## FINAL JEE-MAIN EXAMINATION - JANUARY, 2024

(Held On Saturday 27 ${ }^{\text {th }}$ January, 2024)
TIME:9:00 AM to 12:00 NOON

## MATHEMATICS

## SECTION-A

1. ${ }^{\mathrm{n}-1} \mathrm{C}_{\mathrm{r}}=\left(\mathrm{k}^{2}-8\right){ }^{\mathrm{n}} \mathrm{C}_{\mathrm{r}+1}$ if and only if :
(1) $2 \sqrt{2}<\mathrm{k} \leq 3$
(2) $2 \sqrt{3}<\mathrm{k} \leq 3 \sqrt{2}$
(3) $2 \sqrt{3}<\mathrm{k}<3 \sqrt{3}$
(4) $2 \sqrt{2}<\mathrm{k}<2 \sqrt{3}$

Ans. (1)
2. The distance, of the point $(7,-2,11)$ from the line $\frac{x-6}{1}=\frac{y-4}{0}=\frac{z-8}{3}$ along the line $\frac{x-5}{2}=\frac{y-1}{-3}=\frac{z-5}{6}$, is :
(1) 12
(2) 14
(3) 18
(4) 21

Ans. (2)
3. Let $x=x(t)$ and $y=y(t)$ be solutions of the differential equations $\frac{\mathrm{dx}}{\mathrm{dt}}+\mathrm{ax}=0 \quad$ and $\frac{d y}{d t}+b y=0$ respectively, $a, b \in R$. Given that $x(0)=2 ; y(0)=1$ and $3 y(1)=2 x(1)$, the value of $t$, for which $\mathrm{x}(\mathrm{t})=\mathrm{y}(\mathrm{t})$, is :
(1) $\log _{\frac{2}{3}} 2$
(2) $\log _{4} 3$
(3) $\log _{3} 4$
(4) $\log _{4} 2$

Ans. (4)
4. If $(a, b)$ be the orthocentre of the triangle whose vertices are $(1,2),(2,3)$ and $(3,1)$, and $I_{1}=\int_{a}^{b} x \sin \left(4 x-x^{2}\right) d x, I_{2}=\int_{a}^{b} \sin \left(4 x-x^{2}\right) d x$ , then $36 \frac{\mathrm{I}_{1}}{\mathrm{I}_{2}}$ is equal to :
(1) 72
(2) 88
(3) 80
(4) 66

Ans. (1)

## TEST PAPER WITH ANSWER

5. If A denotes the sum of all the coefficients in the expansion of $\left(1-3 x+10 x^{2}\right)^{n}$ and B denotes the sum of all the coefficients in the expansion of $\left(1+x^{2}\right)^{n}$, then :
(1) $A=B^{3}$
(2) $3 A=B$
(3) $B=A^{3}$
(4) $\mathrm{A}=3 \mathrm{~B}$

Ans. (1)
6. The number of common terms in the progressions $4,9,14,19, \ldots \ldots$. , up to $25^{\text {th }}$ term and $3,6,9,12$, ........, up to $37^{\text {th }}$ term is :
(1) 9
(2) 5
(3) 7
(4) 8

Ans. (3)
7. If the shortest distance of the parabola $y^{2}=4 x$ from the centre of the circle $x^{2}+y^{2}-4 x-16 y+64=0$ is d , then $\mathrm{d}^{2}$ is equal to :
(1) 16
(2) 24
(3) 20
(4) 36

Ans. (3)
8. If the shortest distance between the lines $\frac{x-4}{1}=\frac{y+1}{2}=\frac{z}{-3}$ and $\frac{x-\lambda}{2}=\frac{y+1}{4}=\frac{z-2}{-5}$ is $\frac{6}{\sqrt{5}}$, then the sum of all possible values of $\lambda$ is :
(1) 5
(2) 8
(3) 7
(4) 10

Ans. (2)
9. If $\int_{0}^{1} \frac{1}{\sqrt{3+x}+\sqrt{1+x}} d x=a+b \sqrt{2}+c \sqrt{3}$, where $\mathrm{a}, \mathrm{b}, \mathrm{c}$ are rational numbers, then $2 \mathrm{a}+3 \mathrm{~b}-4 \mathrm{c}$ is equal to :
(1) 4
(2) 10
(3) 7
(4) 8

Ans. (4)
10. Let $S=\{1,2,3, \ldots, 10\}$. Suppose $M$ is the set of all the subsets of $S$, then the relation
$\mathrm{R}=\{(\mathrm{A}, \mathrm{B}): \mathrm{A} \cap \mathrm{B} \neq \phi ; \mathrm{A}, \mathrm{B} \in \mathrm{M}\}$ is :
(1) symmetric and reflexive only
(2) reflexive only
(3) symmetric and transitive only
(4) symmetric only

Ans. (4)
11. If $S=\{z \in C:|z-i|=|z+i|=|z-1|\}$, then, $n(S)$ is:
(1) 1
(2) 0
(3) 3
(4) 2

Ans. (1)
12. Four distinct points $(2 \mathrm{k}, 3 \mathrm{k}),(1,0),(0,1)$ and $(0,0)$ lie on a circle for $k$ equal to :
(1) $\frac{2}{13}$
(2) $\frac{3}{13}$
(3) $\frac{5}{13}$
(4) $\frac{1}{13}$

Ans. (3)
13. Consider the function.

$$
f(x)=\left\{\begin{array}{cc}
\frac{a\left(7 x-12-x^{2}\right)}{b\left|x^{2}-7 x+12\right|} & , \quad x<3 \\
2^{\frac{\sin (x-3)}{x-[x]}} & , \quad x>3 \\
b & , \quad x=3
\end{array}\right.
$$

解 x denotes the greatest integer less than or equal to $x$. If $S$ denotes the set of all ordered pairs $(a, b)$ such that $f(x)$ is continuous at $x=3$, then the number of elements in $S$ is :
(1) 2
(2) Infinitely many
(3) 4
(4) 1

Ans. (4)
14. Let $a_{1}, a_{2}, \ldots . . a_{10}$ be 10 observations such that $\sum_{k=1}^{10} a_{k}=50$ and $\sum_{\forall k<j} a_{k} \cdot a_{j}=1100$. Then the standard deviation of $a_{1}, a_{2}, . ., a_{10}$ is equal to :
(1) 5
(2) $\sqrt{5}$
(3) 10
(4) $\sqrt{115}$

Ans. (2)
15. The length of the chord of the ellipse $\frac{x^{2}}{25}+\frac{y^{2}}{16}=1$, whose mid point is $\left(1, \frac{2}{5}\right)$, is equal to :
(1) $\frac{\sqrt{1691}}{5}$
(2) $\frac{\sqrt{2009}}{5}$
(3) $\frac{\sqrt{1741}}{5}$
(4) $\frac{\sqrt{1541}}{5}$

Ans. (1)
16. The portion of the line $4 x+5 y=20$ in the first quadrant is trisected by the lines $L_{1}$ and $L_{2}$ passing through the origin. The tangent of an angle between the lines $L_{1}$ and $L_{2}$ is :
(1) $\frac{8}{5}$
(2) $\frac{25}{41}$
(3) $\frac{2}{5}$
(4) $\frac{30}{41}$

Ans. (4)
17. Let $\overrightarrow{\mathrm{a}}=\hat{\mathrm{i}}+2 \hat{\mathrm{j}}+\mathrm{k}, \overrightarrow{\mathrm{b}}=3(\hat{\mathrm{i}}-\hat{\mathrm{j}}+\mathrm{k})$. Let $\overrightarrow{\mathrm{c}}$ be the vector such that $\vec{a} \times \vec{c}=\vec{b}$ and $\vec{a} \cdot \vec{c}=3$. Then $\overrightarrow{\mathrm{a}} \cdot((\overrightarrow{\mathrm{c}} \times \overrightarrow{\mathrm{b}})-\overrightarrow{\mathrm{b}}-\overrightarrow{\mathrm{c}})$ is equal to :
(1) 32
(2) 24
(3) 20
(4) 36

Ans. (2)
18. If $\mathrm{a}=\lim _{\mathrm{x} \rightarrow 0} \frac{\sqrt{1+\sqrt{1+\mathrm{x}^{4}}}-\sqrt{2}}{\mathrm{x}^{4}}$ and $b=\lim _{x \rightarrow 0} \frac{\sin ^{2} x}{\sqrt{2}-\sqrt{1+\cos x}}$, then the value of $a b^{3}$ is :
(1) 36
(2) 32
(3) 25
(4) 30

Ans. (2)
19. Consider the matrix $f(x)=\left[\begin{array}{ccc}\cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1\end{array}\right]$.

Given below are two statements :
Statement I: $f(-x)$ is the inverse of the matrix $f(x)$.
Statement II: $f(x) f(y)=f(x+y)$.
In the light of the above statements, choose the correct answer from the options given below
(1) Statement I is false but Statement II is true
(2) Both Statement I and Statement II are false
(3) Statement I is true but Statement II is false
(4) Both Statement I and Statement II are true

Ans. (4)
20. The function $\mathrm{f}: \mathrm{N}-\{1\} \rightarrow \mathrm{N}$; defined by $\mathrm{f}(\mathrm{n})=$ the highest prime factor of $n$, is :
(1) both one-one and onto
(2) one-one only
(3) onto only
(4) neither one-one nor onto

Ans. (4)

## SECTION-B

21. The least positive integral value of $\alpha$, for which the angle between the vectors $\alpha \hat{i}-2 \hat{j}+2 k$ and $\alpha \hat{i}+2 \alpha \hat{j}-2 k$ is acute, is $\qquad$ .

Ans. (5)
22. Let for a differentiable function $f:(0, \infty) \rightarrow R$, $f(x)-f(y) \geq \log _{e}\left(\frac{x}{y}\right)+x-y, \forall x, y \in(0, \infty)$. Then $\sum_{\mathrm{n}=1}^{20} \mathrm{f}^{\prime}\left(\frac{1}{\mathrm{n}^{2}}\right)$ is equal to $\qquad$ .

Ans. (2890)
23. If the solution of the differential equation
$(2 x+3 y-2) d x+(4 x+6 y-7) d y=0, y(0)=3$, is $\alpha x+\beta y+3 \log _{e}|2 x+3 y-\gamma|=6$, then $\alpha+2 \beta+3 \gamma$ is equal to $\qquad$ .

Ans. (29)
24. Let the area of the region $\{(x, y): x-2 y+4 \geq 0$, $\left.x+2 y^{2} \geq 0, x+4 y^{2} \leq 8, y \geq 0\right\}$ be $\frac{m}{n}$, where $m$ and n are coprime numbers. Then $\mathrm{m}+\mathrm{n}$ is equal to
$\qquad$ .

Ans. (119)
25. If
$8=3+\frac{1}{4}(3+p)+\frac{1}{4^{2}}(3+2 p)+\frac{1}{4^{3}}(3+3 p)+\ldots \infty$, then the value of $p$ is $\qquad$ .

Ans. (9)
26. A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required and let $\mathrm{a}=\mathrm{P}(\mathrm{X}=3), \mathrm{b}=\mathrm{P}(\mathrm{X} \geq 3)$ and $\mathrm{c}=$ $P(X \geq 6 \mid X>3)$. Then $\frac{b+c}{a}$ is equal to $\qquad$ $-$

Ans. (12)
27. Let the set of all $a \in R$ such that the equation $\cos 2 x+a \sin x=2 a-7$ has a solution be $[p, q]$ and $r=\tan 9^{\circ}-\tan 27^{\circ}-\frac{1}{\cot 63^{\circ}}+\tan 81^{\circ}$, then pqr is equal to $\qquad$ .

Ans. (48)
28. Let $f(x)=x^{3}+x^{2} f^{\prime}(1)+x f "(2)+f^{\prime \prime \prime}(3), x \in R$. Then $\mathrm{f}^{\prime}(10)$ is equal to $\qquad$ .

Ans. (202)
29. Let $\mathrm{A}=\left\lfloor\begin{array}{lll}2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1\end{array}\right\rfloor, \mathrm{B}=\left[\mathrm{B}_{1}, \mathrm{~B}_{2}, \mathrm{~B}_{3}\right]$, where $\mathrm{B}_{1}$,
$\mathrm{B}_{2}, \mathrm{~B}_{3}$ are column matrices, and $\mathrm{AB}_{1}=\left\lfloor\begin{array}{l}1 \\ 0 \\ 0\end{array}\right\rfloor$,
$\mathrm{AB}_{2}=\left\lfloor\begin{array}{l}2 \\ 3 \\ 0\end{array}\right\rfloor, \mathrm{AB}_{3}=\left\lfloor\begin{array}{l}3 \\ 2 \\ 1\end{array}\right\rfloor$
If $\alpha=|\mathrm{B}|$ and $\beta$ is the sum of all the diagonal elements of $B$, then $\alpha^{3}+\beta^{3}$ is equal to $\qquad$ .

Ans. (28)
30. If $\alpha$ satisfies the equation $x^{2}+x+1=0$ and $(1+\alpha)^{7}=\mathrm{A}+\mathrm{B} \alpha+\mathrm{C} \alpha^{2}, \mathrm{~A}, \mathrm{~B}, \mathrm{C} \geq 0$, then $5(3 \mathrm{~A}-2 \mathrm{~B}-\mathrm{C})$ is equal to $\qquad$ .

Ans. (5)

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