FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

(Held On Monday 29ᵗʰ January, 2024)   **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS	TEST PAPER WITH ANSWER

SECTION-A

1. If in a G.P. of 64 terms, the sum of all the terms is 7 times the sum of the odd terms of the G.P, then the common ratio of the G.P. is equal to

(1) 7 (2) 4

(3) 5 (4) 6

Ans. (4)

2. In an A.P., the sixth terms $a_6 = 2$. If the $a_1 a_4 a_5$ is the greatest, then the common difference of the A.P., is equal to

(1) $\dfrac{3}{2}$ (2) $\dfrac{8}{5}$

(3) $\dfrac{2}{3}$ (4) $\dfrac{5}{8}$

Ans. (2)

3. If $f(x) = \begin{cases} 2+2x, & -1 \le x < 0 \\ 1 - \dfrac{x}{3}, & 0 \le x \le 3 \end{cases}$; $g(x) = \begin{cases} -x, & -3 \le x \le 0 \\ x, & 0 < x \le 1 \end{cases}$,

then range of $(fog(x))$ is

(1) (0, 1] (2) [0, 3)

(3) [0, 1] (4) [0, 1)

Ans. (3)

4. A fair die is thrown until 2 appears. Then the probability, that 2 appears in even number of throws , is

(1) $\dfrac{5}{6}$ (2) $\dfrac{1}{6}$

(3) $\dfrac{5}{11}$ (4) $\dfrac{6}{11}$

Ans. (3)

5. If $z = \dfrac{1}{2} - 2i$, is such that $|z+1| = \alpha z + \beta(1+i), i = \sqrt{-1}$ and $\alpha, \beta \in R$, then $\alpha + \beta$ is equal to

(1) –4 (2) 3

(3) 2 (4) –1

Ans. (2)

6. $\lim\limits_{x \to \frac{\pi}{2}} \left(\dfrac{1}{\left(x - \dfrac{\pi}{2}\right)^2} \int_{x^3}^{\left(\frac{\pi}{2}\right)^3} \cos\left(\dfrac{1}{t^3}\right) dt \right)$ is equal to

(1) $\dfrac{3\pi}{8}$ (2) $\dfrac{3\pi^2}{4}$

(3) $\dfrac{3\pi^2}{8}$ (4) $\dfrac{3\pi}{4}$

Ans. (3)

7. In a $\triangle ABC$, suppose $y = x$ is the equation of the bisector of the angle B and the equation of the side AC is $2x - y = 2$. If $2AB = BC$ and the point A and B are respectively $(4, 6)$ and (α, β), then $\alpha + 2\beta$ is equal to

(1) 42 (2) 39

(3) 48 (4) 45

Ans. (1)

8. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that $\vec{b}$ and $\vec{c}$ are non-collinear .if $\vec{a} + 5\vec{b}$ is collinear with $\vec{c}, \vec{b} + 6\vec{c}$ is collinear with $\vec{a}$ and $\vec{a} + \alpha\vec{b} + \beta\vec{c} = \vec{0}$, then $\alpha + \beta$ is equal to

(1) 35 (2) 30

(3) – 30 (4) –25

Ans. (1)

9. Let $\left(5, \dfrac{a}{4}\right)$, be the circumcenter of a triangle with vertices $A(a, -2)$, $B(a, 6)$ and $C\left(\dfrac{a}{4}, -2\right)$. Let α denote the circumradius, β denote the area and γ denote the perimeter of the triangle. Then $\alpha + \beta + \gamma$ is

(1) 60   (2) 53

(3) 62   (4) 30

Ans. (2)

10. For $x \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$, if

$y(x) = \displaystyle\int \dfrac{\csc x + \sin x}{\csc x \sec x + \tan x \sin^2 x} \, dx$ and

$\displaystyle\lim_{x \to \left(\frac{\pi}{2}\right)^-} y(x) = 0$ then $y\left(\dfrac{\pi}{4}\right)$ is equal to

(1) $\tan^{-1}\left(\dfrac{1}{\sqrt{2}}\right)$ (2) $\dfrac{1}{2}\tan^{-1}\left(\dfrac{1}{\sqrt{2}}\right)$

(3) $-\dfrac{1}{\sqrt{2}}\tan^{-1}\left(\dfrac{1}{\sqrt{2}}\right)$ (4) $\dfrac{1}{\sqrt{2}}\tan^{-1}\left(-\dfrac{1}{2}\right)$

Ans. (4)

11. If $\alpha, -\dfrac{\pi}{2} < \alpha < \dfrac{\pi}{2}$ is the solution of $4\cos\theta + 5\sin\theta = 1$, then the value of $\tan\alpha$ is

(1) $\dfrac{10 - \sqrt{10}}{6}$ (2) $\dfrac{10 - \sqrt{10}}{12}$

(3) $\dfrac{\sqrt{10} - 10}{12}$ (4) $\dfrac{\sqrt{10} - 10}{6}$

Ans. (3)

12. A function $y = f(x)$ satisfies

$f(x)\sin 2x + \sin x - \left(1 + \cos^2 x\right)f'(x) = 0$ with condition $f(0) = 0$. Then $f\left(\dfrac{\pi}{2}\right)$ is equal to

(1) 1   (2) 0

(3) –1   (4) 2

Ans. (1)

13. Let O be the origin and the position vector of A and B be $2\hat{i} + 2\hat{j} + \hat{k}$ and $2\hat{i} + 4\hat{j} + 4\hat{k}$ respectively. If the internal bisector of $\angle AOB$ meets the line AB at C, then the length of OC is

(1) $\dfrac{2}{3}\sqrt{31}$ (2) $\dfrac{2}{3}\sqrt{34}$ (3) $\dfrac{3}{4}\sqrt{34}$ (4) $\dfrac{3}{2}\sqrt{31}$

Ans. (2)

14. Consider the function $f : \left[\dfrac{1}{2}, 1\right] \to R$ defined by $f(x) = 4\sqrt{2}x^3 - 3\sqrt{2}x - 1$. Consider the statements

(I) The curve $y = f(x)$ intersects the x-axis exactly at one point

(II) The curve $y = f(x)$ intersects the x-axis at $x = \cos\dfrac{\pi}{12}$

Then

(1) Only (II) is correct

(2) Both (I) and (II) are incorrect

(3) Only (I) is correct

(4) Both (I) and (II) are correct

Ans. (4)

15. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{bmatrix}$ and $|2A|^3 = 2^{21}$ where $\alpha, \beta \in Z$,

Then a value of α is

(1) 3   (2) 5

(3) 17   (4) 9

Ans. (2)

16. Let PQR be a triangle with $R(-1, 4, 2)$. Suppose $M(2, 1, 2)$ is the mid point of PQ. The distance of the centroid of ΔPQR from the point of intersection of the line $\dfrac{x-2}{0} = \dfrac{y}{2} = \dfrac{z+3}{-1}$ and $\dfrac{x-1}{1} = \dfrac{y+3}{-3} = \dfrac{z+1}{1}$ is

(1) 69   (2) 9

(3) $\sqrt{69}$ (4) $\sqrt{99}$

Ans. (3)

17. Let R be a relation on Z × Z defined by

(a, b)R(c, d) if and only if ad – bc is divisible by 5. Then R is

(1) Reflexive and symmetric but not transitive

(2) Reflexive but neither symmetric not transitive

(3) Reflexive, symmetric and transitive

(4) Reflexive and transitive but not symmetric

Ans. (1)

18. If the value of the integral

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{x^2 \cos x}{1+\pi^x} + \frac{1+\sin^2 x}{1+e^{\sin x^{2023}}} \right) dx = \frac{\pi}{4}(\pi+a) - 2,$$

then the value of a is

(1) 3   (2) $-\frac{3}{2}$

(3) 2   (4) $\frac{3}{2}$

Ans. (1)

19. Suppose

$$f(x) = \frac{\left(2^x + 2^{-x}\right)\tan x \sqrt{\tan^{-1}\left(x^2 - x + 1\right)}}{\left(7x^2 + 3x + 1\right)^3},$$

Then the value of f '(0) is equal to

(1) π   (2) 0

(3) $\sqrt{\pi}$ (4) $\frac{\pi}{2}$

Ans. (3)

20. Let A be a square matrix such that $AA^T = I$. Then

$\frac{1}{2}A\left[\left(A + A^T\right)^2 + \left(A - A^T\right)^2\right]$ is equal to

(1) $A^2 + I$ (2) $A^3 + I$

(3) $A^2 + A^T$ (4) $A^3 + A^T$

Ans. (4)

SECTION-B

21. Equation of two diameters of a circle are

$2x - 3y = 5$ and $3x - 4y = 7$. The line joining the

points $\left(-\frac{22}{7}, -4\right)$ and $\left(-\frac{1}{7}, 3\right)$ intersects the circle

at only one point $P(\alpha, \beta)$. Then $17\beta - \alpha$ is equal to

Ans. (2)

22. All the letters of the word "GTWENTY" are

written in all possible ways with or without

meaning and these words are written as in a

dictionary. The serial number of the word

"GTWENTY" IS

Ans. (553)

23. Let α, β be the roots of the equation $x^2 - x + 2 = 0$

with $\mathrm{Im}(\alpha) > \mathrm{Im}(\beta)$. Then $\alpha^6 + \alpha^4 + \beta^4 - 5\alpha^2$ is

equal to

Ans. (13)

24. Let $f(x) = 2^x - x^2, x \in R$. If m and n are

respectively the number of points at which the

curves y = f(x) and y = f '(x) intersects the x-axis,

then the value of m + n is

Ans. (5)

25. If the points of intersection of two distinct conics

$x^2 + y^2 = 4b$ and $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ lie on the curve

$y^2 = 3x^2$, then $3\sqrt{3}$ times the area of the rectangle

formed by the intersection points is ___

Ans. (432)

26. If the solution curve $y = y(x)$ of the differential equation $(1+y^2)(1+\log_e x)dx + x\,dy = 0$, $x > 0$ passes through the point $(1, 1)$ and

$$y(e) = \frac{\alpha - \tan\left(\dfrac{3}{2}\right)}{\beta + \tan\left(\dfrac{3}{2}\right)}, \text{ then } \alpha + 2\beta \text{ is}$$

Ans. (3)

27. If the mean and variance of the data 65, 68, 58, 44, 48, 45, 60, $\alpha, \beta, 60$ where $\alpha > \beta$ are 56 and 66.2 respectively, then $\alpha^2 + \beta^2$ is equal to

Ans. (6344)

28. The area (in sq. units) of the part of circle $x^2 + y^2 = 169$ which is below the line $5x - y = 13$ is

$$\frac{\pi\alpha}{2\beta} - \frac{65}{2} + \frac{\alpha}{\beta}\sin^{-1}\left(\frac{12}{13}\right) \text{ where } \alpha, \beta \text{ are coprime}$$

numbers. Then $\alpha + \beta$ is equal to

Ans. (171)

29. If $\dfrac{^{11}C_1}{2} + \dfrac{^{11}C_2}{3} + \ldots + \dfrac{^{11}C_9}{10} = \dfrac{n}{m}$ with $\gcd(n, m) = 1$, then $n + m$ is equal to

Ans. (2041)

30. A line with direction ratios 2, 1, 2 meets the lines $x = y + 2 = z$ and $x + 2 = 2y = 2z$ respectively at the point P and Q. if the length of the perpendicular from the point $(1, 2, 12)$ to the line PQ is l, then l^2 is
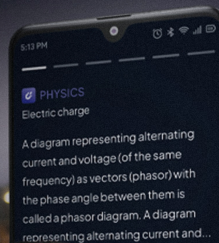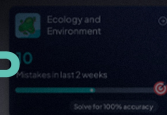
Ans. (65)