

FINAL JEE-MAIN EXAMINATION – JANUARY, 2023
(Held On Tuesday 24th January, 2023)
TIME : 3 : 00 PM to 6 : 00 PM
MATHEMATICS
TEST PAPER WITH ANSWER
SECTION-A

61. Let the six numbers $a_1, a_2, a_3, a_4, a_5, a_6$ be in A.P. and $a_1 + a_3 = 10$. If the mean of these six numbers is $\frac{19}{2}$ and their variance is σ^2 , then $8\sigma^2$ is equal to

- (1) 220
 (2) 210
 (3) 200
 (4) 105

Official Ans. by NTA (2)
Allen Ans. (2)

Sol. $a_1 + a_3 = 10 = a_1 + d \Rightarrow 5$

$$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 = 57$$

$$\Rightarrow \frac{6}{2}[a_1 + a_6] = 57$$

$$\Rightarrow a_1 + a_6 = 19$$

$$\Rightarrow 2a_1 + 5d = 19 \text{ and } a_1 + d = 5$$

$$\Rightarrow a_1 = 2, d = 3$$

Numbers : 2, 5, 8, 11, 14, 17

Variance = $\sigma^2 = \text{mean of squares} - \text{square of mean}$

$$= \frac{2^2 + 5^2 + 8^2 + (11)^2 + (14)^2 + (17)^2}{6} - \left(\frac{19}{2}\right)^2$$

$$= \frac{699}{6} - \frac{361}{4} = \frac{105}{4}$$

$$8\sigma^2 = 210$$

62. Let $f(x)$ be a function such that $f(x+y) = f(x) \cdot f(y)$

for all $x, y \in \mathbb{N}$. If $f(1) = 3$ and $\sum_{k=1}^n f(k) = 3279$,

then the value of n is

- (1) 6
 (2) 8
 (3) 7
 (4) 9

Official Ans. by NTA (3)
Allen Ans. (3)

Sol. $f(x+y) = f(x) \cdot f(y) \quad \forall x, y \in \mathbb{N}, f(1) = 3$

$$f(2) = f^2(1) = 3^2$$

$$f(3) = f(1) f(2) = 3^3$$

$$f(4) = 3^4$$

$$f(k) = 3^k$$

$$\sum_{k=1}^n f(k) = 3279$$

$$f(1) + f(2) + f(3) + \dots + f(k) = 3279$$

$$3 + 3^2 + 3^3 + \dots + 3^k = 3279$$

$$\frac{3(3^k - 1)}{3 - 1} = 3279$$

$$\frac{3^k - 1}{2} = 1093$$

$$3^k - 1 = 2186$$

$$3^k = 2187$$

$$\boxed{k=7}$$

63. The number of real solutions of the equation

$$3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0, \text{ is}$$

- (1) 4 (2) 0
 (3) 3 (4) 2

Official Ans. by NTA (2)
Allen Ans. (2)

Sol. $3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0$

$$3\left[\left(x + \frac{1}{x}\right)^2 - 2\right] - 2\left(x + \frac{1}{x}\right) + 5 = 0$$

$$\text{Let } x + \frac{1}{x} = t$$

$$3t^2 - 2t - 1 = 0$$

$$3t^2 - 3t + t - 1 = 0$$

$$3t(t-1) + 1(t-1) = 0$$

$$(t-1)(3t+1) = 0$$

$$t = 1, -\frac{1}{3}$$

$$x + \frac{1}{x} = 1, -\frac{1}{3} \Rightarrow \text{No solution.}$$

64. If $f(x) = \frac{2^{2x}}{2^{2x} + 2}$, $x \in \mathbb{R}$,
then $f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + \dots + f\left(\frac{2022}{2023}\right)$ is

equal to

- (1) 2011 (2) 1010
(3) 2010 (4) 1011

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. $f(x) = \frac{4^x}{4^x + 2}$

$$f(x) + f(1-x) = \frac{4^x}{4^x + 2} + \frac{4^{1-x}}{4^{1-x} + 2}$$

$$= \frac{4^x}{4^x + 2} + \frac{4}{4 + 2(4^x)}$$

$$= \frac{4^x}{4^x + 2} + \frac{2}{2 + 4^x}$$

$$= 1$$

$$\Rightarrow f(x) + f(1-x) = 1$$

$$\text{Now } f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + f\left(\frac{3}{2023}\right) + \dots +$$

$$\dots + f\left(1 - \frac{3}{2023}\right) + f\left(1 - \frac{2}{2023}\right) + f\left(1 - \frac{1}{2023}\right)$$

Now sum of terms equidistant from beginning and end is 1

$$\text{Sum} = 1 + 1 + 1 + \dots + 1 \text{ (1011 times)}$$

$$= 1011$$

65. If $f(x) = x^3 - x^2f'(1) + xf''(2) - f'''(3)$, $x \in \mathbb{R}$, then

- (1) $3f(1) + f(2) = f(3)$
(2) $f(3) - f(2) = f(1)$
(3) $2f(0) - f(1) + f(3) = f(2)$
(4) $f(1) + f(2) + f(3) = f(0)$

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. $f(x) = x^3 - x^2f'(1) + xf''(2) - f'''(3)$, $x \in \mathbb{R}$

$$\text{Let } f'(1) = a, f''(2) = b, f'''(3) = c$$

$$f(x) = x^3 - ax^2 + bx - c$$

$$f'(x) = 3x^2 - 2ax + b$$

$$f''(x) = 6x - 2a$$

$$f'''(x) = 6$$

$$c = 6, a = 3, b = 6$$

$$f(x) = x^3 - 3x^2 + 6x - 6$$

$$f(1) = -2, f(2) = 2, f(3) = 12, f(0) = -6$$

$$2f(0) - f(1) + f(3) = 2 = f(2)$$

66. The number of integers, greater than 7000 that can be formed, using the digits 3, 5, 6, 7, 8 without repetition, is

- (1) 120
(2) 168
(3) 220
(4) 48

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. Four digit numbers greater than 7000

$$= 2 \times 4 \times 3 \times 2 = 48$$

$$\text{Five digit number} = 5! = 120$$

$$\text{Total number greater than 7000}$$

$$= 120 + 48 = 168$$

67. If the system of equations

$$x + 2y + 3z = 3$$

$$4x + 3y - 4z = 4$$

$$8x + 4y - \lambda z = 9 + \mu$$

has infinitely many solutions, then the ordered pair

(λ, μ) is equal to

(1) $\left(\frac{72}{5}, \frac{21}{5}\right)$ (2) $\left(\frac{-72}{5}, \frac{-21}{5}\right)$

(3) $\left(\frac{72}{5}, \frac{-21}{5}\right)$ (4) $\left(\frac{-72}{5}, \frac{21}{5}\right)$

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. $x + 2y + 3z = 3$ (i)

$$4x + 3y - 4z = 4$$
(ii)

$$8x + 4y - \lambda z = 9 + \mu$$
(iii)

$$(i) \times 4 - (ii) \Rightarrow 5y + 16z = 8$$
 (iv)

$$(ii) \times 2 - (iii) \Rightarrow 2y + (\lambda - 8)z = -1 - \mu$$
(v)

$$(iv) \times 2 - (v) \times 5 \Rightarrow (32 - 5(\lambda - 8))z = 16 - 5(-1 - \mu)$$

$$\text{For infinite solutions} \Rightarrow 72 - 5\lambda = 0 \Rightarrow \lambda = \frac{72}{5}$$

$$21 + 5\mu = 0 \Rightarrow \mu = \frac{-21}{5}$$

$$\Rightarrow (\lambda, \mu) = \left(\frac{72}{5}, \frac{-21}{5}\right)$$

68. The value of $\left(\frac{1 + \sin \frac{2\pi}{9} + i \cos \frac{2\pi}{9}}{1 + \sin \frac{2\pi}{9} - i \cos \frac{2\pi}{9}}\right)^3$ is

- (1) $\frac{-1}{2}(1 - i\sqrt{3})$
- (2) $\frac{1}{2}(1 - i\sqrt{3})$
- (3) $\frac{-1}{2}(\sqrt{3} - i)$
- (4) $\frac{1}{2}(\sqrt{3} + i)$

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. Let $\sin \frac{2\pi}{9} + i \cos \frac{2\pi}{9} = z$

$$\left(\frac{1+z}{1+\bar{z}}\right)^3 = \left(\frac{1+z}{1+\frac{1}{z}}\right)^3 = z^3$$

$$\Rightarrow \left(i\left(\cos \frac{2\pi}{9} - i \sin \frac{2\pi}{9}\right)\right)^3$$

$$= -i \left(\cos \frac{2\pi}{3} - i \sin \frac{2\pi}{3}\right) = -i \left(\frac{-1}{2} - i \frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \frac{-1}{2}(\sqrt{3} - i)$$

69. The equations of the sides AB and AC of a triangle ABC are

$$(\lambda + 1)x + \lambda y = 4 \text{ and } \lambda x + (1 - \lambda)y + \lambda = 0$$

respectively. Its vertex A is on the y-axis and its orthocentre is (1, 2). The length of the tangent from the point C to the part of the parabola $y^2 = 6x$ in the first quadrant is

- (1) $\sqrt{6}$
- (2) $2\sqrt{2}$
- (3) 2
- (4) 4

Official Ans. by NTA (2)

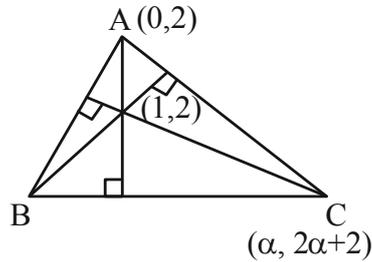
Allen Ans. (2)

Sol. AB: $(\lambda + 1)x + \lambda y = 4$

$$AC: \lambda x + (1 - \lambda)y + \lambda = 0$$

Vertex A is on y-axis

$$\Rightarrow x = 0$$



$$\text{So } y = \frac{4}{\lambda}, y = \frac{\lambda}{\lambda - 1}$$

$$\Rightarrow \frac{4}{\lambda} = \frac{\lambda}{\lambda - 1}$$

$$\Rightarrow \lambda = 2$$

$$AB: 3x + 2y = 4$$

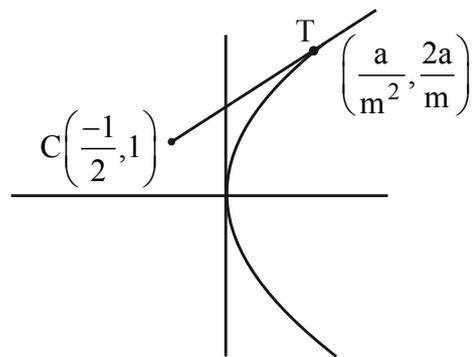
$$AC: 2x - y + 2 = 0$$

$$\Rightarrow A(0,2) \text{ Let } C(\alpha, 2\alpha + 2)$$

$$\text{Now (Slope of Altitude through C)} \left(-\frac{3}{2}\right) = -1$$

$$\left(\frac{2\alpha}{\alpha - 1}\right)\left(-\frac{3}{2}\right) = -1 \Rightarrow \alpha = -\frac{1}{2}$$

$$\text{So } C\left(-\frac{1}{2}, 1\right)$$



$$\text{Let Equation of tangent be } y = mx + \frac{3}{2m}$$

$$m^2 + 2m - 3 = 0$$

$$\Rightarrow m = 1, -3$$

So tangent which touches in first quadrant at T is

$$T \equiv \left(\frac{a}{m^2}, \frac{2a}{m}\right)$$

$$\equiv \left(\frac{3}{2}, 3\right)$$

$$\Rightarrow CT = \sqrt{4 + 4} = 2\sqrt{2}$$

70. The set of all values of a for which $\lim_{x \rightarrow a} ([x-5] - [2x+2]) = 0$, where $[\infty]$ denotes the

greater integer less than or equal to ∞ is equal to

- (1) $(-7.5, -6.5)$ (2) $(-7.5, -6.5]$
 (3) $[-7.5, -6.5]$ (4) $[-7.5, -6.5)$

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. $\lim_{x \rightarrow a} ([x-5] - [2x+2]) = 0$

$$\lim_{x \rightarrow a} ([x] - 5 - [2x] - 2) = 0$$

$$\lim_{x \rightarrow a} ([x] - [2x]) = 7$$

$$[a] - [2a] = 7$$

$$a \in I, a = -7$$

$$a \notin I, a = I + f$$

$$\text{Now, } [a] - [2a] = 7$$

$$-I - [2f] = 7$$

$$\text{Case-I: } f \in \left(0, \frac{1}{2}\right)$$

$$2f \in (0, 1)$$

$$-I = 7$$

$$I = -7 \Rightarrow a \in (-7, -6.5)$$

$$\text{Case-II: } f \in \left(\frac{1}{2}, 1\right)$$

$$2f \in (1, 2)$$

$$-I - 1 = 7$$

$$I = -8 \Rightarrow a \in (-7.5, -7)$$

$$\text{Hence, } a \in (-7.5, -6.5)$$

71. If $({}^{30}C_1)^2 + 2({}^{30}C_2)^2 + 3({}^{30}C_3)^2 + \dots + 30({}^{30}C_{30})^2 = \frac{\alpha 60!}{(30!)^2}$, then α is equal to

- (1) 30 (2) 60
 (3) 15 (4) 10

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. $S = 0 \cdot ({}^{30}C_0)^2 + 1 \cdot ({}^{30}C_1)^2 + 2 \cdot ({}^{30}C_2)^2 + \dots + 30 \cdot ({}^{30}C_{30})^2$

$$S = 30 \cdot ({}^{30}C_0)^2 + 29 \cdot ({}^{30}C_1)^2 + 28 \cdot ({}^{30}C_2)^2 + \dots + 0 \cdot ({}^{30}C_0)^2$$

$$2S = 30 \cdot ({}^{30}C_0)^2 + 30 \cdot ({}^{30}C_1)^2 + \dots + 30 \cdot ({}^{30}C_{30})^2$$

$$S = 15 \cdot 60 C_{30} = 15 \cdot \frac{60!}{(30!)^2}$$

$$\frac{15 \cdot 10!}{(30!)^2} = \frac{\alpha \cdot 60!}{(30!)^2}$$

$$\Rightarrow \alpha = 15$$

72. Let the plane containing the line of intersection of the planes

$$P_1: x + (\lambda + 4)y + z = 1 \text{ and}$$

$P_2: 2x + y + z = 2$ pass through the points $(0, 1, 0)$ and $(1, 0, 1)$. Then the distance of the point $(2\lambda, \lambda, -\lambda)$ from the plane P_2 is

- (1) $5\sqrt{6}$
 (2) $4\sqrt{6}$
 (3) $2\sqrt{6}$
 (4) $3\sqrt{6}$

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. Equation of plane passing through point of intersection of P_1 and P_2

$$P = P_1 + kP_2$$

$$(x + (\lambda + 4)y + z - 1) + k(2x + y + z - 2) = 0$$

Passing through $(0, 1, 0)$ and $(1, 0, 1)$

$$(\lambda + 4 - 1) + k(1 - 2) = 0$$

$$(\lambda + 3) - k = 0 \quad \dots(1)$$

Also passing $(1, 0, 1)$

$$(1 + 1 - 1) + k(2 + 1 - 2) = 0$$

$$1 + k = 0$$

$$k = -1$$

put in (1)

$$\lambda + 3 + 1 = 0$$

$$\lambda = -4$$

Then point $(2\lambda, \lambda, -\lambda)$

$$(-8, -4, 4)$$

$$d = \left| \frac{-16 - 4 + 4 - 2}{\sqrt{6}} \right|$$

$$d = \frac{18}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = 3\sqrt{6}$$

73. Let $\vec{\alpha} = 4\hat{i} + 3\hat{j} + 5\hat{k}$ and $\vec{\beta} = \hat{i} + 2\hat{j} - 4\hat{k}$. Let $\vec{\beta}_1$ be parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ be perpendicular to $\vec{\alpha}$. If

$\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, then the value of $5\vec{\beta}_2 \cdot (\hat{i} + \hat{j} + \hat{k})$ is

- (1) 6
 (2) 11
 (3) 7
 (4) 9

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. Let $\vec{\beta}_1 = \lambda \vec{\alpha}$

Now $\vec{\beta}_2 = \vec{\beta} - \vec{\beta}_1$

$$= (\hat{i} + 2\hat{j} - 4\hat{k}) - \lambda(4\hat{i} + 3\hat{j} + 5\hat{k})$$

$$= (1 - 4\lambda)\hat{i} + (2 - 3\lambda)\hat{j} - (5\lambda + 4)\hat{k}$$

$$\vec{\beta}_2 \cdot \vec{\alpha} = 0$$

$$\Rightarrow 4(1 - 4\lambda) + 3(2 - 3\lambda) - 5(5\lambda + 4) = 0$$

$$\Rightarrow 4 - 16\lambda + 6 - 9\lambda - 25\lambda - 20 = 0$$

$$\Rightarrow 50\lambda = -10$$

$$\Rightarrow \boxed{\lambda = \frac{-1}{5}}$$

$$\vec{\beta}_2 = \left(1 + \frac{4}{5}\right)\hat{i} + \left(2 + \frac{3}{5}\right)\hat{j} - (-1 + 4)\hat{k}$$

$$\vec{\beta}_2 = \frac{9}{5}\hat{i} + \frac{13}{5}\hat{j} - 3\hat{k}$$

$$5\vec{\beta}_2 = 9\hat{i} + 13\hat{j} - 15\hat{k}$$

$$5\vec{\beta}_2 \cdot (\hat{i} + \hat{j} + \hat{k}) = 9 + 13 - 15 = 7$$

74. The locus of the mid points of the chords of the circle $C_1: (x - 4)^2 + (y - 5)^2 = 4$ which subtend an angle θ_1 at the centre of the circle C_1 , is a circle of radius r_1 . If $\theta_1 = \frac{\pi}{3}$, $\theta_3 = \frac{2\pi}{3}$ and $r_1^2 = r_2^2 + r_3^2$, then θ_2 is equal to

(1) $\frac{\pi}{4}$

(2) $\frac{3\pi}{4}$

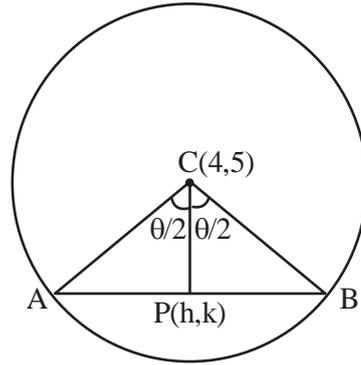
(3) $\frac{\pi}{6}$

(4) $\frac{\pi}{2}$

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. In ΔCPB



$$\cos \frac{\theta}{2} = \frac{PC}{2} \Rightarrow PC = 2 \cos \frac{\theta}{2}$$

$$\Rightarrow (h - 4)^2 + (k - 5)^2 = 4 \cos^2 \frac{\theta}{2}$$

$$\text{Now } (x - 4)^2 + (y - 5)^2 = \left(2 \cos \frac{\theta}{2}\right)^2$$

$$\Rightarrow r_1 = 2 \cos \frac{\pi}{6} = \sqrt{3}$$

$$r_2 = 2 \cos \frac{\theta_2}{2}$$

$$r_3 = 2 \cos \frac{\pi}{3} = 1$$

$$\Rightarrow r_1^2 = r_2^2 + r_3^2$$

$$\Rightarrow 3 = 4 \cos^2 \frac{\theta_2}{2} + 1$$

$$\Rightarrow 4 \cos^2 \frac{\theta_2}{2} = 2$$

$$\Rightarrow \cos^2 \frac{\theta_2}{2} = \frac{1}{2}$$

$$\Rightarrow \boxed{\theta_2 = \frac{\pi}{2}}$$

75. If the foot of the perpendicular drawn from (1, 9, 7) to the line passing through the point (3, 2, 1) and parallel to the planes $x + 2y + z = 0$ and $3y - z = 3$ is (α, β, γ) , then $\alpha + \beta + \gamma$ is equal to

(1) -1

(2) 3

(3) 1

(4) 5

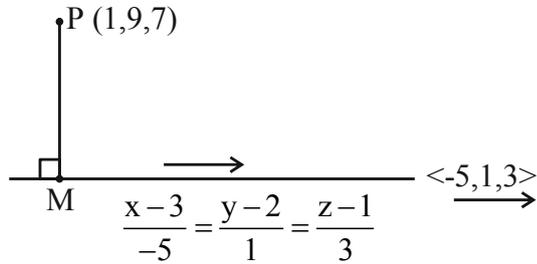
Official Ans. by NTA (4)

Allen Ans. (4)

Sol. Direction ratio of line = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{vmatrix}$

$$= \hat{i}(-5) - \hat{j}(-1) + \hat{k}(3)$$

$$= -5\hat{i} + \hat{j} + 3\hat{k}$$



$$M(-5\lambda + 3, \lambda + 2, 3\lambda + 1)$$

$$\overline{PM} \perp (-5\hat{i} + \hat{j} + 3\hat{k})$$

$$-5(-5\lambda + 2) + (\lambda - 7) + 3(3\lambda - 6) = 0$$

$$\Rightarrow 25\lambda + \lambda + 9\lambda - 10 - 7 - 18 = 0$$

$$\Rightarrow \lambda = 1$$

$$\text{Point } M = (-2, 3, 4) = (\alpha, \beta, \gamma)$$

$$\alpha + \beta + \gamma = 5$$

76. Let $y = y(x)$ be the solution of the differential equation $(x^2 - 3y^2)dx + 3xy dy = 0$, $y(1) = 1$. Then $6y^2(e)$ is equal to

(1) $3e^2$

(2) e^2

(3) $2e^2$

(4) $\frac{3e^2}{2}$

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. $(x^2 - 3y^2) dx + 3xy dy = 0$

$$\frac{dy}{dx} = \frac{3y^2 - x^2}{3xy} \Rightarrow \frac{dy}{dx} = \frac{y}{x} - \frac{1}{3} \frac{x}{y} \quad (1)$$

Put $y = vx$

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$(1) \Rightarrow v + x \frac{dv}{dx} = v - \frac{1}{3} \frac{1}{v}$$

$$\Rightarrow v dv = \frac{-1}{3x}$$

Integrating both side

$$\frac{v^2}{2} = \frac{-1}{3} \ln x + c$$

$$\Rightarrow \frac{y^2}{2x^2} = \frac{-1}{3} \ln x + c$$

$$y(1) = 1$$

$$\Rightarrow \frac{1}{2} = c$$

$$\Rightarrow \frac{y^2}{2x^2} = \frac{-1}{3} \ln x + \frac{1}{2}$$

$$\Rightarrow y^2 = -\frac{2}{3} x^2 \ln x + x^2$$

$$y^2(e) = -\frac{2}{3} e^2 + e^2 = \frac{e^2}{3}$$

$$\Rightarrow 6y^2(e) = 2e^2$$

77. Let p and q be two statements.

Then $\sim(p \wedge (p \Rightarrow \sim q))$ is equivalent to

(1) $p \vee (p \wedge (\sim q))$

(2) $p \vee ((\sim p) \wedge q)$

(3) $(\sim p) \vee q$

(4) $p \vee (p \wedge q)$

Official Ans. by NTA (3)

Allen Ans. (3)

Sol. $\sim(p \wedge (p \rightarrow \sim q))$

$$\equiv \sim p \vee \sim(\sim p \vee \sim q)$$

$$\equiv \sim p \vee (p \wedge q)$$

$$\equiv (\sim p \vee p) \wedge (\sim p \vee q)$$

$$\equiv t \wedge (\sim p \vee q)$$

$$\equiv \sim p \vee q$$

78. The number of square matrices of order 5 with entries from the set $\{0, 1\}$, such that the sum of all the elements in each row is 1 and the sum of all the elements in each column is also 1, is

(1) 225

(2) 120

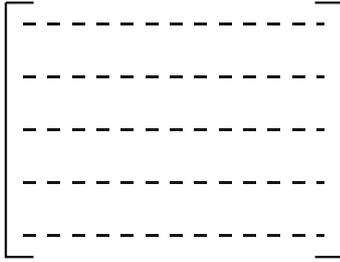
(3) 150

(4) 125

Official Ans. by NTA (2)

Allen Ans. (2)

Sol.



In each row and each column exactly one is to be placed –

$$\therefore \text{No. of such matrices} = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

Alternate :

$$\begin{aligned} \left[\begin{array}{ccccc} 0 & 0 & 1 & 0 & 0 \end{array} \right] &\rightarrow 5 \text{ ways} \\ \left[\begin{array}{ccccc} 0 & 1 & 0 & 0 & 0 \end{array} \right] &\rightarrow 4 \text{ ways} \\ \left[\begin{array}{ccccc} 0 & 0 & 0 & 1 & 0 \end{array} \right] &\rightarrow 3 \text{ ways} \\ \left[\begin{array}{ccccc} 0 & 0 & 0 & 0 & 1 \end{array} \right] &\rightarrow 2 \text{ ways} \\ \left[\begin{array}{ccccc} 1 & 0 & 0 & 0 & 0 \end{array} \right] &\rightarrow 1 \text{ ways} \end{aligned}$$

Step-1 : Select any 1 place for 1's in row 1.

Automatically some column will get filled with 0's.

Step-2 : From next row select 1 place for 1's.

Automatically some column will get filled with 0's.

\Rightarrow Each time one less place will be available for putting 1's.

Repeat step-2 till last row.

$$\text{Req. ways} = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

79. $\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}} dx$ is equal to

- (1) $\frac{\pi}{3}$
- (2) $\frac{\pi}{2}$
- (3) $\frac{\pi}{6}$
- (4) 2π

Official Ans. by NTA (4)

Allen Ans. (4)

Sol. $\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}} dx$

We have $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} \frac{x}{a} + C$

Hence $\int_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}} \frac{48}{\sqrt{9-4x^2}} dx = \frac{48}{2} \times \left[\sin^{-1} \frac{2x}{3} \right]_{\frac{3\sqrt{2}}{4}}^{\frac{3\sqrt{3}}{4}}$

$$= 24 \times \left[\sin^{-1} \left(\frac{2}{3} \times \frac{3\sqrt{3}}{4} \right) - \sin^{-1} \left(\frac{2}{3} \times \frac{3\sqrt{2}}{4} \right) \right]$$

$$= 24 \times \left[\sin^{-1} \frac{\sqrt{3}}{2} - \sin^{-1} \frac{1}{\sqrt{2}} \right]$$

$$= 24 \times \left(\frac{\pi}{3} - \frac{\pi}{4} \right)$$

$$= 24 \times \frac{\pi}{12} = 2\pi$$

80. Let A be a 3×3 matrix such that $|\text{adj}(\text{adj}(\text{adj}A))| = 12^4$. Then $|A^{-1}\text{adj}A|$ is equal to

- (1) $2\sqrt{3}$
- (2) $\sqrt{6}$
- (3) 12
- (4) 1

Official Ans. by NTA (1)

Allen Ans. (1)

Sol. Given $|\text{adj}(\text{adj}(\text{adj}A))| = 12^4$

$$\Rightarrow |A|^{(n-1)^3} = 12^4$$

Given $n = 3$

$$\Rightarrow |A|^8 = 12^4$$

$$\Rightarrow |A|^2 = 12$$

$$|A| = 2\sqrt{3}$$

We are asked

$$|A^{-1} \cdot \text{adj}A|$$

$$= |A^{-1}| \cdot |\text{adj}A|$$

$$= \frac{1}{|A|} \cdot |A|^{3-1}$$

$$= |A| = 2\sqrt{3}$$

81. The urns A, B and C contain 4 red, 6 black; 5 red, 5 black and λ red, 4 black balls respectively. One of the urns is selected at random and a ball is drawn. If the ball drawn is red and the probability that it is drawn from urn C is 0.4 then the square of the length of the side of the largest equilateral triangle, inscribed in the parabola $y^2 = \lambda x$ with one vertex at the vertex of the parabola is

Official Ans. by NTA (432)

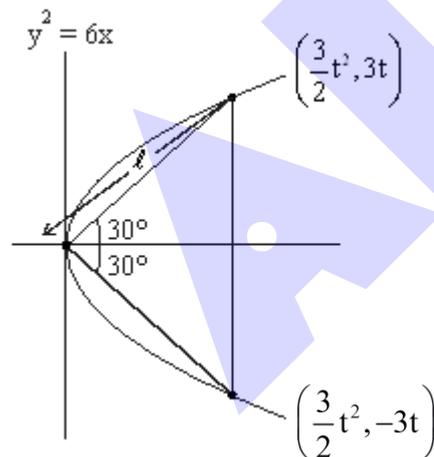
Allen Ans. (432)

Sol.	Urn A		Urn B		Urn C	
	Red	Black	Red	Black	Red	Black
	4	6	5	5	λ	4

$$P\left(\frac{C}{R}\right) = \frac{P(C)P\left(\frac{R}{C}\right)}{P(A)P\left(\frac{R}{A}\right) + P(B)P\left(\frac{R}{B}\right) + P(C)P\left(\frac{R}{C}\right)}$$

$$0.4 = \frac{\frac{1}{3} \times \frac{\lambda}{(\lambda+4)}}{\frac{1}{3} \times \frac{4}{10} + \frac{1}{3} \times \frac{5}{10} + \frac{1}{3} \times \frac{\lambda}{(\lambda+4)}}$$

$$\Rightarrow \lambda = 6$$



$$\tan 30^\circ = 3t = \frac{3}{2}t^2$$

$$\frac{1}{\sqrt{3}} = \frac{2}{t}$$

$$t = 2\sqrt{3}$$

$$\left(\frac{3}{2}t^2, 3t\right) = (18, 6\sqrt{3})$$

$$\ell^2 = 18^2 + (6\sqrt{3})^2$$

$$= 324 + 108$$

$$= 432$$

82. If the area of the region bounded by the curves $y^2 - 2y = -x$, $x + y = 0$ is A, then 8A is equal to

Official Ans. by NTA (36)

Allen Ans. (36)

Sol. $y^2 - 2y = -x$

$$\Rightarrow y^2 - 2y + 1 = -x + 1$$

$$(y - 1)^2 = -(x - 1)$$

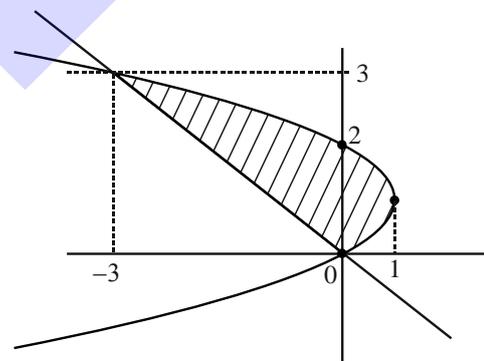
$$y = -x$$

Points of intersection

$$x^2 + 2x = -x$$

$$x^2 + 3x = 0$$

$$x = 0, -3$$



$$A = \int_0^3 (-y^2 + 2y + y) dy$$

$$= \frac{3y^2}{2} - \frac{y^3}{3} \Big|_0^3 = \frac{9}{2}$$

$$8A = 36$$

83. If $\frac{1^3 + 2^3 + 3^3 + \dots \text{upto } n \text{ terms}}{1 \cdot 3 + 2 \cdot 5 + 3 \cdot 7 + \dots \text{upto } n \text{ terms}} = \frac{9}{5}$, then

the value of n is

Official Ans. by NTA (5)

Allen Ans. (5)

Sol. $1^3 + 2^3 + 3^3 + \dots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

$1 \cdot 3 + 2 \cdot 5 + 3 \cdot 7 + \dots + n \text{ terms} =$

$$\sum_{r=1}^n r(2r+1) = \sum_{r=1}^n (2r^2 + r)$$

$$= \frac{2 \cdot n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{6} (2(2n+1) + 3)$$

$$= \frac{n(n+1)}{2} \times \frac{(4n+5)}{3}$$

$$= \frac{\frac{n^2(n+1)^2}{4}}{\frac{n(n+1)}{2} \times \frac{(4n+5)}{3}} = \frac{9}{5}$$

$$\Rightarrow \frac{5n(n+1)}{2} = \frac{9(4n+5)}{3}$$

$$\Rightarrow 15n(n+1) = 18(4n+5)$$

$$\Rightarrow 15n^2 + 15n = 72n + 90$$

$$\Rightarrow 15n^2 - 57n - 90 = 0 \Rightarrow 5n^2 - 19n - 30 = 0$$

$$\Rightarrow (n-5)(5n+6) = 0$$

$$\Rightarrow n = \frac{-6}{5} \text{ or } 5$$

$$\Rightarrow n = 5.$$

84. Let f be a differentiable function defined on $\left[0, \frac{\pi}{2}\right]$ such that $f(x) > 0$ and

$$f(x) + \int_0^x f(t) \sqrt{1 - (\log_e f(t))^2} dt = e, \forall x \in \left[0, \frac{\pi}{2}\right].$$

Then $\left(6 \log_e f\left(\frac{\pi}{6}\right)\right)^2$ is equal to _____.

Official Ans. by NTA (27)

Allen Ans. (27)

Sol. $f(x) + \int_0^x f(t) \sqrt{1 - (\log_e f(t))^2} dt = e$

$$\Rightarrow f(0) = e$$

$$f'(x) + f(x) \sqrt{1 - (\ln f(x))^2} = 0$$

$$f(x) = y$$

$$\frac{dy}{dx} = -y \sqrt{1 - (\ln y)^2}$$

$$\int \frac{dy}{y \sqrt{1 - (\ln y)^2}} = -\int dx$$

Put $\ln y = t$

$$\int \frac{dt}{\sqrt{1-t^2}} = -x + C$$

$$\sin^{-1} t = -x + C \Rightarrow \sin^{-1}(\ln y) = -x + C$$

$$\sin^{-1}(\ln f(x)) = -x + C$$

$$f(0) = e$$

$$\Rightarrow \frac{\pi}{2} = C$$

$$\Rightarrow \sin^{-1}(\ln f(x)) = -x + \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}\left(\ln f\left(\frac{\pi}{6}\right)\right) = \frac{-\pi}{6} + \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}\left(\ln f\left(\frac{\pi}{6}\right)\right) = \frac{\pi}{3}$$

$$\Rightarrow \ln f\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}, \text{ we need } \left(6 \times \frac{\sqrt{3}}{2}\right)^2 = 27.$$

85. The minimum number of elements that must be added to the relation $R = \{(a, b), (b, c), (b, d)\}$ on the set $\{a, b, c, d\}$ so that it is an equivalence relation, is _____.

Official Ans. by NTA (13)

Allen Ans. (13)

Sol. Given $R = \{(a, b), (b, c), (b, d)\}$

In order to make it equivalence relation as per given set, R must be

$\{(a, a), (b, b), (c, c), (d, d), (a, b), (b, a), (b, c), (c, b), (b, d), (d, b), (a, c), (a, d), (c, d), (d, c), (c, a), (d, a)\}$

There already given so 13 more to be added.

86. Let $\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k}$, $\vec{b} = 3\hat{i} - 5\hat{j} - \lambda\hat{k}$, $\vec{a} \cdot \vec{c} = 7$,
 $2\vec{b} \cdot \vec{c} + 43 = 0$, $\vec{a} \times \vec{c} = \vec{b} \times \vec{c}$. Then $|\vec{a} \cdot \vec{b}|$ is equal to

Official Ans. by NTA (8)

Allen Ans. (8)

- Sol.** $\vec{a} = \hat{i} + 2\hat{j} + \lambda\hat{k}$, $\vec{b} = 3\hat{i} - 5\hat{j} - \lambda\hat{k}$, $\vec{a} \cdot \vec{c} = 7$
 $\vec{a} \times \vec{c} - \vec{b} \times \vec{c} = \vec{0}$,
 $(\vec{a} - \vec{b}) \times \vec{c} = \vec{0} \Rightarrow (\vec{a} - \vec{b})$ is paralleled to \vec{c}
 $\vec{a} - \vec{b} = \mu\vec{c}$, where μ is a scalar
 $-2\hat{i} + 7\hat{j} + 2\lambda\hat{k} = \mu \cdot \vec{c}$
Now $\vec{a} \cdot \vec{c} = 7$ gives $2\lambda^2 + 12 = 7\mu$
And $\vec{b} \cdot \vec{c} = -\frac{43}{2}$ gives $4\lambda^2 + 82 = 43\mu$
 $\mu = 2$ and $\lambda^2 = 1$
 $|\vec{a} \cdot \vec{b}| = 8$

87. Let the sum of the coefficients of the first three terms in the expansion of $\left(x - \frac{3}{x^2}\right)^n$, $x \neq 0$, $n \in \mathbb{N}$, be 376. Then the coefficient of x^4 is _____.

Official Ans. by NTA (405)

Allen Ans. (405)

- Sol.** Given Binomial $\left(x - \frac{3}{x^2}\right)^n$, $x \neq 0$, $n \in \mathbb{N}$,
Sum of coefficients of first three terms
 ${}^nC_0 - {}^nC_1 \cdot 3 + {}^nC_2 3^2 = 376$
 $\Rightarrow 3n^2 - 5n - 250 = 0$
 $\Rightarrow (n - 10)(3n + 25) = 0$
 $\Rightarrow n = 10$
Now general term ${}^{10}C_r x^{10-r} \left(\frac{-3}{x^2}\right)^r$
 $= {}^{10}C_r x^{10-r} (-3)^r \cdot x^{-2r}$
 $= {}^{10}C_r (-3)^r \cdot x^{10-3r}$
Coefficient of $x^4 \Rightarrow 10 - 3r = 4$
 $\Rightarrow r = 2$
 ${}^{10}C_2 (-3)^2 = 405$

88. If the shortest between the lines

$$\frac{x + \sqrt{6}}{2} = \frac{y - \sqrt{6}}{3} = \frac{z - \sqrt{6}}{4} \text{ and}$$

$$\frac{x - \lambda}{3} = \frac{y - 2\sqrt{6}}{4} = \frac{z + 2\sqrt{6}}{5} \text{ is } 6, \text{ then the square}$$

of sum of all possible values of λ is

Official Ans. by NTA (384)

Allen Ans. (384)

- Sol.** Shortest distance between the lines

$$\frac{x + \sqrt{6}}{2} = \frac{y - \sqrt{6}}{3} = \frac{z - \sqrt{6}}{4}$$

$$\frac{x - \lambda}{3} = \frac{y - 2\sqrt{6}}{4} = \frac{z + 2\sqrt{6}}{5} \text{ is } 6$$

Vector along line of shortest distance

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}, \Rightarrow -\hat{i} + 2\hat{j} - \hat{k} \text{ (its magnitude is } \sqrt{6})$$

$$\text{Now } \frac{1}{\sqrt{6}} \begin{vmatrix} \sqrt{6} + \lambda & \sqrt{6} & -3\sqrt{6} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = \pm 6$$

$$\Rightarrow \lambda = -2\sqrt{6}, 10\sqrt{6}$$

So, square of sum of these values is 384.

89. Let $S = \{\theta \in [0, 2\pi) : \tan(\pi \cos \theta) + \tan(\pi \sin \theta) = 0\}$.

Then $\sum_{\theta \in S} \sin^2\left(\theta + \frac{\pi}{4}\right)$ is equal to

Official Ans. by NTA (2)

Allen Ans. (2)

- Sol.** $\tan(\pi \cos \theta) + \tan(\pi \sin \theta) = 0$

$$\tan(\pi \cos \theta) = -\tan(\pi \sin \theta)$$

$$\tan(\pi \cos \theta) = \tan(-\pi \sin \theta)$$

$$\pi \cos \theta = n\pi - \pi \sin \theta$$

$$\sin \theta + \cos \theta = n \text{ where } n \in \mathbb{I}$$

possible values are $n = 0, 1$ and -1 because

$$-\sqrt{2} \leq \sin \theta + \cos \theta \leq \sqrt{2}$$

$$\text{Now it gives } \theta \in \left\{0, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{3\pi}{2}, \pi\right\}$$

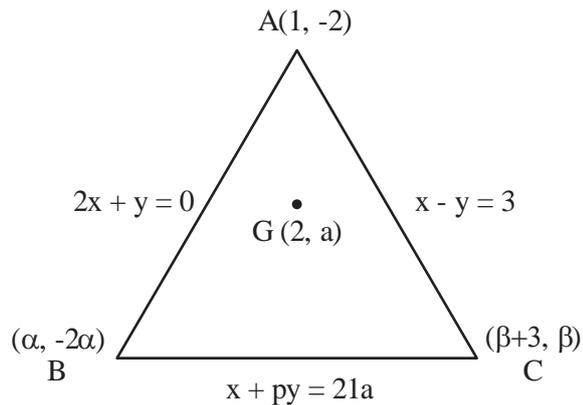
$$\text{So } \sum_{\theta \in S} \sin^2\left(\theta + \frac{\pi}{4}\right) = 2(0) + 4\left(\frac{1}{2}\right) = 2$$

90. The equations of the sides AB, BC and CA of a triangle ABC are: $2x + y = 0$, $x + py = 21a$, ($a \neq 0$) and $x - y = 3$ respectively. Let P (2, a) be the centroid of ΔABC . Then $(BC)^2$ is equal to

Official Ans. by NTA (122)

Allen Ans. (122)

Sol.



Assume B(α, -2α) and C(β + 3, β)

$$\frac{\alpha + \beta + 3 + 1}{3} = 2 \quad \text{also} \quad \frac{-2\alpha - 2 + \beta}{3} = a$$

$$\Rightarrow \alpha + \beta = 2$$

$$-2\alpha - 2 + \beta = 3a$$

$$\Rightarrow \beta = 2 - \alpha \quad -2\alpha - 2 + 2 - \alpha = 3a \Rightarrow \alpha = -a$$

Now both B and C lies as given line

$$\alpha - p \cdot 2\alpha = 21a$$

$$\alpha(1 - 2p) = 21a \quad \dots (1)$$

$$-\alpha(1 - 2p) = 21a \Rightarrow p = 11$$

$$\beta + 3 + p\beta = 21a$$

$$\beta + 3 + 11\beta = 21a$$

$$21\alpha + 12\beta + 3 = 0$$

$$\text{Also } \beta = 2 - \alpha$$

$$21\alpha + 12(2 - \alpha) + 3 = 0$$

$$21\alpha + 24 - 12\alpha + 3 = 0$$

$$9\alpha + 27 = 0$$

$$\alpha = -3, \beta = 5$$

$$\text{So } BC = \sqrt{122} \quad \text{and } (BC)^2 = 122$$