KOTA (RANASTHAN)
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## FINAL JEE-MAIN EXAMINATION - JANUARY, 2024

(Held On Monday 29th January, 2024)
TIME: 3:00 PM to 6:00 PM

## MATHEMATICS

## SECTION-A

1. Let $\mathrm{A}=\left|\begin{array}{ccc}2 & 1 & 2 \\ 6 & 2 & 11 \\ 3 & 3 & 2\end{array}\right|$ and $\mathrm{P}=\left|\begin{array}{lll}1 & 2 & 0 \\ 5 & 0 & 2 \\ 7 & 1 & 5\end{array}\right|$. The sum of the prime factors of $\left|\mathrm{P}^{-1} \mathrm{AP}-2 \mathrm{I}\right|$ is equal to
(1) 26
(2) 27
(3) 66
(4) 23

Ans. (1)
2. Number of ways of arranging 8 identical books into 4 identical shelves where any number of shelves may remain empty is equal to
(1) 18
(2) 16
(3) 12
(4) 15

Ans. (4)
3. Let $\mathrm{P}(3,2,3), \mathrm{Q}(4,6,2)$ and $\mathrm{R}(7,3,2)$ be the vertices of $\triangle \mathrm{PQR}$. Then, the angle $\angle \mathrm{QPR}$ is
(1) $\frac{\pi}{6}$
(2) $\cos ^{-1}\left(\frac{7}{18}\right)$
(3) $\cos ^{-1}\left(\frac{1}{18}\right)$
(4) $\frac{\pi}{3}$

Ans. (4)
4. If the mean and variance of five observations are $\frac{24}{5}$ and $\frac{194}{25}$ respectively and the mean of first four observations is $\frac{7}{2}$, then the variance of the first four observations in equal to
(1) $\frac{4}{5}$
(2) $\frac{77}{12}$
(3) $\frac{5}{4}$
(4) $\frac{105}{4}$

Ans. (3)

## TEST PAPER WITH ANSWER

5. The function $f(x)=2 x+3(x)^{\frac{2}{3}}, x \in \mathbb{R}$, has
(1) exactly one point of local minima and no point of local maxima
(2) exactly one point of local maxima and no point of local minima
(3) exactly one point of local maxima and exactly one point of local minima
(4) exactly two points of local maxima and exactly one point of local minima

Ans. (3)
6. Let $r$ and $\theta$ respectively be the modulus and amplitude of the complex number $z=2-i\left(2 \tan \frac{5 \pi}{8}\right)$, then $(r, \theta)$ is equal to
(1) $\left(2 \sec \frac{3 \pi}{8}, \frac{3 \pi}{8}\right)$
(2) $\left(2 \sec \frac{3 \pi}{8}, \frac{5 \pi}{8}\right)$
(3) $\left(2 \sec \frac{5 \pi}{8}, \frac{3 \pi}{8}\right)$
(4) $\left(2 \sec \frac{11 \pi}{8}, \frac{11 \pi}{8}\right)$

Ans. (1)
7. The sum of the solutions $x \in \mathbb{R}$ of the equation $\frac{3 \cos 2 x+\cos ^{3} 2 x}{\cos ^{6} x-\sin ^{6} x}=x^{3}-x^{2}+6$ is
(1) 0
(2) 1
(3) -1
(4) 3

Ans. (3)
8. Let $\overrightarrow{\mathrm{OA}}=\overrightarrow{\mathrm{a}}, \overrightarrow{\mathrm{OB}}=12 \overrightarrow{\mathrm{a}}+4 \overrightarrow{\mathrm{~b}}$ and $\overrightarrow{\mathrm{OC}}=\overrightarrow{\mathrm{b}}$, where O is the origin. If S is the parallelogram with adjacent sides OA and OC, then $\frac{\text { area of the quadrilateral } \mathrm{OABC}}{\text { area of } \mathrm{S}}$ is equal to $\qquad$
(1) 6
(2) 10
(3) 7
(4) 8

Ans. (4)
9. If $\log _{e} a, \log _{e} b, \log _{e} c$ are in an A.P. and $\log _{e} a-$ $\log _{e} 2 b, \log _{e} 2 b-\log _{e} 3 c, \log _{e} 3 c-\log _{e} a$ are also in an A.P, then $\mathrm{a}: \mathrm{b}: \mathrm{c}$ is equal to
(1) $9: 6: 4$
(2) $16: 4: 1$
(3) $25: 10: 4$
(4) $6: 3: 2$

Ans. (1)
10. If

$$
\int \frac{\sin ^{\frac{2}{2}} x+\cos ^{\frac{3}{2}} x}{\sqrt{\sin ^{3} x \cos ^{3} x \sin (x-\theta)}} d x=A \sqrt{\cos \theta \tan x-\sin \theta}+B \sqrt{\cos \theta-\sin \theta \cot x}+C,
$$

where C is the integration constant, then AB is equal to
(1) $4 \operatorname{cosec}(2 \theta)$
(2) $4 \sec \theta$
(3) $2 \sec \theta$
(4) $8 \operatorname{cosec}(2 \theta)$

Ans. (4)
11. The distance of the point $(2,3)$ from the line $2 x-$ $3 y+28=0$, measured parallel to the line $\sqrt{3} \mathrm{x}-\mathrm{y}+1=0$, is equal to
(1) $4 \sqrt{2}$
(2) $6 \sqrt{3}$
(3) $3+4 \sqrt{2}$
(4) $4+6 \sqrt{3}$

Ans. (4)
12. If $\sin \left(\frac{y}{x}\right)=\log _{e}|x|+\frac{\alpha}{2}$ is the solution of the differential equation $x \cos \left(\frac{y}{x}\right) \frac{d y}{d x}=y \cos \left(\frac{y}{x}\right)+x$ and $y(1)=\frac{\pi}{3}$, then $\alpha^{2}$ is equal to
(1) 3
(2) 12
(3) 4
(4) 9

Ans. (1)
13. If each term of a geometric progression $a_{1}, a_{2}, a_{3}, \ldots$ with $\mathrm{a}_{1}=\frac{1}{8}$ and $\mathrm{a}_{2} \neq \mathrm{a}_{1}$, is the arithmetic mean of the next two terms and $S_{n}=a_{1}+a_{2}+\ldots+a_{n}$, then $\mathrm{S}_{20}-\mathrm{S}_{18}$ is equal to
(1) $2^{15}$
(2) $-2^{18}$
(3) $2^{18}$
(4) $-2^{15}$

Ans. (4)
14. Let A be the point of intersection of the lines $3 \mathrm{x}+$ $2 y=14,5 x-y=6$ and $B$ be the point of intersection of the lines $4 x+3 y=8,6 x+y=5$. The distance of the point $\mathrm{P}(5,-2)$ from the line $A B$ is
(1) $\frac{13}{2}$
(2) 8
(3) $\frac{5}{2}$
(4) 6

Ans. (4)
15. Let $\mathrm{x}=\frac{\mathrm{m}}{\mathrm{n}}$ (m, n are co-prime natural numbers) be a solution of the equation $\cos \left(2 \sin ^{-1} x\right)=\frac{1}{9}$ and let $\alpha, \beta(\alpha>\beta)$ be the roots of the equation $\mathrm{mx}^{2}-\mathrm{nx}-$ $\mathrm{m}+\mathrm{n}=0$. Then the point $(\alpha, \beta)$ lies on the line
(1) $3 x+2 y=2$
(2) $5 x-8 y=-9$
(3) $3 x-2 y=-2$
(4) $5 x+8 y=9$

Ans. (4)
16. The function $f(x)=\frac{x}{x^{2}-6 x-16}, x \in \mathbb{R}-\{-2,8\}$
(1) decreases in $(-2,8)$ and increases in

$$
(-\infty,-2) \cup(8, \infty)
$$

(2) decreases in $(-\infty,-2) \cup(-2,8) \cup(8, \infty)$
(3) decreases in $(-\infty,-2)$ and increases in $(8, \infty)$
(4) increases in $(-\infty,-2) \cup(-2,8) \cup(8, \infty)$

Ans. (2) for Class 10 ${ }^{\text {th }}$ | NEET | JEE
17. Let $y=\log _{\mathrm{e}}\left(\frac{1-\mathrm{x}^{2}}{1+\mathrm{x}^{2}}\right),-1<\mathrm{x}<1$. Then at $\mathrm{x}=\frac{1}{2}$, the value of $225\left(y^{\prime}-y^{\prime \prime}\right)$ is equal to
(1) 732
(2) 746
(3) 742
(4) 736

Ans. (4)
18. If R is the smallest equivalence relation on the set $\{1,2,3,4\}$ such that $\{(1,2),(1,3)\} \subset R$, then the number of elements in R is $\qquad$
(1) 10
(2) 12
(3) 8
(4) 15

Ans. (1)
19. An integer is chosen at random from the integers 1 , $2,3, \ldots, 50$. The probability that the chosen integer is a multiple of atleast one of 4,6 and 7 is
(1) $\frac{8}{25}$
(2) $\frac{21}{50}$
(3) $\frac{9}{50}$
(4) $\frac{14}{25}$

Ans. (2)
20. Let a unit vector $\hat{\mathrm{u}}=x \hat{\mathrm{i}}+y \hat{\mathrm{j}}+\mathrm{zk}$ make angles $\frac{\pi}{2}, \frac{\pi}{3}$ and $\frac{2 \pi}{3}$ with the vectors $\frac{1}{\sqrt{2}} \hat{\mathrm{i}}+\frac{1}{\sqrt{2}} \hat{\mathrm{k}}, \frac{1}{\sqrt{2}} \hat{\mathrm{j}}+\frac{1}{\sqrt{2}} \hat{\mathrm{k}}$ and

$$
\begin{gathered}
\frac{1}{\sqrt{2}} \hat{\mathrm{i}}+\frac{1}{\sqrt{2}} \hat{\mathrm{j}} \quad \text { respectively. } \\
\frac{1}{\sqrt{2}} \hat{\mathrm{j}}+\frac{1}{\sqrt{2}} \hat{\mathrm{k}}, \text { then }|\hat{\mathrm{u}}-\overrightarrow{\mathrm{v}}|^{2} \text { is equal to }
\end{gathered}
$$

(1) $\frac{11}{2}$
(2) $\frac{5}{2}$
(3) 9
(4) 7

Ans. (2)

## SECTION-B

21. Let $\alpha, \beta$ be the roots of the equation $x^{2}-\sqrt{6} x+3=0$ such that $\operatorname{Im}(\alpha)>\operatorname{Im}(\beta)$. Let $a, b$ be integers not divisible by 3 and $n$ be a natural number such that $\frac{\alpha^{99}}{\beta}+\alpha^{98}=3^{n}(a+i b), i=\sqrt{-1}$. Then $\mathrm{n}+\mathrm{a}+\mathrm{b}$ is equal to $\qquad$ .

Ans. 49
22. Let for any three distinct consecutive terms $a, b, c$ of an A.P, the lines $\mathrm{ax}+\mathrm{by}+\mathrm{c}=0$ be concurrent at the point P and $\mathrm{Q}(\alpha, \beta)$ be a point such that the system of equations
$x+y+z=6$,
$2 x+5 y+\alpha z=\beta$ and
$x+2 y+3 z=4$, has infinitely many solutions. Then $(P Q)^{2}$ is equal to $\qquad$ -.

Ans. 113
23. Let $\mathrm{P}(\alpha, \beta)$ be a point on the parabola $y^{2}=4 \mathrm{x}$. If P also lies on the chord of the parabola $x^{2}=8 y$ whose mid point is $\left(1, \frac{5}{4}\right)$. Then $(\alpha-28)(\beta-8)$ is equal to $\qquad$ .

Ans. 192
24. If $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{1-\sin 2 x} d x=\alpha+\beta \sqrt{2}+\gamma \sqrt{3}$, where $\alpha$, $\beta$ and $\gamma$ are rational numbers, then $3 \alpha+4 \beta-\gamma$ is equal to $\qquad$ .

Ans. 6
25. Let the area of the region $\{(x, y): 0 \leq x \leq 3,0 \leq y \leq$ $\left.\min \left\{x^{2}+2,2 x+2\right\}\right\}$ be $A$. Then 12 A is equal to
$\qquad$ .

Ans. 164
26. Let O be the origin, and M and N be the points on the lines $\frac{x-5}{4}=\frac{y-4}{1}=\frac{z-5}{3} \quad$ and $\frac{\mathrm{x}+8}{12}=\frac{\mathrm{y}+2}{5}=\frac{\mathrm{z}+11}{9}$ respectively such that MN is the shortest distance between the given lines. Then $\overrightarrow{\mathrm{OM}} \cdot \overrightarrow{\mathrm{ON}}$ is equal to $\qquad$ .

Ans. 9
27. Let $\left.f(x)=\sqrt{\lim _{r \rightarrow x}\left\{\frac{2 r^{2}\left[(f(r))^{2}-f(x) f(r)\right]}{r^{2}-x^{2}}-r^{3} e^{\frac{f(r)}{r}}\right.}\right\}$ be differentiable in $(-\infty, 0) \cup(0, \infty)$ and $f(1)=1$. Then the value of ea, such that $f(a)=0$, is equal to
$\qquad$ .

Ans. 2
28. Remainder when $64^{32^{32}}$ is divided by 9 is equal to
$\qquad$ .

Ans. 1
29. Let the set $C=\left\{(x, y) \mid x^{2}-2^{y}=2023, x, y \in \mathbb{N}\right\}$.

Then $\sum_{(x, y) \in C}^{\prime}(x+y)$ is equal to $\qquad$ -

Ans. 46
30. Let the slope of the line $45 x+5 y+3=0$ be $27 r_{1}+\frac{9 r_{2}}{2} \quad$ for some $\quad r_{1}, \quad r_{2} \in R$. Then $\operatorname{Lim}_{x \rightarrow 3}\left(\int_{3}^{x} \frac{8 t^{2}}{\frac{3 r_{2} x}{2}-r_{2} x^{2}-r_{1} x^{3}-3 x} d t\right)$ is equal to $\qquad$ .

Ans. 12

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