| CAREER INSTITUTE |
| :--- |
| KOTA (RAJASTHAN] |

- 


## FINAL JEE-MAIN EXAMINATION - JANUARY, 2024

(Held On Saturday 27 ${ }^{\text {th }}$ January, 2024)
TIME: 3:00 PM to 6:00 PM

## MATHEMATICS

## SECTION-A

1. Considering only the principal values of inverse trigonometric functions, the number of positive real values of $x$ satisfying $\tan ^{-1}(x)+\tan ^{-1}(2 x)=\frac{\pi}{4}$ is :
(1) More than 2
(2) 1
(3) 2
(4) 0

Ans. (2)
2. Consider the function $\mathrm{f}:(0,2) \rightarrow \mathrm{R}$ defined by $f(x)=\frac{x}{2}+\frac{2}{x}$ and the function $g(x)$ defined by $\mathrm{g}(\mathrm{x})=\left\{\begin{array}{cc}\min \{\mathrm{f}(\mathrm{t})\}, & 0<\mathrm{t} \leq \mathrm{x} \text { and } 0<\mathrm{x} \leq 1 \\ \frac{3}{2}+\mathrm{x}, & 1<\mathrm{x}<2\end{array}\right.$. Then
(1) g is continuous but not differentiable at $\mathrm{x}=1$
(2) $g$ is not continuous for all $x \in(0,2)$
(3) $g$ is neither continuous nor differentiable at $x=1$
(4) g is continuous and differentiable for all $\mathrm{x} \in(0,2)$

Ans. (1)
3. Let the image of the point $(1,0,7)$ in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ be the point $(\alpha, \beta, \gamma)$. Then which one of the following points lies on the line passing through $(\alpha, \beta, \gamma)$ and making angles $\frac{2 \pi}{3}$ and $\frac{3 \pi}{4}$ with $y$-axis and $z$-axis respectively and an acute angle with x -axis ?
(1) $(1,-2,1+\sqrt{2})$
(2) $(1,2,1-\sqrt{2})$
(3) $(3,4,3-2 \sqrt{2})$
(4) $(3,-4,3+2 \sqrt{2})$

Ans. (3)

## TEST PAPER WITH ANSWER

4. Let R be the interior region between the lines $3 x-y+1=0$ and $x+2 y-5=0$ containing the origin. The set of all values of $a$, for which the points $\left(a^{2}, a+1\right)$ lie in $R$, is :
(1) $(-3,-1) \cup\left(-\frac{1}{3}, 1\right)$
(2) $(-3,0) \cup\left(\frac{1}{3}, 1\right)$
(3) $(-3,0) \cup\left(\frac{2}{3}, 1\right)$
(4) $(-3,-1) \cup\left(\frac{1}{3}, 1\right)$

Ans. (2)
5. The $20^{\text {th }}$ term from the end of the progression $20,19 \frac{1}{4}, 18 \frac{1}{2}, 17 \frac{3}{4}, \ldots .,-129 \frac{1}{4}$ is :-
(1) -118
(2) -110
(3) -115
(4) -100

Ans. (3)
6. Let $f: R-\left\{\frac{-1}{2}\right\} \rightarrow R$ and $g: R-\left\{\frac{-5}{2}\right\} \rightarrow R \quad$ be defined as $\mathrm{f}(\mathrm{x})=\frac{2 \mathrm{x}+3}{2 \mathrm{x}+1}$ and $\mathrm{g}(\mathrm{x})=\frac{|\mathrm{x}|+1}{2 \mathrm{x}+5}$. Then the domain of the function fog is :
(1) $\mathrm{R}-\left\{-\frac{5}{2}\right\}$
(2) R
(3) $R-\left\{-\frac{7}{4}\right\}$
(4) $\mathrm{R}-\left\{-\frac{5}{2},-\frac{7}{4}\right\}$

Ans. (1)
7. For $0<a<1$, the value of the integral $\int_{0}^{\pi} \frac{d x}{1-2 a \cos x+a^{2}}$ is :
(1) $\frac{\pi^{2}}{\pi+a^{2}}$
(2) $\frac{\pi^{2}}{\pi-a^{2}}$
(3) $\frac{\pi}{1-a^{2}}$
(4) $\frac{\pi}{1+a^{2}}$

Ans. (3)
8. Let $g(x)=3 f\left(\frac{x}{3}\right)+f(3-x)$ and $f^{\prime \prime}(x)>0$ for all $x \in(0,3)$. If $g$ is decreasing in $(0, \alpha)$ and increasing in $(\alpha, 3)$, then $8 \alpha$ is
(1) 24
(2) 0
(3) 18
(4) 20

Ans. (3)
9. If $\lim _{x \rightarrow 0} \frac{3+\alpha \sin x+\beta \cos x+\log _{e}(1-x)}{3 \tan ^{2} x}=\frac{1}{3}$, then $2 \alpha-\beta$ is equal to :
(1) 2
(2) 7
(3) 5
(4) 1

Ans. (3)
10. If $\alpha, \beta$ are the roots of the equation, $x^{2}-x-1=0$ and $S_{n}=2023 \alpha^{n}+2024 \beta^{n}$, then
(1) $2 \mathrm{~S}_{12}=\mathrm{S}_{11}+\mathrm{S}_{10}$
(2) $S_{12}=S_{11}+S_{10}$
(3) $2 \mathrm{~S}_{11}=\mathrm{S}_{12}+\mathrm{S}_{10}$
(4) $S_{11}=S_{10}+S_{12}$

Ans. (2)
11. Let $A$ and $B$ be two finite sets with $m$ and $n$ elements respectively. The total number of subsets of the set $A$ is 56 more than the total number of subsets of $B$. Then the distance of the point $\mathrm{P}(\mathrm{m}, \mathrm{n})$ from the point $\mathrm{Q}(-2,-3)$ is
(1) 10
(2) 6
(3) 4
(4) 8

Ans. (1)
12. The values of $\alpha$, for which
$\left|\begin{array}{ccc}1 & \frac{3}{2} & \alpha+\frac{3}{2} \\ 1 & \frac{1}{3} & \alpha+\frac{1}{3} \\ 2 \alpha+3 & 3 \alpha+1 & 0\end{array}\right|=0$, lie in the interval
(1) $(-2,1)$
(2) $(-3,0)$
(3) $\left(-\frac{3}{2}, \frac{3}{2}\right)$
(4) $(0,3)$

Ans. (2)
13. An urn contains 6 white and 9 black balls. Two successive draws of 4 balls are made without replacement. The probability, that the first draw gives all white balls and the second draw gives all black balls, is :
(1) $\frac{5}{256}$
(2) $\frac{5}{715}$
(3) $\frac{3}{715}$
(4) $\frac{3}{256}$

Ans. (3)
14. The integral

$$
\int \frac{\left(x^{8}-x^{2}\right) d x}{\left(x^{12}+3 x^{6}+1\right) \tan ^{-1}\left(x^{3}+\frac{1}{x^{3}}\right)} \text { is }
$$

equal to :
(1) $\log _{e}\left(\left|\tan ^{-1}\left(x^{3}+\frac{1}{x^{3}}\right)\right|\right)^{1 / 3}+C$
(2) $\log _{e}\left(\left|\tan ^{-1}\left(x^{3}+\frac{1}{x^{3}}\right)\right|\right)^{1 / 2}+C$
(3) $\log _{e}\left(\left|\tan ^{-1}\left(x^{3}+\frac{1}{x^{3}}\right)\right|\right)+C$
(4) $\log _{e}\left(\left|\tan ^{-1}\left(x^{3}+\frac{1}{x^{3}}\right)\right|\right)^{3}+C$

Ans. (1)
15. If $2 \tan ^{2} \theta-5 \sec \theta=1$ has exactly 7 solutions in the interval $\left[0, \frac{n \pi}{2}\right]$, for the least value of $n \in N$ then $\sum_{\mathrm{k}=1}^{\mathrm{n}} \frac{\mathrm{k}}{2^{\mathrm{k}}}$ is equal to :
(1) $\frac{1}{2^{15}}\left(2^{14}-14\right)$
(2) $\frac{1}{2^{14}}\left(2^{15}-15\right)$
(3) $1-\frac{15}{2^{13}}$
(4) $\frac{1}{2^{13}}\left(2^{14}-15\right)$

## Ans. (4)

16. The position vectors of the vertices $A, B$ and $C$ of a triangle are $2 \hat{i}-3 \hat{j}+3 \hat{k}, \quad 2 \hat{i}+2 \hat{j}+3 \hat{k}$ and $-\hat{i}+\hat{j}+3 \hat{k}$ respectively. Let $l$ denotes the length of the angle bisector AD of $\angle \mathrm{BAC}$ where D is on the line segment BC , then $2 l^{2}$ equals :
(1) 49
(2) 42
(3) 50
(4) 45

Ans. (4)
17. If $y=y(x)$ is the solution curve of the differential equation $\left(x^{2}-4\right) d y-\left(y^{2}-3 y\right) d x=0$,
$x>2, y(4)=\frac{3}{2}$ and the slope of the curve is never zero, then the value of $y(10)$ equals :
(1) $\frac{3}{1+(8)^{1 / 4}}$
(2) $\frac{3}{1+2 \sqrt{2}}$
(3) $\frac{3}{1-2 \sqrt{2}}$
(4) $\frac{3}{1-(8)^{1 / 4}}$

Ans. (1)
18. Let $e_{1}$ be the eccentricity of the hyperbola $\frac{x^{2}}{16}-\frac{y^{2}}{9}=1$ and $e_{2}$ be the eccentricity of the ellipse $\frac{\mathrm{x}^{2}}{\mathrm{a}^{2}}+\frac{\mathrm{y}^{2}}{\mathrm{~b}^{2}}=1, \mathrm{a}>\mathrm{b}$, which passes through the foci of the hyperbola. If $\mathrm{e}_{1} \mathrm{e}_{2}=1$, then the length of the chord of the ellipse parallel to the $x$-axis and passing through $(0,2)$ is :
(1) $4 \sqrt{5}$
(2) $\frac{8 \sqrt{5}}{3}$
(3) $\frac{10 \sqrt{5}}{3}$
(4) $3 \sqrt{5}$

Ans. (3)
19. Let $\alpha=\frac{(4!)!}{(4!)^{3!}}$ and $\beta=\frac{(5!)!}{(5!)^{4!}}$. Then :
(1) $\alpha \in \mathrm{N}$ and $\beta \notin \mathrm{N}$
(2) $\alpha \notin \mathrm{N}$ and $\beta \in \mathrm{N}$
(3) $\alpha \in \mathrm{N}$ and $\beta \in \mathrm{N}$
(4) $\alpha \notin \mathrm{N}$ and $\beta \notin \mathrm{N}$

Ans. (3)
20. Let the position vectors of the vertices $A, B$ and $C$ of a triangle be $2 \hat{i}+2 \hat{j}+\hat{k}, \quad \hat{i}+2 \hat{j}+2 \hat{k}$ and $2 \hat{i}+\hat{\mathrm{j}}+2 \hat{\mathrm{k}}$ respectively. Let $l_{1}, l_{2}$ and $l_{3}$ be the lengths of perpendiculars drawn from the ortho center of the triangle on the sides $\mathrm{AB}, \mathrm{BC}$ and CA respectively, then $l_{1}^{2}+l_{2}^{2}+l_{3}^{2}$ equals :
(1) $\frac{1}{5}$
(2) $\frac{1}{2}$
(3) $\frac{1}{4}$
(4) $\frac{1}{3}$

Ans. (2)

## SECTION-B

21. The mean and standard deviation of 15 observations were found to be 12 and 3 respectively. On rechecking it was found that an observation was read as 10 in place of 12 . If $\mu$ and $\sigma^{2}$ denote the mean and variance of the correct observations respectively, then $15\left(\mu+\mu^{2}+\sigma^{2}\right)$ is equal to $\qquad$
Ans. (2521)
22. If the area of the region $\left\{(x, y): 0 \leq y \leq \min \left\{2 x, 6 x-x^{2}\right\}\right\}$ is $A$, then $12 A$ is equal to $\qquad$
Ans. (304)
23. Let $A$ be a $2 \times 2$ real matrix and I be the identity matrix of order 2. If the roots of the equation $|A-x I|=0$ be -1 and 3 , then the sum of the diagonal elements of the matrix $A^{2}$ is $\qquad$
Ans. (10)
24. If the sum of squares of all real values of $\alpha$, for which the lines $2 x-y+3=0,6 x+3 y+1=0$ and $\alpha x+2 y-2=0$ do not form a triangle is $p$, then the greatest integer less than or equal to p is $\qquad$
Ans. (32)
25. The coefficient of $x^{2012}$ in the expansion of $(1-x)^{2008}\left(1+x+x^{2}\right)^{2007}$ is equal to

Ans. (0)
26. If the solution curve, of the differential equation $\frac{d y}{d x}=\frac{x+y-2}{x-y}$ passing through the point $(2,1)$ is $\tan ^{-1}\left(\frac{y-1}{x-1}\right)-\frac{1}{\beta} \log _{e}\left(\alpha+\left(\frac{y-1}{x-1}\right)^{2}\right)=\log _{e}|x-1|$, then $5 \beta+\alpha$ is equal to

Ans. (11)
27. Let $f(x)=\int_{0}^{x} g(t) \log _{e}\left(\frac{1-t}{1+t}\right) d t$, where $g$ is a continuous odd function. If $\int_{-\pi / 2}^{\pi / 2}\left(f(x)+\frac{x^{2} \cos x}{1+e^{x}}\right) d x=\left(\frac{\pi}{\alpha}\right)^{2}-\alpha$, then $\alpha$ is equal to. $\qquad$
Ans. (2)
28. Consider a circle $(x-\alpha)^{2}+(y-\beta)^{2}=50$, where $\alpha, \beta>0$. If the circle touches the line $y+x=0$ at the point $P$, whose distance from the origin is $4 \sqrt{2}$ , then $(\alpha+\beta)^{2}$ is equal to $\qquad$

Ans. (100) for Class 10 ${ }^{\text {th }}$ | NEET | JEE
29. The lines $\frac{x-2}{2}=\frac{y}{-2}=\frac{z-7}{16}$ and
$\frac{x+3}{4}=\frac{y+2}{3}=\frac{z+2}{1}$ intersect at the point $P$. If the distance of P from the line $\frac{\mathrm{x}+1}{2}=\frac{\mathrm{y}-1}{3}=\frac{\mathrm{z}-1}{1}$ is $l$, then $14 l^{2}$ is equal to $\qquad$
Ans. (108)
30. Let the complex numbers $\alpha$ and $\frac{1}{\bar{\alpha}}$ lie on the circles $\left|\mathrm{z}-\mathrm{z}_{0}\right|^{2}=4$ and $\left|\mathrm{z}-\mathrm{z}_{0}\right|^{2}=16$ respectively, where $\mathrm{z}_{0}=1+\mathrm{i}$. Then, the value of $100|\alpha|^{2}$ is. $\qquad$
Ans. (20)

# ALIEX AIPOWERED APP 

$\square$

## SCALE UP YOUR SCORE!

 with ALLEN SCORE TEST PAPERS
## 管筌 <br> Total 10 Full syllabus papers



Paper Analysis of JEE Advanced 2023

8By A놎N Subject Experts
(1) Answer key with Solutions

## Scan QR to Buy

## ALLEX: <br> SCORE <br> TEST PAPERS with SOLUTIONS

Key Features:
P Full Syllabus Papers
Including Answer key
JEE (Adv.) 2023 Paper Analysis
Prepared by ALLEN Expert Faculties


