# FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

**(Held On Thursday 01st February, 2024)** **TIME : 3 : 00 PM to 6 : 00 PM**

## MATHEMATICS

## TEST PAPER WITH ANSWER

### SECTION-A

**1.** Let $f(x) = |2x^2+5|x|-3|$, $x \in R$. If m and n denote the number of points where $f$ is not continuous and not differentiable respectively, then m + n is equal to :

(1) 5 (2) 2

(3) 0 (4) 3

**Ans. (4)**

**2.** Let $\alpha$ and $\beta$ be the roots of the equation $px^2 + qx - r = 0$, where $p \neq 0$. If p, q and r be the consecutive terms of a non-constant G.P and $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{3}{4}$, then the value of $(\alpha - \beta)^2$ is :

(1) $\frac{80}{9}$ (2) 9

(3) $\frac{20}{3}$ (4) 8

**Ans. (1)**

**3.** The number of solutions of the equation $4\sin^2 x - 4\cos^3 x + 9 - 4\cos x = 0$; $x \in [-2\pi, 2\pi]$ is :

(1) 1 (2) 3

(3) 2 (4) 0

**Ans. (4)**

**4.** The value of $\int_0^1 (2x^3 - 3x^2 - x + 1)^{\frac{1}{3}}\,dx$ is equal to:

(1) 0 (2) 1

(3) 2 (4) –1

**Ans. (1)**

**5.** Let P be a point on the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$. Let the line passing through P and parallel to y-axis meet the circle $x^2 + y^2 = 9$ at point Q such that P and Q are on the same side of the x-axis. Then, the eccentricity of the locus of the point R on PQ such that PR : RQ = 4 : 3 as P moves on the ellipse, is :

(1) $\frac{11}{19}$ (2) $\frac{13}{21}$

(3) $\frac{\sqrt{139}}{23}$ (4) $\frac{\sqrt{13}}{7}$

**Ans. (4)**

**6.** Let m and n be the coefficients of seventh and thirteenth terms respectively in the expansion of $\left(\frac{1}{3}x^{\frac{1}{3}}+\frac{1}{2x^{\frac{2}{3}}}\right)^{18}$. Then $\left(\frac{n}{m}\right)^{\frac{1}{3}}$ is :

(1) $\frac{4}{9}$ (2) $\frac{1}{9}$

(3) $\frac{1}{4}$ (4) $\frac{9}{4}$

**Ans. (4)**

**7.** Let $\alpha$ be a non-zero real number. Suppose $f : R \to R$ is a differentiable function such that $f(0) = 2$ and $\lim_{x\to-\infty} f(x) = 1$. If $f'(x) = \alpha f(x) + 3$, for all $x \in R$, then $f(-\log_e 2)$ is equal to____.

(1) 3 (2) 5

(3) 9 (4) 7

**Ans. (Bonus)**

**8.** Let P and Q be the points on the line $\frac{x+3}{8}=\frac{y-4}{2}=\frac{z+1}{2}$ which are at a distance of 6 units from the point R (1,2,3). If the centroid of the triangle PQR is $(\alpha, \beta, \gamma)$, then $\alpha^2+\beta^2+\gamma^2$ is:

(1) 26 (2) 36

(3) 18 (4) 24

**Ans. (3)**

**9.** Consider a $\Delta ABC$ where A(1,3,2), B(–2,8,0) and C(3,6,7). If the angle bisector of $\angle BAC$ meets the line BC at D, then the length of the projection of the vector $\vec{AD}$ on the vector $\vec{AC}$ is:

(1) $\frac{37}{2\sqrt{38}}$ (2) $\frac{\sqrt{38}}{2}$

(3) $\frac{39}{2\sqrt{38}}$ (4) $\sqrt{19}$

**Ans. (1)**

**10.** Let $S_n$ denote the sum of the first n terms of an arithmetic progression. If $S_{10} = 390$ and the ratio of the tenth and the fifth terms is 15 : 7, then $S_{15} - S_5$ is equal to:

(1) 800 (2) 890

(3) 790 (4) 690

**Ans. (3)**

**11.** If $\int_0^{\frac{\pi}{3}} \cos^4 x \, dx = a\pi + b\sqrt{3}$, where a and b are rational numbers, then 9a + 8b is equal to :

(1) 2 (2) 1

(3) 3 (4) $\frac{3}{2}$

**Ans. (1)**

**12.** If z is a complex number such that $|z| \geq 1$, then the minimum value of $\left|z + \frac{1}{2}(3+4i)\right|$ is:

(1) $\frac{5}{2}$ (2) 2

(3) 3 (4) $\frac{3}{2}$

**Ans. (Bonus)**

**13.** If the domain of the function $f(x) = \frac{\sqrt{x^2-25}}{(4-x^2)} + \log_{10}(x^2 + 2x - 15)$ is $(-\infty, \alpha) \cup [\beta, \infty)$, then $\alpha^2 + \beta^3$ is equal to :

(1) 140 (2) 175

(3) 150 (4) 125

**Ans. (3)**

**14.** Consider the relations $R_1$ and $R_2$ defined as $aR_1b \Leftrightarrow a^2 + b^2 = 1$ for all a , b, $\in$ R and (a, b) $R_2$(c, d) $\Leftrightarrow a + d = b + c$ for all (a,b), (c,d) $\in N \times N$. Then

(1) Only $R_1$ is an equivalence relation

(2) Only $R_2$ is an equivalence relation

(3) $R_1$ and $R_2$ both are equivalence relations

(4) Neither $R_1$ nor $R_2$ is an equivalence relation

**Ans. (2)**

**15.** If the mirror image of the point P(3,4,9) in the line $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{1}$ is $(\alpha, \beta, \gamma)$, then $14(\alpha + \beta + \gamma)$ is :

(1) 102 (2) 138

(3) 108 (4) 132

**Ans. (3)**

**16.** Let $f(x) = \begin{cases} x-1, & x \text{ is even}, \\ 2x, & x \text{ is odd}, \end{cases}$ $x \in N$. If for some $a \in N$, $f(f(f(a))) = 21$, then $\lim\limits_{x \to a^-} \left\{\frac{|x|^3}{a} - \left[\frac{x}{a}\right]\right\}$, where [t] denotes the greatest integer less than or equal to t, is equal to :

(1) 121

(2) 144

(3) 169

(4) 225

**Ans. (2)**

**17.** Let the system of equations x + 2y +3z = 5, 2x + 3y + z = 9, 4x + 3y + $\lambda$z = $\mu$ have infinite number of solutions. Then $\lambda + 2\mu$ is equal to :

(1) 28 (2) 17

(3) 22 (4) 15

**Ans. (2)**

**18.** Consider 10 observation $x_1, x_2, \ldots, x_{10}$. such that $\sum_{i=1}^{10} (x_i - \alpha) = 2$ and $\sum_{i=1}^{10} (x_i - \beta)^2 = 40$, where $\alpha, \beta$ are positive integers. Let the mean and the variance of the observations be $\frac{6}{5}$ and $\frac{84}{25}$ respectively. The $\frac{\beta}{\alpha}$ is equal to :

(1) 2

(2) $\frac{3}{2}$

(3) $\frac{5}{2}$

(4) 1

**Ans. (1)**

**19.** Let Ajay will not appear in JEE exam with probability $p = \frac{2}{7}$, while both Ajay and Vijay will appear in the exam with probability $q = \frac{1}{5}$. Then the probability, that Ajay will appear in the exam and Vijay will not appear is :

(1) $\frac{9}{35}$ (2) $\frac{18}{35}$

(3) $\frac{24}{35}$ (4) $\frac{3}{35}$

**Ans. (2)**

**20.** Let the locus of the mid points of the chords of circle $x^2+(y-1)^2=1$ drawn from the origin intersect the line $x+y=1$ at P and Q. Then, the length of PQ is :

(1) $\frac{1}{\sqrt{2}}$ (2) $\sqrt{2}$

(3) $\frac{1}{2}$ (4) 1

**Ans. (1)**

## SECTION-B

**21.** If three successive terms of a G.P. with common ratio $r(r > 1)$ are the lengths of the sides of a triangle and [r] denotes the greatest integer less than or equal to r, then $3[r] + [-r]$ is equal to :

**Ans. (1)**

**22.** Let $A = I_2 - 2MM^T$, where M is real matrix of order $2 \times 1$ such that the relation $M^T M = I_1$ holds. If $\lambda$ is a real number such that the relation $AX = \lambda X$ holds for some non-zero real matrix X of order $2 \times 1$, then the sum of squares of all possible values of $\lambda$ is equal to :

**Ans. (2)**

**23.** Let $f : (0, \infty) \to R$ and $F(x) = \int_0^x tf(t)dt$. If $F(x^2) = x^4 + x^5$, then $\sum_{r=1}^{12} f(r^2)$ is equal to :

**Ans. (219)**

**24.** If $y = \frac{\left(\sqrt{x}+1\right)\left(x^2-\sqrt{x}\right)}{x\sqrt{x}+x+\sqrt{x}} + \frac{1}{15}(3\cos^2 x - 5)\cos^3 x$, then $96y'\left(\frac{\pi}{6}\right)$ is equal to :

**Ans. (105)**

**25.** Let $\vec{a} = \hat{i}+\hat{j}+\hat{k}$, $\vec{b} = -\hat{i}-8\hat{j}+2\hat{k}$ and $\vec{c} = 4\hat{i}+c_2\hat{j}+c_3\hat{k}$ be three vectors such that $\vec{b}\times\vec{a} = \vec{c}\times\vec{a}$. If the angle between the vector $\vec{c}$ and the vector $3\hat{i}+4\hat{j}+\hat{k}$ is $\theta$, then the greatest integer less than or equal to $\tan^2\theta$ is :

**Ans. (38)**

**26.** The lines $L_1, L_2, \ldots, L_{20}$ are distinct. For $n = 1, 2, 3, \ldots, 10$ all the lines $L_{2n-1}$ are parallel to each other and all the lines $L_{2n}$ pass through a given point P. The maximum number of points of intersection of pairs of lines from the set $\{L_1, L_2, \ldots, L_{20}\}$ is equal to :

**Ans. (101)**

**27.** Three points $O(0,0)$, $P(a, a^2)$, $Q(-b, b^2)$, $a > 0$, $b > 0$, are on the parabola $y = x^2$. Let $S_1$ be the area of the region bounded by the line PQ and the parabola, and $S_2$ be the area of the triangle OPQ. If the minimum value of $\frac{S_1}{S_2}$ is $\frac{m}{n}$, $\gcd(m, n) = 1$, then $m + n$ is equal to :

**Ans. (7)**

**28.** The sum of squares of all possible values of k, for which area of the region bounded by the parabolas $2y^2 = kx$ and $ky^2 = 2(y - x)$ is maximum, is equal to :

**Ans. (8)**

**29.** If $\frac{dx}{dy} = \frac{1 + x - y^2}{y}$, $x(1) = 1$, then $5x(2)$ is equal to :

**Ans. (5)**

**30.** Let ABC be an isosceles triangle in which A is at $(-1, 0)$, $\angle A = \frac{2\pi}{3}$, AB = AC and B is on the positive x-axis. If $BC = 4\sqrt{3}$ and the line BC intersects the line $y = x + 3$ at $(\alpha, \beta)$, then $\frac{\beta^4}{\alpha^2}$ is :

**Ans. (36)**

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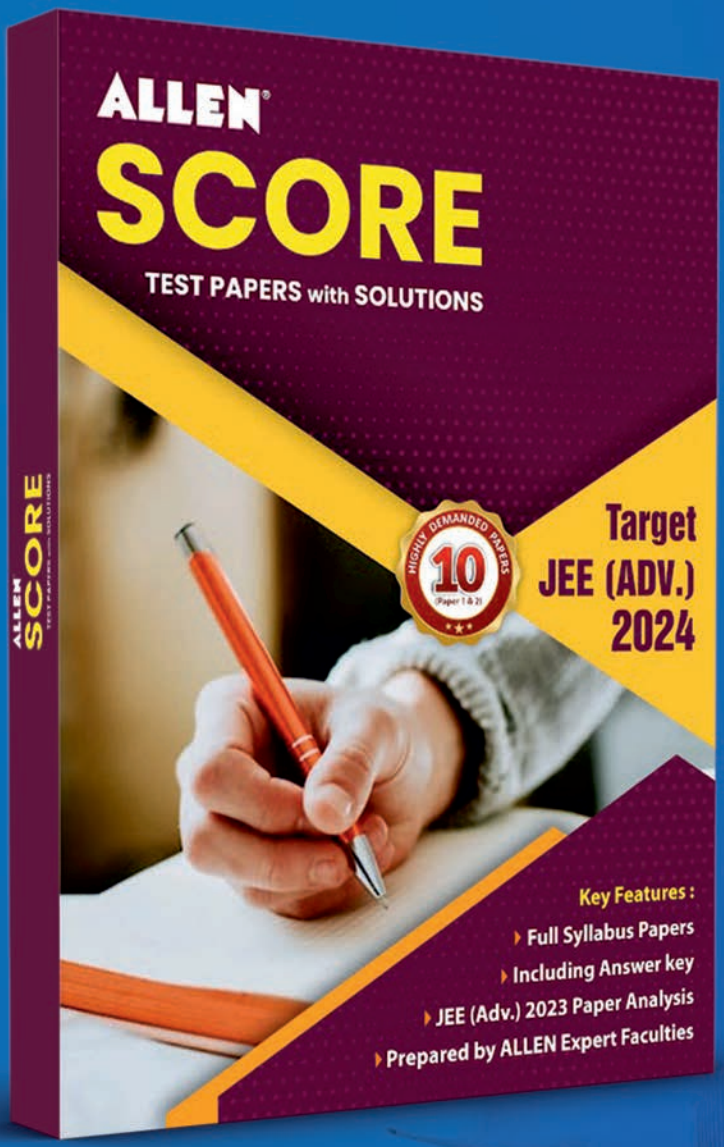


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