

# FINAL JEE-MAIN EXAMINATION - JANUARY, 2023

(Held On Monday 30th January, 2023)

# **TEST PAPER WITH SOLUTION**

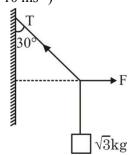
# TIME: 3:00 PM to 6:00 PM

### **SECTION-A**

**PHYSICS** 

A block of  $\sqrt{3}$  kg is attached to a string whose 1. other end is attached to the wall. An unknown force F is applied so that the string makes an angle of 30° with the wall. The tension T is:

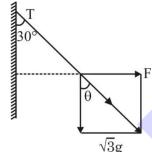
(Given  $g = 10 \text{ ms}^{-2}$ )



- (1) 20 N
- (2) 25 N
- (3) 10 N
- (4) 15 N

Official Ans. by NTA (1)

Allen Ans. (1)



Sol.

$$\theta = 30^{\circ}$$

$$\cos \theta = \frac{\sqrt{3}g}{T}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{\sqrt{3}g}{T}$$

$$\Rightarrow$$
 T = 20N

- A flask contains hydrogen and oxygen in the ratio 2. of 2:1 by mass at temperature 27°C. The ratio of average kinetic energy per molecule of hydrogen and oxygen respectively is:
  - (1) 2 : 1
- (2) 1:1
- (3)1:4
- (4) 4:1

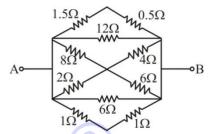
Official Ans. by NTA (2)

Allen Ans. (2)

**Sol.** 
$$K_{av} = \frac{5}{2}kT$$

Ratio = 1:1

3. The equivalent resistance between A and B is .....



Official Ans. by NTA (1) Allen Ans. (1)

Sol. 
$$\stackrel{A}{\circ}$$

$$\begin{array}{c}
12\Omega \\
1.6\Omega & 2.4\Omega \\
\hline
6\Omega \\
\hline
20
\end{array}$$

$$\frac{1}{R_{eq}} = \frac{1}{2} + \frac{1}{12} + \frac{1}{4} + \frac{1}{6} + \frac{1}{2}$$

$$=\frac{6+1+3+2+6}{12}=\frac{18}{12}=\frac{3}{2}$$

$$\Rightarrow R_{eq} = \frac{2}{3}\Omega$$

4. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**. **Assertion A:** The nuclear density of nuclides  $^{10}_5\,B,\,^6_3Li,\,^{56}_{26}Fe,\,^{20}_{10}Ne\,$  and  $^{209}_{83}Bi\,$  can be arranged as  $\rho_{Ri}^{N} > \rho_{Fe}^{N} > \rho_{Ne}^{N} > \rho_{R}^{N} > \rho_{Li}^{N}$ .

> **Reason R:** The radius R of nucleus is related to its mass number A as  $R = R_0 A^{1/3}$ , where  $R_0$  is a constant.

> In the light of the above statement, choose the **correct** answer from the options given below:

- (1) Both A and R are true and R is the correct explanation of A
- (2) A is false but R is true
- (3) **A** is true but **R** is false
- (4) Both A and R are true but R is NOT the correct explanation of A

Official Ans. by NTA (2)

Allen Ans. (2)

**Sol.** Nuclear density is independent of A.



- A thin prism  $P_1$  with an angle  $6^{\circ}$  and made of glass of refractive index 1.54 is combined with another prism P<sub>2</sub> made from glass of refractive index 1.72 to produce dispersion without average deviation. The angle of prism  $P_2$  is:
  - $(1) 6^{\circ}$

- $(2) 1.3^{\circ}$
- $(3) 7.8^{\circ}$
- $(4) 4.5^{\circ}$

# Official Ans. by NTA (4)

Allen Ans. (4)

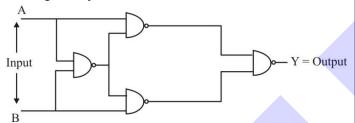
**Sol.**  $\delta_1 = \delta_2$  [for no average deviation]

$$\Rightarrow$$
 6° (1.54 – 1) = A(1.72 – 1)

$$\Rightarrow A = \frac{6^{\circ} \times 0.54}{0.72}$$

$$=\frac{18^{\circ}}{4}=4.5^{\circ}$$

6. The output Y for the inputs A and B of circuit is given by



Truth table of the shown circuit is:

A	В	Y
0	0	1

- (1) 00
  - 0 0
- (2) 01 0 0 1 0 1 1 1
  - $B \mid$ A Y  $\overline{0}$ 0 0
- (3) 01 1 1 0 1  $1 \mid 1$ 1
  - $_{\rm B}$  $\overline{0}$ 0 0
- (4) 01 1 1 0 1 1 0

Official Ans. by NTA (4)

Allen Ans. (4)

Given circuit represent XOR. Sol.

- 7. A vehicle travels 4 km with speed of 3 km/h and another 4 km with speed of 5 km/h, then its average speed is:
  - (1) 4.25 km/h
- (2) 3.50 km/h
- (3) 4.00 km/h
- (4) 3.75 km/h

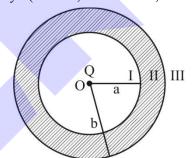
Official Ans. by NTA (4)

Allen Ans. (4)

**Sol.** 
$$\frac{2}{V_{av}} = \frac{1}{3} + \frac{1}{5} = \frac{8}{15}$$

$$\Rightarrow$$
  $V_{av} = \frac{15}{4} = 3.75 \text{ km/h}$ 

As shown in the figure, a point charge Q is placed 8. at the centre of conducting spherical shell of inner radius a and outer radius b. The electric field due to charge Q in three different regions I, II and III is given by : (I : r < a, II : a < r < b, III : r > b)



(1) 
$$E_I = 0$$
,  $E_{II} = 0$ ,  $E_{III} \neq 0$ 

(2) 
$$E_I \neq 0$$
,  $E_{II} = 0$ ,  $E_{III} \neq 0$ 

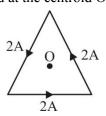
(3) 
$$E_I \neq 0$$
,  $E_{II} = 0$ ,  $E_{III} = 0$ 

(4) 
$$E_I = 0$$
,  $E_{II} = 0$ ,  $E_{III} = 0$ 

Official Ans. by NTA (2)

Allen Ans. (2)

- Sol. Electric field inside material of conductor is zero.
- As shown in the figure, a current of 2A flowing in 9. an equilateral triangle of side  $4\sqrt{3}$  cm. The magnetic field at the centroid O of the triangle is:



(Neglect the effect of earth's magnetic field.)

(1) 
$$4\sqrt{3} \times 10^{-4} \text{ T}$$
 (2)  $4\sqrt{3} \times 10^{-5} \text{ T}$ 

(2) 
$$4\sqrt{3} \times 10^{-5} \text{ T}$$

(3) 
$$\sqrt{3} \times 10^{-4} \,\mathrm{T}$$

(4) 
$$3\sqrt{3} \times 10^{-5} \text{ T}$$

Official Ans. by NTA (4)

Allen Ans. (4)

# Final JEE-Main Exam January, 2023/30-01-2023/Evening Session



**Sol.** 
$$d \tan 60^{\circ} = 2\sqrt{3}$$

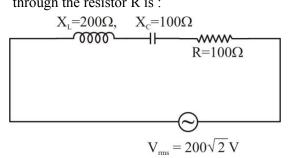
d = 2 cm

$$B = 3 \times \frac{\mu_0 i}{2\pi d} \sin 60^{\circ}$$

$$= 3 \times \frac{2 \times 10^{-7} \times 2}{2 \times 10^{-2}} \times \frac{\sqrt{3}}{2}$$

$$=3\sqrt{3}\times10^{-5}$$

10. In the given circuit, rms value of current  $(I_{rms})$  through the resistor R is:



- (1) 2A
- (2)  $\frac{1}{2}$  A
- (3) 20 A
- (4)  $2\sqrt{2}A$

#### Official Ans. by NTA (1)

Allen Ans. (1)

**Sol.** 
$$z = \sqrt{100^2 + (200 - 100)^2}$$

$$=100\sqrt{2}\,\Omega$$

$$i_{rms} = \frac{V_{rms}}{z} = \frac{200\sqrt{2}}{100\sqrt{2}}$$

$$=2A$$

- 11. A machine gun of mass 10 kg fires 20 g bullets at the rate of 180 bullets per minute with a speed of 100 m s<sup>-1</sup> each. The recoil velocity of the gun is:
  - (1) 0.02 m/s
  - (2) 2.5 m/s
  - (3) 1.5 m/s
  - (4) 0.6 m/s

Official Ans. by NTA (4)

Allen Ans. (4)

**Sol.** 
$$20 \times 10^{-3} \times \frac{180}{60} \times 100 = 10 \text{ V}$$

$$\Rightarrow$$
 v = 0.6 m/s

- 12. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R. Assertion A: Efficiency of a reversible heat engine will be highest at -273°C temperature of cold reservoir.
  - **Reason R:** The efficiency of Carnot's engine depends not only on temperature of cold reservoir but it depends on the temperature of hot reservoir

too and is given as 
$$\eta = \left(1 - \frac{T_2}{T_1}\right)$$
.

In the light of the above statements, choose the **correct** answer from the options given below:

- (1) A is true but R is false
- (2) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**
- (3) A is false but R is true
- (4) Both **A** and **R** are true and **R** is the correct explanation of **A**

Official Ans. by NTA (4)

Allen Ans. (4)

- **Sol.** Both A and R are true and R is the correct explanation of A
- 13. Match List I with List II.

	List I		List II	
A	Torque	I.	$kg m^{-1} s^{-2}$	
В	Energy density	II.	kg ms <sup>-1</sup>	
C	Pressure gradient	III.	$kg m^{-2} s^{-2}$	
D	Impulse	IV.	$kg m^2 s^{-2}$	

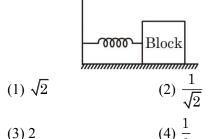
Choose the **correct** answer from the options given below:

- (1) A-IV, B-III, C-I, D-II
- (2) A-I, B-IV, C-III, D-II
- (3) A-IV, B-I, C-II, D-III
- (4) A-IV, B-I, C-III, D-II

Official Ans. by NTA (4)

Allen Ans. (4)

14. For a simple harmonic motion in a mass spring system shown, the surface is frictionless. When the mass of the block is 1 kg, the angular frequency is  $\omega_1$ . When the mass block is 2 kg the angular frequency is  $\omega_2$ . The ratio  $\omega_2/\omega_1$  is:



Official Ans. by NTA (2)

Allen Ans. (2)



Sol. 
$$\omega = \sqrt{\frac{k}{m}}$$

$$\frac{\omega_2}{\omega_1} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{2}}$$

15. An electron accelerated through a potential difference  $V_1$  has a de-Broglie wavelength of  $\lambda$ . When the potential is changed to  $V_2$ , its de-Broglie wavelength increases by 50%. The value of  $\left(\frac{V_1}{V_2}\right)$ 

is equal to:

(2) 
$$\frac{9}{4}$$

(3) 
$$\frac{3}{2}$$

Official Ans. by NTA (2) Allen Ans. (2)

**Sol.** 
$$KE = \frac{P^2}{2m}$$
,  $P = \frac{h}{\lambda}$ 

$$eV_1 = \frac{\left(\frac{h}{\lambda}\right)^2}{2m}$$

$$eV_2 = \frac{\left(\frac{h}{1.5\lambda}\right)^2}{2m}$$

$$\frac{V_1}{V_2} = (1.5)^2 = \frac{9}{4}$$

**16.** Match List I with List II:

	List I		List II
A.	Attenuation	Ι	Combination of a
			receiver and
			transmitter.
B.	Transducer	II	Process of retrieval of
			information from the
			carrier wave at received
C.	Demodulation	III	Converts one form of
			energy into another
D.	Repeater	IV	Loss of strength of a
	\		signal while
			propagating through a
			medium

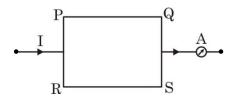
Choose the correct answer from the options given below:

- (1) A-I, B-II, C-III, D-IV
- (2) A-II, B-III, C-IV, D-I
- (3) A-IV, B-III, C-I, D-II
- (4) A-IV, B-III, C-II, D-I

Official Ans. by NTA (4)

Allen Ans. (4)

17. A current carrying rectangular loop PQRS is made of uniform wire. The length PR = QS = 5 cm and PQ = RS = 100 cm. If ammeter current reading changes from I to 2I, the ratio of magnetic forces per unit length on the wire PQ due to wire RS in the two cases respectively  $f_{PO}^{I}$ :  $f_{PO}^{2I}$  is:



(1) 1 : 2

(2) 1 : 4

(3) 1:5

(4) 1:3

Official Ans. by NTA (2)

Allen Ans. (2)

Sol.  $F \propto I_1I_2$ 

 $F_{I}: F_{2I} = 1:4$ 

- 18. A force is applied to a steel wire 'A', rigidly clamped at one end. As a result elongation in the wire is 0.2 mm. If same force is applied to another steel wire 'B' of double the length and a diameter 2.4 times that of the wire 'A', the elongation in the wire 'B' will be (wires having uniform circular cross sections)
  - (1)  $6.06 \times 10^{-2}$  mm
  - (2)  $2.77 \times 10^{-2}$  mm
  - $(3) 3.0 \times 10^{-2} \,\mathrm{mm}$
  - $(4) 6.9 \times 10^{-2} \text{ mm}$

Official Ans. by NTA (4)

Allen Ans. (4)

$$Y = \frac{F/A}{\frac{\Delta \ell}{\ell}}$$

$$\Rightarrow F = \frac{YA}{\ell} \Delta \ell$$

$$\left(\frac{A\Delta \ell}{\ell}\right)_1 = \left(\frac{A\Delta \ell}{\ell}\right)_2$$

$$\Rightarrow \frac{\Delta \ell_2}{\Delta \ell_1} = \frac{A_1}{A_2} \times \frac{\ell_2}{\ell_1}$$

$$\Rightarrow \frac{\Delta \ell_2}{0.2} = \frac{1}{2.4 \times 2.4} \times \frac{2}{1}$$

$$\Rightarrow \Delta \ell_2 = 6.9 \times 10^{-2} \text{ mm}$$

# Final JEE-Main Exam January, 2023/30-01-2023/Evening Session

- 19. An object is allowed to fall from a height R above the earth, where R is the radius of earth. Its velocity when it strikes the earth's surface, ignoring air resistance, will be:
  - (1)  $2\sqrt{gR}$
- (2)  $\sqrt{gR}$
- (3)  $\sqrt{\frac{gR}{2}}$
- $(4) \sqrt{2gR}$

Official Ans. by NTA (2)

Allen Ans. (2)

**Sol.** Loss in PE = Gain in KE

$$\left(-\frac{GMm}{2R}\right) - \left(-\frac{GMm}{R}\right) = \frac{1}{2}mv^2$$

$$\Rightarrow v^2 = \frac{GM}{R} = gR$$

$$\Rightarrow v = \sqrt{gR}$$

- **20.** A point source of 100 W emits light with 5% efficiency. At a distance of 5 m from the source, the intensity produced by the electric field component is:
  - (1)  $\frac{1}{2\pi} \frac{W}{m^2}$ 
    - (2)  $\frac{1}{40\pi} \frac{W}{m^2}$
  - (3)  $\frac{1}{10\pi} \frac{W}{m^2}$
- (4)  $\frac{1}{20\pi} \frac{W}{m^2}$

Official Ans. by NTA (2)

Allen Ans. (2)

Sol.  $I_{EF} = \frac{1}{2} \times \frac{5}{4\pi \times 5^2}$  $= \frac{1}{40\pi} \text{W/m}^2$ 

#### **SECTION-B**

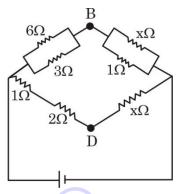
21. A faulty thermometer reads 5°C in melting ice and 95°C in steam. The correct temperature on absolute scale will be....... K when the faulty thermometer reads 41°C.

Official Ans. by NTA (313)

Allen Ans. (313)

Sol. 
$$\frac{41^{\circ} - 5^{\circ}}{95^{\circ} - 5^{\circ}} = \frac{C - 0^{\circ}}{100^{\circ} - 0^{\circ}}$$
  
 $\Rightarrow C = \frac{36}{90} \times 100 = 40^{\circ}C = 313 \text{ K}$ 

22. If the potential difference between B and D is zero, the value of x is  $\frac{1}{n}\Omega$ . The value of n is .......



Official Ans. by NTA (2)

Allen Ans. (2)

Sol. 
$$\frac{2}{3} = \frac{\overline{x+1}}{x}$$

$$\Rightarrow \frac{2}{3} = \frac{1}{x+1}$$

$$\Rightarrow x = 0.5 = \frac{1}{2}$$

$$n = 2$$

23. The velocity of a particle executing SHM varies with displacement (x) as  $4v^2 = 50 - x^2$ . The time period of oscillations is  $\frac{x}{7}$ s. The value of x is

$$\dots \left( \text{Take } \pi = \frac{22}{7} \right)$$

Official Ans. by NTA (88)

Allen Ans. (88)

Sol. 
$$4v^2 = 50 - x^2$$
  

$$\Rightarrow v = \frac{1}{2}\sqrt{50 - x^2}$$

$$\omega = \frac{1}{2}$$

$$T = \frac{2\pi}{\omega} = 4\pi = \frac{88}{7}$$

$$x = 88$$

24. In a Young's double slit experiment, the intensities at two points, for the path difference  $\frac{\lambda}{4}$  and  $\frac{\lambda}{3}$  ( $\lambda$  being the wavelength of light used) are  $I_1$  and  $I_2$  respectively. If  $I_0$  denotes the intensity produced by each one of the individual slits, then  $\frac{I_1+I_2}{I_0}=\dots$ 

Official Ans. by NTA (3) Allen Ans. (3)



$$\begin{aligned} \textbf{Sol.} \quad & I = 4I_0\cos^2\left(\frac{\Delta\phi}{2}\right) \\ & I_1 = 4I_0\cos^2\left(\frac{\pi}{4}\right) = 2I_0 \\ & I_2 = 4I_0\cos^2\left(\frac{2\pi}{3}\right) = I_0 \\ \\ & \Rightarrow \quad & \frac{I_1 + I_2}{I_0} = 3 \end{aligned}$$

25. A radioactive nucleus decays by two different process. The half life of the first process is 5 minutes and that of the second process is 30s. The effective half-life of the nucleus is calculated to be  $\frac{\alpha}{11}$ s. The value of  $\alpha$  is \_\_\_\_\_.

# Official Ans. by NTA (300) Allen Ans. (300)

$$\begin{aligned} \textbf{Sol.} \quad & \frac{dN_1}{dt} = -\lambda_1 N & \frac{dN_2}{dt} = -\lambda_2 N \\ & \frac{dN}{dt} = -\left(\lambda_1 + \lambda_2\right) N \\ & \Rightarrow \quad & \lambda_{eq} = \lambda_1 + \lambda_2 \\ & \Rightarrow \quad & \frac{1}{t_{1/2}} = \frac{1}{300} + \frac{1}{30} = \frac{11}{300} \\ & \Rightarrow \quad & t_{1/2} = \frac{300}{11} \end{aligned}$$

26. A body of mass 2 kg is initially at rest. It starts moving unidirectionally under the influence of a source of constant power P. Its displacement in 4s is  $\frac{1}{3}\alpha^2\sqrt{P}$  m. The value of  $\alpha$  will be .....

# Official Ans. by NTA (4)

#### Allen Ans. (4)

Sol. 
$$\frac{1}{2}mV^{2} = Pt$$

$$V = \sqrt{\frac{2Pt}{m}}$$

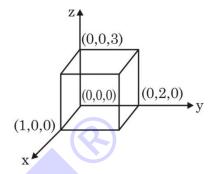
$$\frac{dx}{dt} = \sqrt{\frac{2Pt}{m}}$$

$$x = \sqrt{\frac{2P}{m}} \frac{2}{3} \left[t^{3/2}\right]_{0}^{4}$$

$$x = \frac{16\sqrt{P}}{3} = \frac{1}{3} \times 16\sqrt{P}$$

$$\alpha = 4$$

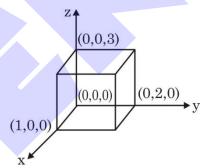
27. As shown in figure, a cuboid lies in a region with electric field  $E=2x^2\hat{i}-4y\hat{j}+6\hat{k}$  N/C. The magnitude of charge within the cuboid is  $n\in_0 C$ . The value of n is \_\_\_\_\_ (if dimension of cuboid is  $1\times 2\times 3$  m³)



#### Official Ans. by NTA (12)

Allen Ans. (12)

Sol. 
$$\vec{E} = 2x^2\hat{i} - 4y\hat{j} + 6\hat{k}$$



$$\phi_{net} = -8 \times 3 + 2 \times 6 = -12$$

$$-12 = \frac{q}{\epsilon_0}$$

$$|q| = 12 \in_0$$

28. In an ac generator, a rectangular coil of 100 turns each having area  $14 \times 10^{-2}$  m<sup>2</sup> is rotated at 360 rev/min about an axis perpendicular to a uniform magnetic field of magnitude 3.0 T. The maximum value of the emf produced will be

$$\underline{\qquad} V. \left( Take \ \pi = \frac{22}{7} \right)$$

#### Official Ans. by NTA (1584)

Allen Ans. (1584)

Sol. 
$$\xi_{max} = NAB\omega$$
  
=  $100 \times 14 \times 10^{-2} \times 3 \times \frac{360 \times 2\pi}{60}$   
=  $1584V$ 

### Final JEE-Main Exam January, 2023/30-01-2023/Evening Session



29. A stone tied to 180 cm long string at its end is making 28 revolutions in horizontal circle in every minute. The magnitude of acceleration of stone is

$$\frac{1936}{x}$$
 ms<sup>-2</sup>. The value of x \_\_\_\_\_.

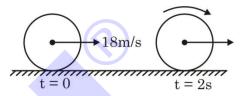
$$\left(Take \ \pi = \frac{22}{7}\right)$$

Official Ans. by NTA (125)

Allen Ans. (125)

Sol. 
$$a = \omega^2 R = \left(\frac{28 \times 2\pi}{60}\right)^2 \times 1.8$$
  
 $= \left(\frac{56}{60} \times \frac{22}{7}\right)^2 \times 1.8$   
 $= \frac{(44)^2}{225} \times 1.8$   
 $= \frac{1936 \times 1.8}{225}$   
 $x = 125$ 

30. A uniform disc of mass 0.5 kg and radius r is projected with velocity 18 m/s at t=0 s on a rough horizontal surface. It starts off with a purely sliding motion at t=0 s. After 2s it acquires a purely rolling motion (see figure). The total kinetic energy of the disc after 2s will be \_\_\_\_\_ J (given, coefficient of friction is 0.3 and  $g=10 \text{ m/s}^2$ ).



Official Ans. by NTA (54)

Allen Ans. (54)

Sol. 
$$a = -\mu_k g = -3$$

$$V = 18 - 3 \times 2$$

$$V = 12 \text{ m/s}$$

$$KE = \frac{1}{2}mv^2 + \frac{1}{2}\frac{mr^2}{2}\frac{v^2}{r^2}$$

$$KE = \frac{3}{4} mv^2$$

$$KE = 3 \times 18 = 54 J$$