KOTA (RAJASTHAN)
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## FINAL JEE-MAIN EXAMINATION - JANUARY, 2024

(Held On Thursday 01st February, 2024)
TIME:9:00 AM to 12:00 NOON

## MATHEMATICS

## SECTION-A

1. A bag contains 8 balls, whose colours are either white or black. 4 balls are drawn at random without replacement and it was found that 2 balls are white and other 2 balls are black. The probability that the bag contains equal number of white and black balls is:
(1) $\frac{2}{5}$
(2) $\frac{2}{7}$
(3) $\frac{1}{7}$
(4) $\frac{1}{5}$

Ans. (2)
2. The value of the integral $\int_{0}^{\frac{\pi}{4}} \frac{x d x}{\sin ^{4}(2 x)+\cos ^{4}(2 x)}$ equals:
(1) $\frac{\sqrt{2} \pi^{2}}{8}$
(2) $\frac{\sqrt{2} \pi^{2}}{16}$
(3) $\frac{\sqrt{2} \pi^{2}}{32}$
(4) $\frac{\sqrt{2} \pi^{2}}{64}$

Ans. (3)
3. If $\mathrm{A}=\left[\begin{array}{cc}\sqrt{2} & 1 \\ -1 & \sqrt{2}\end{array}\right], \mathrm{B}=\left[\begin{array}{ll}1 & 0 \\ 1 & 1\end{array}\right], \mathrm{C}=\mathrm{ABA}^{\mathrm{T}}$ and X
$=A^{T} C^{2} A$, then $\operatorname{det} X$ is equal to :
(1) 243
(2) 729
(3) 27
(4) 891

Ans. (2)

## TEST PAPER WITH ANSWER

4. If $\tan \mathrm{A}=\frac{1}{\sqrt{x\left(x^{2}+x+1\right)}}, \tan B=\frac{\sqrt{x}}{\sqrt{x^{2}+x+1}}$
and
$\tan C=\left(x^{-3}+x^{-2}+x^{-1}\right)^{\frac{1}{2}}, 0<A, B, C<\frac{\pi}{2}$, then
$\mathrm{A}+\mathrm{B}$ is equal to :
(1) C
(2) $\pi-C$
(3) $2 \pi-C$
(4) $\frac{\pi}{2}-C$

Ans. (1)
5. If n is the number of ways five different employees can sit into four indistinguishable offices where any office may have any number of persons including zero, then n is equal to:
(1) 47
(2) 53
(3) 51
(4) 43

Ans. (3)
6. Let $S=\{z \in C:|z-1|=1$ and
$(\sqrt{2}-1)(z+\bar{z})-i(z-\bar{z})=2 \sqrt{2}\}$. Let $\mathrm{z}_{1}, \quad \mathrm{z}_{2}$ $\in S$ be such that $\left|z_{1}\right|=\max _{z \in S}|z|$ and $\left|z_{2}\right|=\min _{z \in S}|z|$.

Then $\left|\sqrt{2} z_{1}-z_{2}\right|^{2}$ equals :
(1) 1
(2) 4
(3) 3
(4) 2

Ans. (4)
7. Let the median and the mean deviation about the median of 7 observation $170,125,230,190,210$, a, b be 170 and $\frac{205}{7}$ respectively. Then the mean deviation about the mean of these 7 observations is :
(1) 31
(2) 28
(3) 30
(4) 32

Ans. (3)
8. Let $\vec{a}=-5 \hat{i}+\hat{j}-3 \hat{k}, \vec{b}=\hat{i}+2 \hat{j}-4 \hat{k}$ and $\vec{c}=(((\vec{a} \times \vec{b}) \times \hat{i}) \times \hat{i}) \times \hat{i}$. Then $\vec{c} \cdot(-\hat{i}+\hat{j}+\hat{k})$ is equal to
(1) -12
(2) -10
(3) -13
(4) -15

Ans. (1)
9. Let $S=\left\{x \in R:(\sqrt{3}+\sqrt{2})^{x}+(\sqrt{3}-\sqrt{2})^{x}=10\right\}$. Then the number of elements in S is :
(1) 4
(2) 0
(3) 2
(4) 1

Ans. (3)
10. The area enclosed by the curves $x y+4 y=16$ and $x+y=6$ is equal to :
(1) $28-30 \log _{e} 2$
(2) $30-28 \log _{e} 2$
(3) $30-32 \log _{e} 2$
(4) $32-30 \log _{e} 2$

Ans. (3)
11. Let $\mathrm{f}: \mathbf{R} \rightarrow \mathbf{R}$ and $\mathrm{g}: \mathbf{R} \rightarrow \mathbf{R}$ be defined as $f(x)=\left\{\begin{array}{cll}\log _{e} x & , & x>0 \\ e^{-x} & , & x \leq 0\end{array}\right.$ and
$g(x)=\left\{\begin{array}{cl}x & , \\ e^{x} \geq 0 \\ e^{x} & , \quad x<0\end{array}\right.$. Then, gof $: \mathbf{R} \rightarrow \mathbf{R}$ is :
(1) one-one but not onto
(2) neither one-one nor onto
(3) onto but not one-one
(4) both one-one and onto

Ans. (2)
12. If the system of equations
$2 x+3 y-z=5$
$x+\alpha y+3 z=-4$
$3 x-y+\beta z=7$
has infinitely many solutions, then $13 \alpha \beta$ is equal to
(1) 1110
(2) 1120
(3) 1210
(4) 1220

Ans. (2)
13. For $0<\theta<\pi / 2$, if the eccentricity of the hyperbola $x^{2}-y^{2} \operatorname{cosec}^{2} \theta=5$ is $\sqrt{7}$ times eccentricity of the ellipse $x^{2} \operatorname{cosec}^{2} \theta+y^{2}=5$, then the value of $\theta$ is :
(1) $\frac{\pi}{6}$
(2) $\frac{5 \pi}{12}$
(3) $\frac{\pi}{3}$
(4) $\frac{\pi}{4}$

Ans. (3)
14. Let $y=y(x)$ be the solution of the differential equation $\frac{d y}{d x}=2 x(x+y)^{3}-x(x+y)-1, y(0)=1$.

Then, $\left(\frac{1}{\sqrt{2}}+y\left(\frac{1}{\sqrt{2}}\right)\right)^{2}$ equals :
(1) $\frac{4}{4+\sqrt{\mathrm{e}}}$
(2) $\frac{3}{3-\sqrt{e}}$
(3) $\frac{2}{1+\sqrt{\mathrm{e}}}$
(4) $\frac{1}{2-\sqrt{\mathrm{e}}}$

Ans. (4)
15. Let $\mathrm{f}: \mathrm{R} \rightarrow \mathrm{R}$ be defined as $f(x)=\left\{\begin{array}{ccc}\frac{a-b \cos 2 x}{x^{2}} & ; & x<0 \\ x^{2}+c x+2 & ; & 0 \leq x \leq 1 \\ 2 x+1 & ; & x>1\end{array}\right.$

If $f$ is continuous everywhere in $\mathbf{R}$ and m is the number of points where $f$ is NOT differential then $\mathrm{m}+\mathrm{a}+\mathrm{b}+\mathrm{c}$ equals :
(1) 1
(2) 4
(3) 3
(4) 2

Ans. (4)
16. Let $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1, a>b$ be an ellipse, whose eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latus rectum is $\sqrt{14}$. Then the square of the eccentricity of $\frac{x^{2}}{a^{2}}-\frac{y^{2}}{b^{2}}=1$ is :
(1) 3
(2) $7 / 2$
(3) $3 / 2$
(4) $5 / 2$

Ans. (3)
17. Let $3, a, b, c$ be in A.P. and $3, a-1, b+1, c+9$ be in G.P. Then, the arithmetic mean of $a, b$ and $c$ is :
(1) -4
(2) -1
(3) 13
(4) 11

Ans. (4)
18. Let $C: x^{2}+y^{2}=4$ and $C^{\prime}: x^{2}+y^{2}-4 \lambda x+9=0$ be two circles. If the set of all values of $\lambda$ so that the circles C and $\mathrm{C}^{\prime}$ intersect at two distinct points, is $\mathbf{R}-[a, b]$, then the point $(8 a+12,16 b-20)$ lies on the curve :
(1) $x^{2}+2 y^{2}-5 x+6 y=3$
(2) $5 x^{2}-y=-11$
(3) $x^{2}-4 y^{2}=7$
(4) $6 x^{2}+y^{2}=42$

Ans. (4)
19. If $5 f(x)+4 f\left(\frac{1}{x}\right)=x^{2}-2, \forall x \neq 0$ and $y=9 x^{2} f(x)$, then y is strictly increasing in :
(1) $\left(0, \frac{1}{\sqrt{5}}\right) \cup\left(\frac{1}{\sqrt{5}}, \infty\right)$
(2) $\left(-\frac{1}{\sqrt{5}}, 0\right) \cup\left(\frac{1}{\sqrt{5}}, \infty\right)$
(3) $\left(-\frac{1}{\sqrt{5}}, 0\right) \cup\left(0, \frac{1}{\sqrt{5}}\right)$
(4) $\left(-\infty, \frac{1}{\sqrt{5}}\right) \cup\left(0, \frac{1}{\sqrt{5}}\right)$

Ans. (2)
20. If the shortest distance between the lines $\frac{\mathrm{x}-\lambda}{-2}=\frac{\mathrm{y}-2}{1}=\frac{\mathrm{z}-1}{1}$ and $\frac{x-\sqrt{3}}{1}=\frac{y-1}{-2}=\frac{z-2}{1}$
is 1 , then the sum of all possible values of $\lambda$ is :
(1) 0
(2) $2 \sqrt{3}$
(3) $3 \sqrt{3}$
(4) $-2 \sqrt{3}$

Ans. (2)

## SECTION-B

21. If $x=x(t)$ is the solution of the differential equation $(t+1) d x=\left(2 x+(t+1)^{4}\right) d t, x(0)=2$, then, $x(1)$ equals $\qquad$ .

Ans. (14)
22. The number of elements in the set
$S=\{(x, y, z): x, y, z \in \mathbf{Z}, x+2 y+3 z=42, x, y, z$ $\geq 0\}$ equals $\qquad$ .
Ans. (169)
23. If the Coefficient of $x^{30}$ in the expansion of $\left(1+\frac{1}{x}\right)^{6}\left(1+x^{2}\right)^{7}\left(1-x^{3}\right)^{8} ; x \neq 0$ is $\alpha$, then $|\alpha|$ equals $\qquad$ .

Ans. (678)
24. Let $3,7,11,15, \ldots ., 403$ and $2,5,8,11, \ldots, 404$ be two arithmetic progressions. Then the sum, of the common terms in them, is equal to $\qquad$ .

Ans. (6699)
25. Let $\{x\}$ denote the fractional part of $x$ and $f(x)=\frac{\cos ^{-1}\left(1-\{x\}^{2}\right) \sin ^{-1}(1-\{x\})}{\{x\}-\{x\}^{3}}, x \neq 0$. If $L$ and R respectively denotes the left hand limit and the right hand limit of $f(x)$ at $x=0$, then $\frac{32}{\pi^{2}}\left(L^{2}+R^{2}\right)$ is equal to $\qquad$ .
Ans. (18)
26. Let the line $\mathrm{L}: \sqrt{2} \mathrm{x}+\mathrm{y}=\alpha$ pass through the point of the intersection P (in the first quadrant) of the circle $x^{2}+y^{2}=3$ and the parabola $x^{2}=2 y$. Let the line $L$ touch two circles $C_{1}$ and $C_{2}$ of equal radius $2 \sqrt{3}$. If the centres $Q_{1}$ and $Q_{2}$ of the circles $C_{1}$ and $C_{2}$ lie on the $y$-axis, then the square of the area of the triangle $\mathrm{PQ}_{1} \mathrm{Q}_{2}$ is equal to $\qquad$ .

Ans. (72)
27. Let $P=\{z \in \mathbb{C}:|z+2-3 i| \leq 1\}$ and $\mathrm{Q}=\{\mathrm{z} \in \mathbb{C}: \mathrm{z}(1+\mathrm{i})+\overline{\mathrm{z}}(1-\mathrm{i}) \leq-8\}$. Let in $\mathrm{P} \cap \mathrm{Q},|\mathrm{z}-3+2 \mathrm{i}|$ be maximum and minimum at $z_{1}$ and $z_{2}$ respectively. If $\left|z_{1}\right|^{2}+2|z|^{2}=\alpha+\beta \sqrt{2}$, where $\alpha, \beta$ are integers, then $\alpha+\beta$ equals
$\qquad$ -.
Ans. (36)
28. If $\int_{-\pi / 2}^{\pi / 2} \frac{8 \sqrt{2} \cos x d x}{\left(1+e^{\sin x}\right)\left(1+\sin ^{4} x\right)}=\alpha \pi+\beta \log _{e}(3+2$ $\sqrt{2}$ ), where $\alpha, \beta$ are integers, then $\alpha^{2}+\beta^{2}$ equals
$\qquad$ -.

Ans. (8)

29. Let the line of the shortest distance between the lines
$L_{1}: \overrightarrow{\mathrm{r}}=(\hat{\mathrm{i}}+2 \hat{\mathrm{j}}+3 \hat{\mathrm{k}})+\lambda(\hat{\mathrm{i}}-\hat{\mathrm{j}}+\hat{\mathrm{k}})$ and
$L_{2}: \overrightarrow{\mathrm{r}}=(4 \hat{\mathrm{i}}+5 \hat{\mathrm{j}}+6 \hat{\mathrm{k}})+\mu(\hat{\mathrm{i}}+\hat{\mathrm{j}}-\hat{\mathrm{k}})$
intersect $L_{1}$ and $L_{2}$ at $P$ and $Q$ respectively. If $(\alpha, \beta, \gamma)$ is the mid point of the line segment PQ , then $2(\alpha+\beta+\gamma)$ is equal to $\qquad$ .
Ans. (21)
30. Let $A=\{1,2,3, \ldots 20\}$. Let $R_{1}$ and $R_{2}$ two relation on A such that
$R_{1}=\{(a, b): b$ is divisible by $a\}$
$R_{2}=\{(a, b): a$ is an integral multiple of $b\}$.
Then, number of elements in $R_{1}-R_{2}$ is equal to $\qquad$ .
Ans. (46)

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