FINAL JEE–MAIN EXAMINATION – JANUARY, 2023

(Held On Sunday 29th January, 2023) TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS | **TEST PAPER WITH ANSWER**

SECTION-A

61. The statement $B \Rightarrow ((\sim A) \vee B)$ is equivalent to :

(1) $B \Rightarrow (A \Rightarrow B)$

(2) $A \Rightarrow (A \Leftrightarrow B)$

(3) $A \Rightarrow ((\sim A) \Rightarrow B)$

(4) $B \Rightarrow ((\sim A) \Rightarrow B)$

Official Ans. by NTA (1,3,4)

Allen Ans. (1 or 3 or 4)

62. The shortest distance between the lines $\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}$ and $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$ is

(1) $2\sqrt{3}$

(2) $4\sqrt{3}$

(3) $3\sqrt{3}$

(4) $5\sqrt{3}$

Official Ans. by NTA (2)

Allen Ans. (2)

63. If $\vec{a}=\hat{i}+2\hat{k},\ \vec{b}=\hat{i}+\hat{j}+\hat{k},\ \vec{c}=7\hat{i}-3\hat{k}+4\hat{k}$, $\vec{r}\times\vec{b}+\vec{b}\times\vec{c}=\vec{0}$ and $\vec{r}.\vec{a}=0$. Then $\vec{r}.\vec{c}$ is equal to :

(1) 34 (2) 32

(3) 36 (4) 30

Official Ans. by NTA (1)

Allen Ans. (1)

64. Let $S=\{w_1, w_2, ...\}$ be the sample space associated to a random experiment. Let $P(w_n)=\frac{P(w_{n-1})}{2}, n \geq 2$.

Let $A=\{2k+3\ell;\ k,\ell \in \mathbb{N}\}$ and $B=\{w_n;\ n \in A\}$.

Then P(B) is equal to

(1) $\frac{3}{32}$ (2) $\frac{3}{64}$

(3) $\frac{1}{16}$ (4) $\frac{1}{32}$

Official Ans. by NTA (2)

Allen Ans. (2)

65. The value of the integral $\int_1^2\left(\frac{t^4+1}{t^6+1}\right)dt$ is :

(1) $\tan^{-1}\frac{1}{2}+\frac{1}{3}\tan^{-1}8-\frac{\pi}{3}$

(2) $\tan^{-1}2-\frac{1}{3}\tan^{-1}8+\frac{\pi}{3}$

(3) $\tan^{-1}2+\frac{1}{3}\tan^{-1}8-\frac{\pi}{3}$

(4) $\tan^{-1}\frac{1}{2}-\frac{1}{3}\tan^{-1}8+\frac{\pi}{3}$

Official Ans. by NTA (3)

Allen Ans. (3)

66. Let K be the sum of the coefficients of the odd powers of x in the expansion of $(1+x)^{99}$. Let a be the middle term in the expansion of $\left(2+\frac{1}{\sqrt{2}}\right)^{200}$.

If $\frac{{}^{200}C_{99}K}{a}=\frac{2^{\ell}m}{n}$, where m and n are odd numbers, then the ordered pair (ℓ, n) is equal to :

(1) (50, 51)

(2) (51, 99)

(3) (50, 101)

(4) (51, 101)

Official Ans. by NTA (3)

Allen Ans. (3)

67. Let f and g be twice differentiable functions on $\mathbb{R}$ such that

$f''(x)=g''(x)+6x$

$f'(1)=4g'(1)-3=9$

$f(2)=3g(2)=12$

Then which of the following is **NOT** true ?

(1) $g(-2)-f(-2)=20$

(2) If $-1<x<2$, then $|f(x)-g(x)|<8$

(3) $|f'(x)-g'(x)|<6 \Rightarrow -1<x<1$

(4) There exists $x_0 \in \left(1, \frac{3}{2}\right)$ such that $f(x_0)=g(x_0)$

Official Ans. by NTA (2)

Allen Ans. (2)

68. The set of all values of $t \in \mathbb{R}$, for which the matrix

$$\begin{bmatrix} e^t & e^{-t}(\sin t - 2\cos t) & e^{-t}(-2\sin t - \cos t) \\ e^t & e^{-t}(2\sin t + \cos t) & e^{-t}(\sin t - 2\cos t) \\ e^t & e^{-t}\cos t & e^{-t}\sin t \end{bmatrix}$$ is

invertible, is

(1) $\left\{(2k+1)\frac{\pi}{2}, k \in \mathbb{Z}\right\}$ (2) $\left\{k\pi + \frac{\pi}{4}, k \in \mathbb{Z}\right\}$

(3) $\{k\pi, k \in \mathbb{Z}\}$ (4) $\mathbb{R}$

Official Ans. by NTA (4)

Allen Ans. (4)

69. The area of the region

$A = \left\{(x, y) : |\cos x - \sin x| \le y \le \sin x, 0 \le x \le \frac{\pi}{2}\right\}$ is

(1) $1 - \frac{3}{\sqrt{2}} + \frac{4}{\sqrt{5}}$ (2) $\sqrt{5} + 2\sqrt{2} - 4.5$

(3) $\frac{3}{\sqrt{5}} - \frac{3}{\sqrt{2}} + 1$ (4) $\sqrt{5} - 2\sqrt{2} + 1$

Official Ans. by NTA (4)

Allen Ans. (4)

70. The set of all values of λ for which the equation $\cos^2 2x - 2\sin^4 x - 2\cos^2 x = \lambda$ has a real solution x, is :-

(1) [–2, –1] (2) $\left[-2, -\frac{3}{2}\right]$

(3) $\left[-1, -\frac{1}{2}\right]$ (4) $\left[-\frac{3}{2}, -1\right]$

Official Ans. by NTA (4)

Allen Ans. (4)

71. The letters of the word OUGHT are written in all possible ways and these words are arranged as in a dictionary, in a series. Then the serial number of the word TOUGH is :

(1) 89 (2) 84

(3) 86 (4) 79

Official Ans. by NTA (1)

Allen Ans. (1)

72. The plane $2x - y + z = 4$ intersects the line segment joining the points A(a, –2, 4) and B(2, b, –3) at the point C in the ratio 2 : 1 and the distance of the point C from the origin is $\sqrt{5}$. If $ab < 0$ and P is the point $(a - b, b, 2b - a)$ then CP^2 is equal to :

(1) $\frac{17}{3}$

(2) $\frac{16}{3}$

(3) $\frac{73}{3}$

(4) $\frac{97}{3}$

Official Ans. by NTA (1)

Allen Ans. (1)

73. Let $\vec{a} = 4\hat{i} + 3\hat{j}$ and $\vec{b} = 3\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c}$ is a vector such that $\vec{c}.(\vec{a} \times \vec{b}) + 25 = 0$, $\vec{c}.(\hat{i} + \hat{j} + \hat{k}) = 4$, and projection of $\vec{c}$ on $\vec{a}$ is 1, then the projection of $\vec{c}$ on $\vec{b}$ equals :

(1) $\frac{5}{\sqrt{2}}$ (2) $\frac{1}{5}$

(3) $\frac{1}{\sqrt{2}}$ (4) $\frac{3}{\sqrt{2}}$

Official Ans. by NTA (1)

Allen Ans. (1)

74. If the lines $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z+3}{1}$ and $\frac{x-a}{2} = \frac{y+2}{3} = \frac{z-3}{1}$ intersects at the point P, then the distance of the point P from the plane z = a is :

(1) 16 (2) 28

(3) 10 (4) 22

Official Ans. by NTA (2)

Allen Ans. (2)

75. The value of the integral $\int_{1/2}^{2} \frac{\tan^{-1} x}{x} dx$ is equal to

(1) $\pi \log_e 2$ (2) $\frac{1}{2}\log_e 2$

(3) $\frac{\pi}{4}\log_e 2$ (4) $\frac{\pi}{2}\log_e 2$

Official Ans. by NTA (4)

Allen Ans. (4)

76. If the tangent at a point P on the parabola $y^2 = 3x$ is parallel to the line $x + 2y = 1$ and the tangents at the points Q and R on the ellipse $\frac{x^2}{4}+\frac{y^2}{1}=1$ are perpendicular to the line $x - y = 2$, then the area of the triangle PQR is:

(1) $\frac{9}{\sqrt{5}}$

(2) $5\sqrt{3}$

(3) $\frac{3}{2}\sqrt{5}$

(4) $3\sqrt{5}$

Official Ans. by NTA (4)

Allen Ans. (4)

77. Let $y = y(x)$ be the solution of the differential equation $x\log_e x\frac{dy}{dx}+y=x^2\log_e x, \ (x>1)$.

If $y(2) = 2$, then $y(e)$ is equal to

(1) $\frac{4+e^2}{4}$ (2) $\frac{1+e^2}{4}$

(3) $\frac{2+e^2}{2}$ (4) $\frac{1+e^2}{2}$

Official Ans. by NTA (1)

Allen Ans. (1)

78. The number of 3 digit numbers, that are divisible by either 3 or 4 but not divisible by 48, is

(1) 472

(2) 432

(3) 507

(4) 400

Official Ans. by NTA (2)

Allen Ans. (2)

79. Let R be a relation defined on $\mathbb{N}$ as a R b if $2a + 3b$ is a multiple of 5, $a, b \in \mathbb{N}$. Then R is

(1) not reflexive

(2) transitive but not symmetric

(3) symmetric but not transitive

(4) an equivalence relation

Official Ans. by NTA (4)

Allen Ans. (4)

80. Consider a function $f : \mathbb{N} \to \mathbb{R}$, satisfying

$f(1) + 2f(2) + 3f(3) + \ldots + xf(x) = x(x + 1) f(x); x \geq 2$

with f(1)=1. Then $\frac{1}{f(2022)}+\frac{1}{f(2028)}$ is equal to

(1) 8200

(2) 8000

(3) 8400

(4) 8100

Official Ans. by NTA (4)

Allen Ans. (4)

SECTION-B

81. The total number of 4-digit numbers whose greatest common divisor with 54 is 2, is _____.

Official Ans. by NTA (3000)

Allen Ans. (3000)

82. A triangle is formed by the tangents at the point (2, 2) on the curves $y^2 = 2x$ and $x^2 + y^2 = 4x$, and the line $x + y + 2 = 0$. If r is the radius of its circumcircle, then r^2 is equal to _______.

Official Ans. by NTA (10)

Allen Ans. (10)

83. A circle with centre (2, 3) and radius 4 intersects the line $x + y = 3$ at the points P and Q. If the tangents at P and Q intersect at the point $S(\alpha, \beta)$, then $4\alpha - 7\beta$ is equal to _________.

Official Ans. by NTA (11)

Allen Ans. (11)

84. Let $a_1 = b_1 = 1$ and $a_n = a_{n-1} + (n-1)$, $b_n = b_{n-1} + a_{n-1}$, $\forall\ n \geq 2$. If $S = \sum_{n=1}^{10} \frac{b_n}{2^n}$ and $T = \sum_{n=1}^{8} \frac{n}{2^{n-1}}$, then $2^7(2S - T)$ is equal to ______.

Official Ans. by NTA (461)

Allen Ans. (461)

85. If the equation of the normal to the curve $y = \frac{x-a}{(x+b)(x-2)}$ at the point $(1, -3)$ is $x - 4y = 13$, then the value of $a + b$ is equal to _____.

Official Ans. by NTA (4)

Allen Ans. (4)

86. Let A be a symmetric matrix such that $|A| = 2$ and $\begin{bmatrix} 2 & 1 \\ 3 & \frac{3}{2} \end{bmatrix} A = \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$. If the sum of the diagonal elements of A is s, then $\frac{\beta s}{\alpha^2}$ is equal to ______.

Official Ans. by NTA (5)

Allen Ans. (5)

87. Let $\{a_k\}$ and $\{b_k\}$, $k \in \mathbb{N}$, be two G.P.s with common ratios r_1 and r_2 respectively such that $a_1 = b_1 = 4$ and $r_1 < r_2$. Let $c_k = a_k + b_k$, $k \in \mathbb{N}$. If $c_2 = 5$ and $c_3 = \frac{13}{4}$ then $\sum_{k=1}^{\infty} c_k - (12a_6 + 8b_4)$ is equal to _______.

Official Ans. by NTA (9)

Allen Ans. (9)

88. Let $X = \{11, 12, 13, \ldots, 40, 41\}$ and $Y = \{61, 62, 63, \ldots, 90, 91\}$ be the two sets of observations. If $\bar{x}$ and $\bar{y}$ are their respective means and σ^2 is the variance of all the observations in $X \cup Y$, then $\left|\bar{x} + \bar{y} - \sigma^2\right|$ is equal to ______.

Official Ans. by NTA (603)

Allen Ans. (603)

89. Let $\alpha = 8 - 14i$, $A = \left\{z \in \mathbb{C} : \frac{\alpha z - \bar{\alpha}\bar{z}}{z^2 - (\bar{z})^2 - 112i} = 1\right\}$ and $B = \left\{z \in \mathbb{C} : |z + 3i| = 4\right\}$.

Then $\sum_{z \in A \cap B} (\operatorname{Re} z - \operatorname{Im} z)$ is equal to _____.

Official Ans. by NTA (14)

Allen Ans. (14)

90. Let $\alpha_1, \alpha_2, \ldots, \alpha_7$ be the roots of the equation $x^7 + 3x^5 - 13x^3 - 15x = 0$ and $|\alpha_1| \geq |\alpha_2| \geq \ldots \geq |\alpha_7|$.

Then $\alpha_1\alpha_2 - \alpha_3\alpha_4 + \alpha_5\alpha_6$ is equal to______.

Official Ans. by NTA (9)

Allen Ans. (9)