I) Trace (R) = 0

(II) If trace (adj(adj(R)) = 0, then R has exactly one non-zero entry.

(1) Both (I) and (II) are true

(2) Neither (I) nor (II) is true

(3) Only (II) is true

(4) Only (I) is true

Ans. (3)

SECTION-B

21. Let Y = Y(X) be a curve lying in the first quadrant such that the area enclosed by the line $Y - y = Y'(x)(X - x)$ and the co-ordinate axes, where (x, y) is any point on the curve, is always $\frac{-y^2}{2Y'(x)}+1$, $Y'(x) \neq 0$. If Y(1) = 1, then 12Y(2) equals ______ .

Ans. (20)

22. Let a line passing through the point (–1, 2, 3) intersect the lines $L_1 : \frac{x-1}{3}=\frac{y-2}{2}=\frac{z+1}{-2}$ at $M(\alpha,\beta,\gamma)$ and $L_2 : \frac{x+2}{-3}=\frac{y-2}{-2}=\frac{z-1}{4}$ at N(a, b, c). Then the value of $\frac{(\alpha+\beta+\gamma)^2}{(a+b+c)^2}$ equals ____.

Ans. (196)

23. Consider two circles $C_1 : x^2 + y^2 = 25$ and $C_2 : (x - \alpha)^2 + y^2 = 16$, where $\alpha \in (5, 9)$. Let the angle between the two radii (one to each circle) drawn from one of the intersection points of C_1 and C_2 be $\sin^{-1}\left(\frac{\sqrt{63}}{8}\right)$. If the length of common chord of C_1 and C_2 is β, then the value of $(\alpha\beta)^2$ equals _____ .

Ans. (1575)

24. Let $\alpha = \sum_{k=0}^{n}\left(\frac{({}^nC_k)^2}{k+1}\right)$ and $\beta = \sum_{k=0}^{n-1}\left(\frac{{}^nC_k\,{}^nC_{k+1}}{k+2}\right)$.

If 5α = 6β, then n equals ___________ .

Ans. (10)

25. Let S_n be the sum to n-terms of an arithmetic progression 3, 7, 11, ... If $40 < \left(\frac{6}{n(n+1)}\sum_{k=1}^{n} S_k\right) < 42$, then n equals _____ .

Ans. (9)

26. In an examination of Mathematics paper, there are 20 questions of equal marks and the question paper is divided into three sections : A, B and C . A student is required to attempt total 15 questions taking at least 4 questions from each section. If section A has 8 questions, section B has 6 questions and section C has 6 questions, then the total number of ways a student can select 15 questions is _______ .

Ans. (11376)

27. The number of symmetric relations defined on the set {1, 2, 3, 4} which are not reflexive is _____ .

Ans. (960)

28. The number of real solutions of the equation $x\left(x^2+3|x|+5|x-1|+6|x-2|\right)=0$ is ______ .

Ans. (1)

29. The area of the region enclosed by the parabola $(y-2)^2 = x-1$, the line $x-2y+4=0$ and the positive coordinate axes is ______.

Ans. (5)

30. The variance σ^2 of the data

x_i	0	1	5	6	10	12	17
f_i	3	2	3	2	6	3	3

Is _______ .

Ans. (29)

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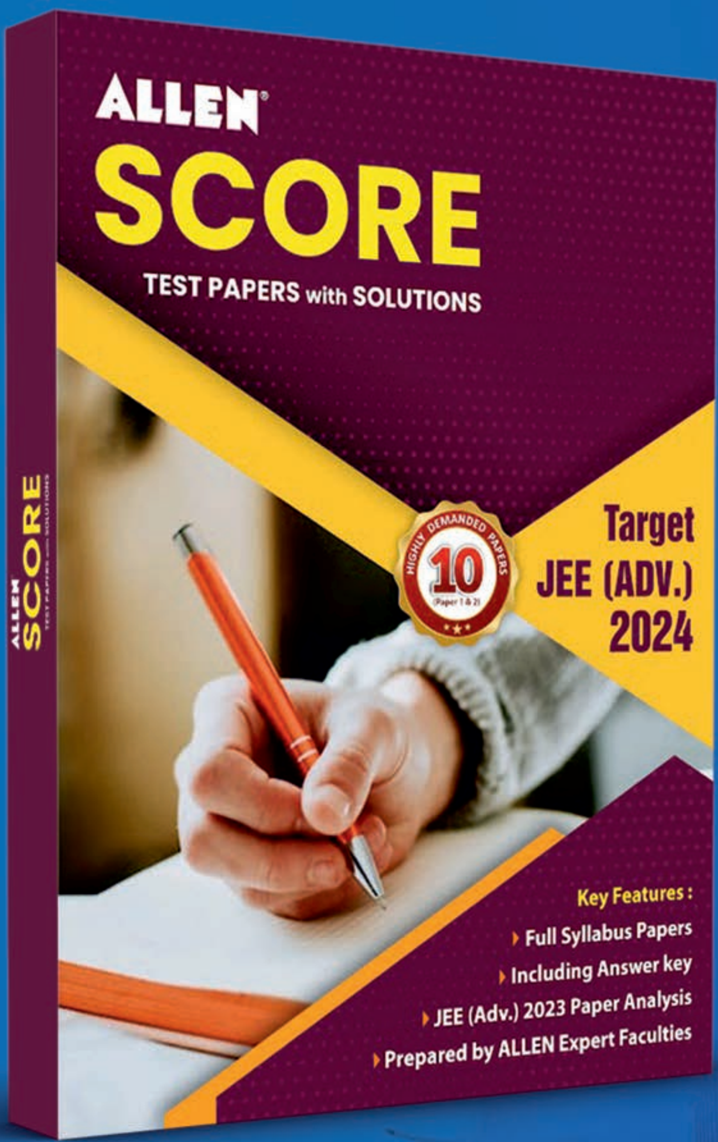

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